over the azimuthal circle of the theodolite, Z for the number of the zero settings of the circle, N for the number of graduations brought under the microscopes, A = 360°÷N, the arc between the graduations, R the prescribed number of rounds of measures, and P = R × Z, the minimum number of telescope pointings to any station, excluding repetitions for discrepant observations :—

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Period. | M | Z | N | A | R | P |
| 1830-45 | 5 | 8 | 40 | 9° 0' | 3 | 24 |
| 1845-55 | 5 | 10 | 50 | 7° 12' | 2 | 20 |
| 1855-80 |  | 10 | 50 | 7° 12' | 3 | 30 |
| 3 | 12 | 36 | 10° 0' | 3 | 36 |

Under this system of procedure the instrumental and ordinary errors are practically cancelled and any remaining error is most probably due to lateral refraction, more especially when the rays of light graze the surface of the ground. The three angles of every triangle were always measured.

10. *Vertical Angles. Refraction.—*The apparent alti­tude of a distant point is liable to considerable variations during the twenty-four hours, under the influence of changes in the density of the lower strata of the atmosphere. Ter­restrial refraction is very capricious, more particularly when the rays of light graze the surface of the ground, passing through a medium which is liable to extremes of rarefac­tion and condensation, under the alternate influence of the sun’s heat radiated from the surface of the ground and of chilled atmospheric vapour. When the back and forward verticals at a pair of stations are equally refracted, their difference gives an exact measure of the difference of height. But the atmospheric conditions are not always identical at the same moment everywhere on long rays which graze the surface of the ground, and the ray between two recipro­cating stations is liable to be differently refracted at its extremities, each end being influenced in a greater degree by the conditions prevailing around it than by those at a distance ; thus instances are on record of a station A being invisible from another B, while B was visible from A.

When the great arc entered the plains of the Gangetic valley, simultaneous reciprocal verticals were at first adopted with the hope of eliminating refraction ; but it was soon found that they did not do so sufficiently to justify the expense of the additional instruments and observers. After­wards the back and forward verticals were observed as the stations were visited in succession, the back angles at as nearly as possible the same time of the day as the forward angles, and always during the so-called “time of minimum refraction,” which ordinarily commences about an hour after apparent noon and lasts from two to three hours. The apparent zenith distance is always greatest then, but the refraction is a minimum only at stations which are well elevated above the surface of the ground ; at stations on plains the refraction is liable to pass through zero and attain a considerable negative magnitude during the heat of the day, for the lower strata of the atmosphere are then less dense than the strata immediately above and the rays are refracted downwards. On plains the greatest positive refractions are also obtained,—maximum values, both positive and negative, usually occurring, the former by night, the latter by day, when the sky is most free from clouds. The values actually met with were found to range from + 1·21 down to - 0·09 parts of the contained arc on plains ; the normal “ coefficient of refraction ” for free rays between hill stations below 6000 feet was about ·07, which diminished to ·04 above 18,000 feet, broadly varying in­versely as the temperature and directly as the pressure, but much influenced also by local climatic conditions.

In measuring the vertical angles with the great theodo­lites, graduation errors were regarded as insignificant com­pared with errors arising from uncertain refraction ; thus no arrangement was made for effecting changes of zero in

the circle settings. The observations were always taken in pairs, face right and left, to eliminate index errors, only a few daily, but some on as many days as possible, for the variations from day to day were found to be greater than the diurnal variations during the hours of minimum refraction.

11. *Results deduced from Observations of Horizontal Angles ; Weights.—*In the Ordnance and other surveys the bearings of the surrounding stations are deduced from the actual observations, but from the “included angles” in the Indian Survey. The observations of every angle are tabulated vertically in as many columns as the number of circle settings face left and face right, and the mean for each setting is taken. For several years the general mean of these was adopted as the final result; but subsequently a “ concluded angle ” was obtained by combining the single means with weights inversely proportional to *g*2 + *o*2÷*n*, *—g* being a value of the *e.m.s.@@*1 of graduation derived empirically from the differences between the general mean and the mean for each setting, *o* the *e.m.s.* of observa­tion deduced from the differences between the individual measures and their respective means, and *n* the number of measures at each setting. Thus, putting *w*1, *w*2*,, . .* . for the weights of the single means, *w* for the weight of the con­cluded angle, *M* for the general mean, *C* for the concluded angle, and *d*1, *d*2, . . . for the differences between *M* and the single means, we have

(1)

W1 + W⅛ +

and to=w1 + w2 + (2).

*C - M* vanishes when *n* is constant ; it is inappreciable when *g* is much larger than *o* ; it is significant only when the graduation errors are more minute than the errors of observation; but it was always small, not exceeding 0"·14 with the system of two rounds of measures and 0"·05 with the system of three rounds.

The weights of the concluded angles thus obtained were employed in the primary reductions of the angles of single triangles and polygons which were made to satisfy the geometrical conditions of each figure, because they were strictly relative for all angles measured with the same instrument and under similar circumstances and conditions, as was almost always the case for each single figure. But in the final reductions, when numerous chains of triangles composed of figures executed with different instruments and under different circumstances came to be adjusted simultaneously, it was necessary to modify the original weights, on such evidence of the precision of the angles as might be obtained from other and more reliable sources than the actual measures of the angles. This treatment will now be described.

12. *Determination of Theoretical Absolute Errors of Observed Angles.—*Values of theoretical error for groups of angles measured with the same instrument and under similar conditions may be obtained in three ways,—(i.) from the squares of the reciprocals of the weight *w* deduced as above from the measures of such angle, (ii.) from the magni­tudes of the excess of the sum of the angles of each triangle above 180° + the spherical excess, and (iii.) from the magni­tudes of the corrections which it is necessary to apply to the angles of polygonal figures and networks to satisfy the several geometrical conditions (indicated in the next sec­tion). Let *e*1*, e*2*,* and *e*3 be the values of the *e.m.s.* thus obtained ; then, putting *n* for the number of angles grouped

together, we have

*e*12=*n/[w]* and *e*22=[squares of triangular errors]/*n*

also, putting *W* for the mean of the weights of the *t* angles

@@@1 The theoretical “ error of mean square” = 1·48 × “ probable error.”