where *a* is the actual radius of the pin. Any line drawn tangent to this circle will make the angle *φ* with the radius of the pin at the surface of the pin. The thrust of the connecting-rod must be tangent to both

circles ; it is drawn as in fig.

110, so that it resists the rota­tion of the pins relatively to the rod. The direction of ro­tation of the pins is shown

by curved arrows in the figure, where the friction-circles are drawn to a greatly exaggerated scale. Finally, P (after allowing for the friction of piston-packing and stuffing-box) is resolved into Q, and Q·CM, the moment of Q on the shaft, is determined. This gives a diagram of crank-effort, correct so far as friction affects it, whose area is no longer equal to that of the indicator diagram. The difference, however, does not represent the whole work lost through friction of the mechanism, since the friction of the shaft itself, and of those parts of the engine which it drives, has still to be allowed for if the frictional efficiency of the engine as a whole is in question.

186. The diagram of crank-effort is further modified when we take account of the inertia of the piston and connecting-rod. For the purpose of investigating the effects of inertia, we may assume that the crank is revolving at a sensibly uniform rate of *n* turns per second. Let M be the mass of the piston, piston-rod, and crosshead in pounds, and *a* its acceleration at any instant in feet per second per second. The force required to accelerate it is *Ma/g,* in pounds-weight, and this is to be deducted in estimating the effective value of P. The effect is to reduce P during the first part of the stroke and to increase it towards the end, thereby compensating to some extent for the variation which P undergoes in consequence of an early cut off. If the connecting-rod is so long that its obliquity may be neglected the piston has simple harmonic motion, and

*a=*-4π2*n*2*r*cosα =-*4π2n2x,*

where *x* is the distance of the piston from its middle position. More generally, whatever ratio the length *l* of the connecting-rod bears to that of the crank *r*,

. „ ∕ rZ2cos2α + r3sin4αλ

*a =* - 4π⅜⅛ cos a 4 5— I .

∖ (Z2-r2sin2a)^ ∕

The effect is to make, on the diagram of P, a correction of the character shown in fig. 111, where the broken line *cd* refers to the case of an indefinitely long con­

necting-rod and the full line *ab*

to the case of a connecting-rod

31/2 times the length of the crank.

In a vertical engine the weight

of the piston and piston-rod is

to be added to or subtracted

from P.

To allow for the inertia of the connecting-rod is a matter of somewhat greater difficulty.

Its motion may conveniently be analysed as consisting of translation with the velocity of the crosshead, combined with rota­tion about the crosshead as centre. Hence the force required for its acceleration is the resultant of three components—F1, the force required for the linear ac­celeration *a* (which is the same as that of the piston);

F2, the force required to cause angular acceleration about the crosshead; and F3, the force towards the

centre of rotation, which depends on the angular velocity, and is equal and opposite to the so-called centrifugal force. Let *θ* be the angle BAC (fig. 112), *6* the angular velocity of the rod about A, and *θ* its angular acceleration, and let M’ be the mass of the rod. Then

F1=M'*a*∕*gr*,

and acts through the centre of gravity G, parallel to AC ;

F2=M'. AG.θ/*g,*

and acts at right angles to the rod through the centre of per­cussion H; F3 = M'. AG. θ*2∣g,*

and acts along the rod towards A. Also,

o 2πnrcosα *U ~~~ — .. — ,*

^√72 - r2sin2<x θ\_ ~ 4π2n2rsin*a(l2 - r2)*

and (I2-r2sin2α)⅞

The moments of these forces about C are next to be found, anil to be deducted from the moment of the thrust in the connecting-rod (and, if the weight of the rod is to be considered, its moment about C is to be added) in finding the resultant moment of crank-effort.

187. If, however, the friction at the crosshead and crank-pin is to be taken account of the whole group of forces acting on the rod must be considered as follows. Compound forces equal and oppo­site to F1, F2, and F3 into a single force R (fig. 113), which may be called the resultant resistance to acceleration of the connecting- rod. If the weight of the rod is to be considered, let it also be taken

as a component in reckoning R, Then the rod may in any position be regarded as in equilibrium under the action of the forces Q, R, and S, where Q and S are the forces exerted on it by the crosshead and crank-pin respectively. These three forces meet in a point *p* in R, which is to be found by trial, the condition being that in the diagram of forces, fig. 114, after the triangle POQ has been drawn, and the force R set out, the force-line S shall be parallel to a line drawn from *p* tangent to the friction-circle of the crank-pin, as in fig. 113. When this condition has been satisfied by trial, the value of S, which is the thrust on the crank-pin, is determined, and S. CM is the moment of crank-effort. This method is due to the late Prof. Fleeming Jenkin, who has applied it with great generality to the determination of the frictional efficiency of ma­chinery in two important papers,@@1 the second of which deals in detail with the dynamics of the steam-engine. Fig. 115, taken

from that paper, shows the diagram of crank-effort in a horizontal direct-acting engine,—the full line with friction, and the dotted line without friction,—the inertia of the piston and connecting rod being taken account of, as well as the weight of the latter. It exhibits well the influence which the inertia of the reciprocating parts has in equalizing the crank-effort in the case of an early cut­off. The cut-off is supposed to occur pretty sharply at about one- sixth of the stroke. The engine considered is of practical propor­tions, and makes four turns per second ; and the initial steam pressure is 50 lb per square inch. It appears from the diagram that, with a slightly higher speed, or with heavier rods, a better balance of crank-effort might be secured, especially as regards the stroke towards the

crank, which comes

first in the dia­

gram; on the other

hand, by unduly

increasing the mass

of the reciprocat­

ing pieces or their

speed the inequal­

ity due to expan­

sion would be over-corrected and a new inequality would come in.

188. When two or more cranks act on the same shaft, the joint moment of crank-effort is found by combining the diagrams for the separate cranks, in the manner illustrated by fig. 116, which refers to the case of two cranks at right angles.

Another graphic method of exhibiting the variations of moment exerted on the crank-shaft during a revolution is to draw a circular diagram of crank-effort, in which lines proportional to the moment are set off radially from a circular line which represents the zero of moment. An example of this plan is given in fig. 117, which shows the resultant moment determined by Mr A. C. Kirk for one of his triple-expansion engines with three cranks set at 120° from each other. Curves are drawn for various speeds, giving in each case the resultant moment due to the steam pressure (as

@@@1 *Trans. Roy. Soc. Edin.,* vol. xxviii. p. 1 and p. 703.