expansion the external work is done entirely at the expense of the substance’s stock of internal energy. Hence in the adiabatic expan­sion of a gas the temperature falls, and in adiabatic compression it rises. To find the change of temperature in a gas when expanded or compressed adiabatically we have only to combine the equations

P1v1v-wa∏d^-n,

and we find

τ2 = τ1 (V1∕V2)γ-1.

It is clear from the above that if, during expansion, *n* is less than γ the fluid is taking in heat, and if *n* is greater than γ the fluid is rejecting heat.

39. Another very important mode of expansion or compression is that called *isothermal,* in which the temperature of the working substance is kept constant during the process.

In the case of a gas the curve of isothermal expansion is a rectangulaχ∙ hyperbola, having the equation

PV=constant=*cτ.*

When a gas expands (or is compressed) isothermally at tempera­ture τ from V1 to V2 the work done by (or upon) it (per lb) is

∕\*V2 ∕\*v2<ZV

2PdV≈Py(ι ^-=PVloger=cτloger,

where *r* is the ratio V2∕V1 as before.@@1

During isothermal expansion or compression a gas suffers no change of internal energy (by § 34, since τ is constant). Hence during isothermal expansion a gas must

take in an amount of heat just equal to

the work it does, and during isothermal

compression it must reject an amount

of heat just equal to the work spent

upon it. The expression cτlogε*r* con­

sequently measures, not only the work

done by or upon the gas, but also the

heat taken in during isothermal expan­

sion or given out during isothermal

compression. In the diagram, fig. 11,

the line AB is an example of a curve of isothermal expansion for a perfect gas, called for brevity an isothermal line, while AC is an adiabatic line starting from the same point A.

40. We shall now consider the action of an ideal engine in which the working substance is a perfect gas, and is caused to pass through a cycle of changes

each of which is either isother­

mal or adiabatic. The cycle

to be described was first exa­

mined by Carnot, and is spoken

of as Carnot’s cycle of opera­

tions. Imagine a cylindeι- and

piston composed of a perfectly

non - conducting material,

except as regards the bottom

of the cylinder, which is a

conductor. Imagine also a

hot body or indefinitely ca­

pacious source of heat A,

kept always at a tempera­

ture τ1, a perfectly non­

conducting cover B, and a

cold body or indefinitely

capacious receiver of heat C, kept always at a temperature τ2, which is lower than τ1. It is supposed that A, B, or C can be applied to the bottom of the cylinder. Let the cylinder contain 1 lb of a perfect gas, at temperature τ1, volume V*a*, and pressure P*a*to begin with. The suffixes refer to the points on the indicator diagram, fig- 12.

(1) Apply A, and allow the piston to rise. The gas expands isothermally at τ1, taking heat from A and doing work. The pressure changes to P*b* and the volume to V*b*.

(2) Remove A and apply B. Allow the piston to go on rising. The gas expands adiabatically, doing work at the expense of its internal energy, and the temperature falls. Let this go on until the temperature is τ2, The pressure is then P*c*, and the volume V*c*.

(3) Remove B and apply C. Force the piston down. The gas is compressed isothermally at τ2, since the smallest increase of temperature above τ2 causes heat to pass into C. Work is spent upon the gas, and heat is rejected to C. Let this be continued until a certain point *d* (fig. 12) is reached, such that the fourth operation will complete the cycle.

(4) Remove C and apply B. Continue the compression, which is now adiabatic. The pressure and temperature rise, and if the point *d* has been properly chosen, when the pressure is restored to its original value P*a*, the temperature will also have risen to its

original value τ1. [In other words, the third operation must be stopped when a point *d* is reached such that an adiabatic line drawn through *d* will pass through *a.*] This completes the cycle.

To find the proper place at which to stop the third operation, we have, by § 38, τ1∕τ2 = (V*c*∕V*b*)γ-1 in the second operation, and again τ1∕τ2=(V*d*∕V*a*)γ-1 in the fourth operation. Hence V*c* ∕V*b* = V*d*∕V*a*, and V*b*∕V*a*, the ratio of isothermal expansion, is equal to V*c*∕V*a*, the ratio of isothermal compression. For brevity we shall denote either of these last ratios by *r.*

41. The following are the transfers of heat to and from the working fluid, in successive stages of the cycle :—

(1) Heat taken in from A = *c*τ1logε*r* (by § 39).

(2) No heat taken in or rejected.

(3) Heat rejected to C=*c*τ2logε*r* (by § 39).

(4) No heat taken in or rejected.

Hence, by the first law of thermodynamics, the net external work done by the gas is

*c*(τ1-τ2) logε*r*;

and the efficiency of the engine (§ 23) is

cHl ~ T2)1°ger τl ~ τ2

cτ1loger - τ1

This is the fraction of the whole heat given to it which an engine following Carnot’s cycle converts into work. The engine takes in an amount of heat, at the temperature of the source, pro­portional to τ1 ; it rejects an amount of heat, at the temperature of the receiver, proportional to τ2. It works within a range of tem­perature extending from τ1 to τ2, by letting down heat from τ1 to τ2 (§ 24), and in the process it converts into work a fraction of that heat, which fraction will be greater the lower the temperature τ2 at which heat is rejected is below the temperature τ1 at which heat is received.

42. Next let us consider what will happen if we reverse Carnot’s cycle, that is to say, if we force this engine to act so that the same indicator diagram as before is traced out, but in the direction opposite to that followed in § 40. Starting as before from the point *a* and with the gas at τ1, we shall require the following four operations :—

(1) Apply B and allow the piston to rise. The gas expands adiabatically, the curve traced is *ad,* and when *d* is reached the temperature has fallen to τ2.

(2) Remove B and apply C. Allow the piston to go on rising. The gas expands isothermally at τ2, taking heat from C, and the curve *dc* is traced.

(3) Remove C and apply B. Compress the gas. The process is adiabatic. The curve traced is *cb,* and when *b* is reached the temperature has risen to τl.

(4) Remove B and apply A. Continue the compression, which is now isothermal, at τ1. Heat is now rejected to A, and the cycle is completed by the curve *ba.*

In this process the engine is not doing work ; on the contrary, work is spent upon it equal to the area of the diagram, or *c*(τ1 — τ2) logε*r*. Heat is taken in from C in the first operation, to the amount *c*τ2logε*r*. Heat is rejected to A in the fourth operation, to the amount *c*τ1logε*r*. In the first and third opera­tions there is no transfer of heat.

The action is now in every respect the reverse of what it was before. The same work is now spent upon the engine as was formerly done by it. The same amount of heat is now given to the hot body A as was formerly taken from it. The same amount of heat is now taken from the cold body C as was formerly given to it, as will be seen by the following scheme :—

*Carnot's Cycle, Direct.*

Work done by the engine = *c*(τ1 - τ2) logε*r* ;

Heat taken from A = *c*τ1logε*r*;

Heat rejected to C=*c*τ2logε*r*.

*Carnot’s Cycle, Reversed.*

Work spent upon the engine=*c*(τ1 - τ2) logε*r* ;

Heat rejected to A = *c*τ1logε*r* ;

Heat taken from C = *c*τ2logε*r*.

The reversal of the work has been accompanied by an exact reversal of each of the transfers of heat.

43. An engine in which this is possible is called, from the thermodynamic point of view, a *reversible* engine. In other words, a reversible heat-engine is one which, if forced to trace out its indicator diagram reversed in direction, so that the work which would be done by the engine, when running direct, is actually spent upon it, will reject to the source of heat the same quantity of heat as, when running direct, it would take from the source, and will take from the receiver of heat the same quantity as, when running direct, it would reject to the receiver. By “the source of heat” is meant the hot body which acts as source, and by “ the receiver ” is meant the cold body which acts as receiver, when the

@@@1 In calculations where this expression is involved it is convenient to remem­ber that logε, the hyperbolic logarithm, of any number is 2∙3026 times the common logarithm of the number.