, C . ! \*,

— sin *A* tan X cosec 1"

V

1 *en-* l . o , ,. 1 e2cos2 X ) . *s,A"* or + y — ) 1 + 2 tan- X + ——--2 1 sin *2 A* cosec 1

Z-(ιr + √4)=- c3^5 ∖tanX . . . „ ^(11)∙

— ( - + tan- X I—s- sin *2 A* cos *A* cosec 1

r3∖ 6 *∕ I*

+ - A\* sin3 J tan X (1 + 2 tan2 X) cosec 1"

„ 6 *v3*

Each Δ is the sum of four terms symbolized by δ1, δ2, *δ3,* and δ4 ; the calculations are so arranged as to produce these terms in the order δλ, δ*L*, and *δA,* each term entering as a factor in calculating the following term. The arrangement is shown below in equations in which the symbols *P, Q,. . . Z* represent the factors which depend on the adopted geodetic constants, and vary with the latitude ; the logarithms of their numerical values are tabulated in the *Auxiliary Tables to Facilitate the Calculations of the Indian Survey, δ,∖ = - P.cοsA.c* δ1L = +δ∣λ.<2.secλ.tan *A* δ1∠∕ = +δ1Z.sinλλ

δ.,λ= + *δlA.BsinA.c δ2L = - δ.2λ.S.cotA δ2A=+δ2L.T* I ,-ζ>.

δ3λ=-δsJ.r.cot∠i δ3Z = + δ3λ.f∕.sin∠∕.c *δ3A = + δ3L.JV* Pιz>∙

δ4λ = -δ3Λ.A.tan∠∕ *δ4L =* + δ4λ. F. tan√f *δ4A=+δ4L.Z J*

By this artifice the calculations are rendered less laborious and made susceptible of being readily performed by any persons who are acquainted with the use of logarithm tables.

16. *Limits within which Geodetic Formulæ may be em­ployed without Sensible Error.—*Each Δ is expressed as a series of ascending differentials in which all terms above the third order are neglected ; for the side length *c* in no case exceeded 70 miles, nor was the latitude ever higher than 36°, and for these extreme values the maximum magni­tudes of the fourth differential are only 0"·002 in latitude and 0"·004 in longitude and azimuth.

Far greater error may arise from uncertainties regard­ing the elements of the earth’s figure, which was assumed to be spheroidal, with semi-axes *a* = 20,922,932 feet and *b =* 20,853,375 feet. The changes in ∆λ, Δ*L*, and *ΔA* which would arise from errors *da* and *db* in *a* and *b* are indicated by the following formulæ :—

*, .. .. dp „ . dv -,(dv 2de ∖ . dv^*

d.∆λ = - ∆λ. — - δ2λ. —-δ .λ( - 5r I - 2δ4λ.-

*p 2 V i ∖v* (1 - e2)e∕ *v*

*d.ΔL = -ΔL.^- δ2L.^*- (δ3Z + δ4Z)2^

*d∆A=*- ∆∠i- δu ⅛ - -⅛>j—1— i ",(13)’

" l" ⅛∕ ^2tan2λ + 4

-(δ3J + δ4J)2- *Pj J*

in which

⅛= - O00,000,0478 *{dα - 2db - ⅛{dα - db)* sin2 λ} ' y = + '000,000,0478 *{da + {da - db)* sin2 λ} . ...(14).

, 2de- = + ‘000,0145 *{da - db}*

(1 - *e2)e 7*

The adopted values of the semi-axes were determined by Colonel Everest in an investigation of the figure of the earth from such data as were available in 1826. Forty years afterwards an investigation was made by Captain (now Colonel) A. R. Clarke with additional data, which gave new values, both exceeding the former.@@1 Accepting these as exact, the errors of the first values are *da* = -3130 feet and *db=* - 1746 feet, the former being 150, the latter 84 millionth parts of the semi-axis. The corresponding changes in arcs of 1° of latitude and longitude, expressed in seconds of arc and in millionth parts (*μ*) of arc-length, are as follows :—

In lat. 5° rf.∆λ= - "Ό69 or 19 g and *dΛL=* -"-540or 150 g ;

„ 15° „ -"∙113 ,, 31,, „ „ -"∙554 ,, 154,, ;

,, 25° „ -"∙195,,54,, „ ,, -"∙581 ,, 161;

„ 35° „ -"∙303 „ 84,, „ „ -",617 „ 171 „.

These assumed errors in the geodetic latitudes and longi­tudes are of service when comparisons are made between independent astronomical and geodetic determinations at

any points for which both may be available : they indi­cate the extent to which differences may be attributable to errors in the adopted geodetic constants, as distinct from errors in the trigonometrical or the astronomical operations.

17. *Final Reduction of Principal Triangulations—*The calculations described so far suffice to make the angles of the several trigonometrical figures consistent *inter se,* and to give preliminary values of the lengths and azimuths of the sides and the latitudes and longitudes of the stations. The results are amply sufficient for the requirements of the topographer and land surveyor, and they are published in preliminary charts, which give full numerical details of latitude, longitude, azimuth, and side-length, and of height also, for each portion of the triangulation—secondary as well as principal—as executed year by year. But on the completion of the several chains of triangles further reduc­tions became necessary, to make the triangulation every­where consistent *inter se* and with the verificatory base­lines, so that the lengths and azimuths of common sides and the latitudes and longitudes of common stations should be identical at the junctions of chains, and that the measured and computed lengths of the base-lines should also be identical.

How this was done will now be set forth. But first it must be noted that the triangulation might at the same time have been made consistent with any values of latitude, longitude, or azimuth which had been determined by astronomical observations at either of the trigonometrical stations. This, however, was undesirable, because such observations are liable to errors from deflexion of the plumb-line from the true normal under the influence of local attraction, and these errors are of a much greater magnitude than those that would be generated in triangu­lating between astronomical stations which are not a great distance apart. The trigonometrical elements could not be forced into accordance with the astronomical without altering the angles by amounts much larger than their probable errors, and the results would be useless for in­vestigations of the figure of the earth. The only inde­pendent facts of observation which could be legitimately combined with the angular adjustments were the base-lines, and all these were employed, while the several astronomical determinations—of latitude, differential longitude, and azimuth—were held in reserve for future geodetic investi­gations.

As an illustration of the problem for treatment, suppose a com­bination of three meridional and two longitudinal chains comprising seventy-two single triangles, with a base-line at each corner, as shown in the accompanying diagram (fig. 2) ; suppose the three angles of every triangle to have been

measured and made con­

sistent. Let A be the ori­

gin, with its latitude and

longitude given, and also

the length and azimuth of

the adjoining base-line.

With these data processes

of calculation are carried

through the triangulation

to obtain the lengths and

azimuths of the sides and

the latitudes and longi­

tudes of the stations, say in the following order :—from A through B to E, through F to E, through F to D, through F and E to C, and through F and D to C. Then there are two values of side, azimuth, latitude, and longitude at E,—one from the right­hand chains *via* B, the other from the left-hand chains *via* F ; similarly there are two sets of values at C ; and each of the base­lines at B, C, and D has a calculated as well as a measured value. Thus eleven absolute errors are presented for dispersion over the triangulation by the application of the most appropriate correction to each angle, and, as a preliminary to the determination of these corrections, equations must be constructed between each of the absolute errors and the unknown errors of the angles from which

@@@1 See *Account of the Principal Triangulation of the Ordnance Sur­vey,* 1858, and *Comparisons of Standards of Length,* 1866.