engine is running direct. Carnot’s engine is one example of a reversible engine. The idea of thermodynamic reversibility is highly important, for the reason that no heat-engine can be more efficient than a reversible engine, if both take in and reject heat at the same pair of temperatures.

44. To prove this, let it be supposed that we have two engines M and N, of which N is reversible in the above sense, and that we have a hot body A capable of acting as a source of heat, and a cold body C capable of acting as a receiver of heat. The engine M is set to work as a heat-engine, taking heat from A and rejecting heat to C. To prove that M cannot be more efficient than the reversible engine N, we shall assume that it *is* more efficient, and trace the consequences of that assumption.

Let M, working direct, be coupled so as to work N reversed ; if we suppose that the engines are without mechanical friction, and can be coupled up without loss of power, the work represented by the indicator diagram of M is spent on N, and N will therefore reject to A an amount of heat which we will call Qa and take from B an amount of heat which we will call Qb. Now, since N is reversible, if it worked direct, taking Qa from A, it would do the same amount of work as, in the supposed circumstances, is spent upon it. Hence, if M is more efficient than N it is taking from A an amount of heat *less* than Qa, and consequently also is giving to B an amount of heat correspondingly less than Qb. The joint effect, therefore, of M working direct and N working reversed is that the heat taken from A by M is less than the heat given to A by N, while the heat given to B by M is less (to an equal extent) than the heat taken from B by N. The consequence is that the hot body A is gaining heat on the whole, and the cold body B is losing an equal amount of heat ; in other words, with the continued action of the double system heat passes, in indefinitely large quantity, from a cold body to a hot body, by means of an agency which, it is to be observed, is purely self-acting, for if we suppose there is no mechanical friction the system requires no help from without. Now this result is, by the second law of thermodynamics (§ 23), contrary to all experience ; and we are forced to conclude that the assumption that M is more efficient than the reversible engine N, when both take in and reject heat at the same two temperatures, is false. Hence, with given temperatures of source and receiver of heat no engine is more efficient than a reversible engine.

Next, let M and N both be reversible and both work between the same limits, but be different in any other respect. Then by the foregoing argument M cannot be more efficient than N, neither can N be more efficient than M. Hence all reversible heat- engines taking in and rejecting heat at the same temperatures are equally efficient.

45. These results imply that reversibility, in the thermodynamic sense, is the criterion of what may be called perfection in a heat- engine. A reversible engine is perfect in the sense that it cannot be improved on as regards efficiency : no other engine, taking in and rejecting heat at the same temperatures, will convert into work a greater fraction of the heat which it takes in. Moreover, if this criterion be satisfied, it is as regards efficiency a matter of complete indifference what is the nature of the working substance, or what, in other respects, is the mode of the engine’s action.

46. Further, since all engines that arc reversible are equally efficient, provided they work between the same temperatures, an expression for the efficiency of one will apply equally to all. Now, the engine whose efficiency we have found in § 41 is one example of a reversible engine. Hence its efficiency

(τ1-τ2)∕τ1

is the efficiency of any reversible heat-engine whatsoever taking in heat at τ1 and rejecting heat at τ2. And, as no engine can be more efficient than one that is reversible, this expression is the measure of *perfect efficiency.* We have thus arrived at the immensely important conclusion that no heat-engine can convert into work a greater fraction of the heat which it receives than is expressed by the excess of the temperature of reception above that of rejection divided by the absolute temperature of reception.

47. Briefly recapitulated, the steps of the argument by which this result has been reached are as follows. After stating the experimental laws to which gases conform, we examined the action of a heat-engine in which the working substance took in heat when at the temperature of the source and rejected heat when at the temperature of the receiver, the change of temperature from one to the other of these limits being accomplished by adiabatic expansion and adiabatic compression. Taking a special case in which this engine had for its working substance a perfect gas, we found that its efficiency was (τ1-τ2)∕ τ1 (§ 41). We also observed that it was, in the thermodynamic sense, a reversible engine (§ 43). Then we found, by an application of the second law of thermodynamics, that no heat-engine can have a higher efficiency than a reversible engine, when taking in and giving out heat at the same two temperatures τ1 and τ2 ; this was shown by the fact that a contrary assumption leads to a violation of the second law (§ 44). Hence, we concluded that all reversible heat- engines receiving and rejecting heat at the same temperatures τ1

and τ2 respectively are equally efficient, and hence that the efficiency (τ1 - τ2) ∕τ1, already determined for one particular reversible engine, measures the efficiency of any reversible engine, and is a limit of efficiency which no engine whatever can exceed.

48. Tho second law of thermodynamics, on which (along with the first law) this conclusion rests has been given in many different forms. The statement of it in § 23 is that of Clausius, and is very similar to that of Sir W. Thomson. Rankine, to whom with Thomson and Clausius is due the development of the theory of heat- engines from the point at which Carnot left it, has stated the second law in a form which is neither easy to understand, nor obvious, as an experimental result, when understood. His statement runs :—

“ If the absolute temperature of any uniformly hot substance he divided into any number of equal parts, the effects of those parts in causing work to be performed are equal.”@@1

To make this intelligible we may suppose that any quantity *q* of heat from a source at temperature τ1 is taken by the first of a series of perfect heat-engines, and that this engine rejects heat at a temperature τ2 less than τ1 by a certain interval ∆τ. Let the heat so rejected by the first engine form the heat supply of a second perfect engine working from τ2 to τ3 through an equal inter­val ∆τ ; let the heat which it in turn rejects form the heat-supply of a third perfect engine working again through an equal interval from τ3 to τ4 ; and so on. The efficiencies of the several engines are (by § 46) — , — , — , &c. The amounts of heat supplied to τι τa τ3

them are *q , qτ⅛ , qπ⅛ ,* &c. Hence the amount of work done by *τl*

each engine is the same, namely, 7— . Thus Rankine’s statement

is to be understood as meaning that each of the equal intervals into which any range of temperature may be divided is equally effective in allowing work to be produced from heat when heat is made to pass, doing work in the most efficient possible way, through all the intervals from the top to the bottom of the range.

49. A point of much theoretical interest may be noted in pass­ing. In place of measuring temperature, as we have done, by the expansion of a perfect gas, a scale of temperature might be formed thus. Starting from any one temperature, let a series of intervals be taken such that a series of reversible engines, each working with one of the intervals for its range, in the manner described in § 48 (so that the heat rejected by the first forms the supply of the second, and so on), will each do the same amount of work ; then *call these intervals equal.* This gives a scale of temperature (origin­ally suggested by Sir W. Thomson) which is truly absolute in the sense of being independent of the properties of any substance ; it coincides, as is evident from § 48, with the scale we have been using, in which equal intervals of temperature are defined as those corresponding to equal amounts of expansion of a perfect gas under constant pressure ; and it coincides approximately with the scale of a mercury thermometer when that is graduated to read from the absolute zero by the addition of a suitable constant (§ 29).

50. The availability of heat for transformation into work depends essentially on the range of temperature through which the heat is let down from the hot source to the cold body into which heat is rejected ; it is only in virtue of a difference of temperature between bodies that conversion of any part of their heat into work becomes possible. If τ1 and τ2 are the highest and lowest tem­peratures of the range through which a heat-engine works, it is clear that the maximum of efficiency can be reached only when the engine takes in all its heat at τ1 and rejects at τ2 all that is rejected. With respect to every portion of heat taken in and rejected the greatest ideal efficiency is

\_ Temperature of reception - temperature of rejection Temperature of reception

Any heat taken in at a temperature below τ1 or rejected at a temperature above τ2 will have less availability for conversion into work than if taken in at τ1 and rejected at τ2, and hence, with a given pair of limiting temperatures, it is essential to maximum efficiency that uo heat be taken in by the engine except at the top of the range, and no heat rejected except at the bottom of the range. Further, as we have seen in § 45, when the temperatures at which heat is received and rejected are assigned, an engine attains the maximum of efficiency if it be reversible,

51. It is therefore important to inquire more particularly what kinds of action are reversible in the thermodynamic sense. A little consideration will show that a transfer of heat from the source or to the receiver is reversible only when the working substance is at sensibly the same temperature as the source or the receiver, as the case may be, and an expansion is reversible only when it occurs by the gradual displacement of some part of the containing envelope in such a manner that the expanding fluid does external work on the envelope, and does not waste energy to any sensible extent in setting itself in motion. This excludes what may be termed free expansion,

@@@1 *Manual of the Steam Engine and other prime Movers,* § 243.