called principal stresses, and their directions are called the axes of principal stress. These axes have the important property that the intensity of stress along one of them is greater, and

along another it is less, than in any other direction.

These are called respectively the axes of greatest and

least principal stress.

5. Returning now to the case of a single simple longitudinal stress, let AB (fig. 1) be a portion of a tie or a strut which is being pulled or pushed in the direction of the axis AB with a total stress P. On any plane CD taken at right angles to the axis we have a normal pull or push of intensity *p*=P∕S, S being the area of the normal cross-section. On a plane EF whose normal is inclined to the axis at an angle θ we have a stress still in the direction of the axis, and therefore oblique to the plane EF, of intensity P∕S', where S' is the area of the surface EF, or S/cos *θ.* The whole stress P on EF may be resolved into two com­ponents, one normal to EF, and the other a shearing stress tangential to EF. The normal component (Pn, fig. 2) is PcosØ; the tangential component (P*t*) is Psin*θ*. Hence the inten­sity of normal pull or push on EF, or *pn,* is *p*cos2*θ*, and the intensity of shearing stress EF, or *pt*, is *p*sin*θ*cos*θ*.

This expression makes *pt* a maximum when 0 = 45°: planes inclined at 45° to tho axis are called planes of maximum shearing stress ; the intensity of shearing stress on them is 1/2*p*.

6. Shearing stress in one direction is necessarily ac­companied by an equal intensity of shearing stress in another direction at right angles to the first. To prove this it is sufficient to consider the equilibrium of an indefinitely small cube (fig. 3), with one pair of sides parallel to the direc­tion of the shearing stress P*t*. This stress, act­ing on two opposite sides, produces a couple which tends to rotate the cube. No arrange­ment of normal stresses on any of the three pairs of sides of the cube can balance this couple ; that can be done only by a shearing stress Q*t* whose direction is at right angles to the first stress P*t*and to the surface on which P*t* acts, and whose intensity is the same as that of P*t*. The shearing stresses P*t* and Q*t* may exist alone, or as components of oblique stress.

7. If they exist alone, the material is said to be in a state of simple shearing stress. This state of stress may be otherwise described by reference to the stresses on diagonal planes of the cube ABCD. Thus P*t* aud Q*t* produce a normal stress R on a diagonal plane, and the equilibrium of

the triangular prism

(fig. 4) requires that

R=P*t*√2. But R acts

on a surface which is

greater than each of

the sides in the ratio

of √2 : 1. The inten­

sity of normal stress

on the diagonal plane

AC is therefore the same as the intensity of shearing stress on AB or BC. The same considerations apply to the other diagonal plane BD at right angles to AC, with this difference, that the stress on it is normal pull instead of push. Hence we may regard a state of simple shearing stress as compounded of two simple longitudinal stresses, one of push and one of pull, at right angles to each other, of equal intensity, and inclined at 45° to the direction of the shear­ing stress.

8. Strain is the change of shape produced by stress. If the stress is a simple longitudinal pull, the strain consists of lengthen­ing in the direction of the pull, accompanied by contraction in both directions at right angles to the pull. If the stress is a simple push, the strain consists of shortening in the direction of the push and expansion in both directions at right angles to that ; the stress and the strain are then exactly the reverse of what they are in the case of simple pull. If the stress is one of simple shearing, the strain consists of a distortion such as would be produced by the sliding of layers in the direction of the shearing stresses.

A material is elastic with regard to any applied stress if the strain disappears when the stress is removed. Strain which per­sists after the stress that produced it is removed is called perma­nent set. For brevity, it is convenient to speak of strain which disappears when the stress is removed as elastic strain.

9. Actual materials are generally very perfectly elastic with regard to small stresses, and very imperfectly elastic with regard to great stresses. If the applied stress is less than a certain limit, the strain is in general small in amount, and disappears wholly or almost wholly when the stress is removed. If the applied stress exceeds this limit, the strain is, in general, much greater than before, and the principal part of it is found, when the stress

is removed, to consist of permanent set. The limits of stress within which strain is wholly or almost wholly elastic are called limits of elasticity.

For any particular mode of stress the limit of elasticity is much more sharply defined in some materials than in others. When well defined it may readily be recognized in the testing of a sample from the fact that after the stress exceeds the limit of elasticity the strain begins to increase in a much more rapid ratio to the stress than before. This characteristic goes along with the one already mentioned, that up to the limit the strain is wholly or almost wholly elastic.

10. Within the limits of elasticity the strain produced by a stress of any one kind is proportional to the stress producing it. This is Hooke’s law, enunciated by him in 1676.

In applying Hooke’s law to the case of simple longitudinal stress, —such as the case of a bar stretched by simple longitudinal pull,— we may measure the state of strain by the change of length per unit of original length which the bar undergoes when stressed. Let the original length be *l,* and let the whole change of length be δ*l* when a stress is applied whose intensity *p* is within the elastic limit. Then the strain is measured by *Sl∕l,* and this by Hooke’s law is proportional to *ρ.* This may be written

*δl : l :: p* : E,

where E is a constant for the particular material considered. The same value of E applies to push and to pull, these modes of stress being essentially continuous, and differing only in sign.

11. This constant E is called the modulus of longitudinal extensibility, or Young’s modulus. Its value, which is expressed in the same units as are used to express intensity of stress, may be measured directly by exposing a long sample of the material to longitudinal pull and noting the extension, or indirectly by measuring the flexure of a loaded beam of the material, or by ex­periments on the frequency of vibrations. It is frequently spoken of by engineers simply as the modulus of elasticity, but this name is too general, as there are other moduluses applicable to other modes of stress. Since E *=pl/δl,* the modulus may be defined as the ratio of the intensity of stress *p* to the longitudinal strain *δl∕l*.

12. In the case of simple shearing stress, the strain may be measured by the angle by which each of the four originally right angles in the square prism of fig. 3 is altered by the distortion of the prism. Let this angle be *φ* in radians; then by Hooke’s law *p∣φ=C,* where *p* is the intensity of shearing stress and C is a con­stant which measures the rigidity of the material. C is called the modulus of rigidity, and is usually determined by experiments on torsion.

13. When three simple stresses of equal intensity *p* and of the same sign (all pulls or all pushes) are applied in three directions, the material (provided it be isotropic, that is to say, provided its properties are the same in all directions) suffers change of volume only, without distortion of form. If the volume is V and the change of volume δV, the ratio of the stress *p* to the strain δV∕Y is called the modulus of cubic compressibility, and will be denoted by K. The state of stress here considered is the only one possible in a fluid at rest. The intensity of stress is equal in all directions.

14. Of these three moduluses the one of most importance in engineering applications is Young’s modulus E. When a simple longitudinal pull or push of intensity *p* is applied to a piece, the longitudinal strain of extension or compression is *p*∕E. This is accompanied by a lateral contraction or expansion, in each trans­verse direction, whose amount may be written *p*∕σE, where σ is the ratio of longitudinal to lateral strain. It is shown in the article Elasticity, § 47, that E=9CK/3K+C and σ=2(3K+C)/3K-2C

15. Beyond the limits of elasticity the relation of strain to stress becomes very indefinite. Materials then exhibit, to a greater or less degree, the property of plasticity. The strain is much affected by the length of time during which the stress has been in opera­tion, and reaches its maximum, for any assigned stress, only after a long (probably an indefinitely long) time. Finally, when the stress is sufficiently increased, the ratio of the increment of strain to the increment of stress becomes indefinitely great if time is given for the stress to take effect. In other words, the substance then assumes what may be called a completely plastic state ; it *flows* under the applied stress like a viscous liquid.

16. The *ultimate strength* of a material with regard to any stated mode of stress is the stress required to produce rupture. In reckon­ing ultimate strength, however, engineers take, not the actual in­tensity of stress at which rupture occurs, but the value which this intensity would have reached had rupture ensued without previous alteration of shape. Thus, if a bar whose original cross-section is 2 square inches breaks under a uniformly distributed pull of 60 tons, the ultimate tensile strength of the material is reckoned to be 30 tons per square inch, although the actual intensity of stress which produced rupture may have been much greater than this, owing to the con­traction of the section previous to fracture. The convenience of this usage will be obvious from an example. Suppose that a piece