sectional area and of different lengths, provided the length of both were great enough to prevent tho action described in § 40 from affect­ing the result. But, since no material is perfectly homogeneous, the longer rod will in general be the weaker, offering as it does more chances of a weak place ; and the probable defect of strength in the long rod will depend on the degree of variability of the material. When this has been established by numerous tests of short samples, the strength which a rod of any assigned length may be expected to possess can be calculated by an appli­

cation of the theory of probabilities. A theory of the

strength of long bars has been worked out on this

basis by Prof. Chaplin,@@1 and has been experimentally

confirmed by tests of long and short samples of wire.

The theory does not apply when the length is so small

that the action of § 40 enters into the case, and the

experimental data on which it is based must be taken

from tests of samples long enough to exclude that

action.

42. In tension tests, rupture may occur, as in fig.

19, by direct separation over a surface which is nearly plane and normal to the line of stress. This is usual in hard steel aud other comparatively non-ductile mate­rials. Or it may occur by shearing along an oblique

plane, as in fig. 20, which shows the fracture of a piece of steel softer than the speci­men of fig. 19. In very duc­tile samples these two modes of rupture are frequently found in com­bination, as in fig. 21, where a central core is broken by direct tension while round it is a ring over which separation has taken place by shearing. In this instance the ring is in two parts, one above and one

below the surface of rupture of the central flat core. In other instances, such as that of the sample shown in fig. 16, the shorn ring forms a continuous cone or crater round a flat core.

43. In compression tests of a plastic material, such as mild steel, a process of flow may go on without limit ; the piece (which must of course be short, to avoid buckling) shortens and bulges out in the form of a cask. This is illustrated

by fig. 22 (from one of Fairbairn’s expe­

riments), which shows the compression

of a round block of steel (the original

height and diameter of which are shown

by the dotted lines) by a load equal to

100 tons per square inch of original sec­

tional area. The surface over which the

stress is distributed becomes enlarged,

and the total load must be increased in

a corresponding degree to maintain the

process of flow.@@2 The bulging often produces longitudinal cracks, as in the figure, especially when the material is fibrous as well as plastic (as in the case of wrought-iron). A brittle material, such as cast-iron brick, or stone, yields by shearing on inclined planes

as in figs. 23 and 24, which are taken from Hodgkinson’s experiments on cast-iron.@@3 The simplest fracture of this kind is exem­plified by fig. 23, where a single surface (ap­proximately a plane) of shear divides the compressed block into two wedges. With cast-iron the slope of the plane is such that this simple mode of fracture can take place only if the height of the block is not less than about 3/2 the width of the base. When the height is less the action is more complex. Shearing must then take place over more than one plane, as in fig. 24, so that cones or wedges are formed by which the surrounding portions of the block are split off. The stress required to crush the block is con­

sequently greater than if the height were sufficient for shearing in a single plane.

44. The inclination of the surfaces of shear, when fracture takes place by shearing under a simple stress of pull or push, is a matter of much interest, throwing some light on the question of how the resistance whieh a material exerts to stress of one kind is affected by the presence of stress of another kind,—a question scarcely touched by direct experiment. At the shorn surface there is, in the case of tension tests, a normal pull as well as a shearing stress, and in the case of compression tests a normal push as well as shearing stress. If this normal component were absent the material (assuming it to be isotropic) would shear in the surface of greatest shearing stress, which, as we have seen in § 5, is a surface inclined at 45° to the axis. In fact, however, it does not shear on this surface. Hodgkinson’s experiments on the compression of cast- iron give surfaces of shear whose normal is inclined at about 55° to the axis of stress,@@4 and Kirkaldy’s, on the tension of steel, show that when rupture takes place by shear the normal to the surface is inclined at about 25° to the axis.4 These results show that normal pull diminishes resistance to shearing and normal push increases resistance to shearing. In the case of cast-iron under compression, the material prefers to shear on a section where the intensity of shearing stress is only 0·94 of its value on the surface of maximum shearing stress (inclined at 45°), but where the normal push is reduced to 0·66 of its value on the surface of maxi­mum shearing stress.

45. *Fatigue of Metals.—*A matter of great practical as well as scientific interest is the weakening which materials undergo by repeated changes in their state of stress. It appears that in some if not in all materials a limited amount of stress-variation may be repeated time after time without appreciable deterioration in the strength of the piece; in the balance-spring of a watch, for instance, tension and compression succeed each other some 150 millions of times in a year, and the spring works for years without apparent injury. In such cases the stresses lie well within the elastic limits. On the other hand, the toughest bar breaks after a small number of bendings to and fro, when these pass the elastic limits, although the stress may have a value greatly short of the normal ultimate strength. A laborious research by Wohler,@@5 ex­tending over twelve years, has given much important information regarding the effects on iron and steel of very numerous repeated alternations of stress from positive to negative, or between a higher and a lower value without change of sign. By means of ingeniously contrived machines he submitted test-pieces to direct pull, alter­nated with complete or partial relaxation from pull, to repeated bending in one direction and also in opposite directions, and to re­peated twisting towards one side and towards opposite sides. The results show that a stress greatly less than the ultimate strength (as tested in the usual way by a single application of load continued to rupture) is sufficient to break a piece if it be often enough removed and restored, or even alternated with a less stress of the same kind. In that case, however, the variation of stress being less, the number of repetitions required to produce rupture is greater. In general, the number of repetitions required to produce rup­ture is increased by reducing the range through which the stress is varied, or by lowering the upper limit of that range. If the greatest stress be chosen small enough, it may be reduced, re­moved, or even reversed many million times without destroying the piece. Wohler’s results are best shown by quoting a few figures selected from his experiments. The stresses are stated in centners per square zoll;@@6 in the case of bars subjected to bending they refer to the top and bottom sides, which are the most stressed parts of the bar.

|  |  |  |  |
| --- | --- | --- | --- |
| I. Iron bar in direct tension :— | | | |
| Stress. Number of Applications | | Stress. | Number of Applications |
| Max. Min | causing Rupture. | Max. Min. | causing Rupture. |
| 480 0 | 800 | 320 0 | 10,141,645 |
| 440 0 | 106,901 | — |  |
| 400 0 | 340,853 | 440 200 | 2,373,424 |
| 360 0 | 480,852 | 440 240 | Not broken with 4 millions. |
| II. Iron bar bent by transverse load :— | | |  |
| Stress. | Number of Bendings | Stress. | Number of Bendings |
| Max. Min. | causing Rupture. | Max. Min. | causing Rupture. |
| 530 0 | 169.750 | 400 0 | 1,320,000 |
| 500 0 | 420,000 | 350 0 | 4,035,400 |
| 450 0 | 481.950 | 300 0 | Not broken with 48 millions. |
| III. Steel bar bent by transverse load: — | | |  |
| Stress. | Number of Bendings | Stress. | Number of Bendings |
| Max. Min. | causing Rupture. | Max. Min | causing Rupture. |
| 900 0 | 72,459 | 900 400 | 225,300 |
| 900 200 | 81,200 | 900 500 | 764,900—mean of two trials. |
| 900 300 | 156,200 | 900 600 | Not broken with 33 J mills. |

@@@1 *Van Nostrand's Engineering Magazine,* Dec. 1880 ; *Proc. Engineers' Club of Philadelphia,* March, 1882.

@@@2 For examples, see Fairbairn’s experiments on steel, *Pep. Brit. Ass.,* 1867.

@@@3 *Report of the Royal Commissioners on the Application of Iron to Railway Structures,* 1849; see also *Brit. Ass. Rep.,* 1837.

@@@4 *Op. cit.*

@@@5 *Die Festigkeits-Versuche mit Eisen und Stahl,* Berlin, 1870, or *Zeitschr. für Bauwesen,* 1860-70; see also *Engineering,* vol. xi., 1871. For early experiments by Fairbairn on the same subject, see *Phil. Trans.,* 1864.

@@@6 According to Bauschinger (*loc*. *cit.,* p. 44), the centner per square zoll in which Wöhler gives his results is equivalent to 6·837 kilos per square cm., or 0·0434 ton per square inch.