which are very laborious and out of place for the deter­mination of a secondary point. The following is a de­scription of the application of this method to points on a plane surface in the calculations of the Ordnance Survey. Let *s*1, *s*2, . . . be stations whose rectangular coordinates, *x1****,*** x2,... perpendicular, and *y1, y2, . . .* parallel, to the meri­dian of the origin are given ; let a1,a2, . . . be the bearings —here the direction-inclinations with the meridian of the origin—of any point *P, as* observed at the several stations; and let *p* be an approximate position of *P,* with coor­dinates *xp****,*** *yp,* as determined by graphical projection on a district map or by rough calculation. Construct a diagram of the rays converging around *p,* by taking a point to represent *p* and drawing two lines through it at right angles to each other to indicate the directions of north, south, east, and west. Calculate accurately *(yp - y1)* tan *a*p and compare with *(xp* - *x1*) ; the difference will show how far the direction of the ray from s1 falls to the east or west of *p.* Or calculate *(xp — x1)* cot *a1*, and compare with *(yp-y1)* to find how far the direction falls to the north or south of *p.* Set off the distance on the corresponding axis of *p,* and through the point thus fixed draw the direction *a1* with a com­

mon protract­

or. All the

other rays

around *p* may

be drawn in

like manner;

they will in­

tersect each

other in a number of points,

the centre of which may be

adopted as the most prob­

able position of *P.* The co­

ordinates of *P* will then be

readily obtained from those

of *p* ± the distances on the

meridian and perpendicular.

In the annexed diagram

(fig. 6) *P* is supposed to have been observed from five sta­tions, giving as many intersecting rays, (1, 1), (2, 2), . . . ; there are ten points of intersection, the mean position of which gives the true position of *P,* the assumed position being *p.* The advantages claimed for the method are that, the bearings being independent, an erroneous bearing may be redrawn without disturbing those that are correct; similarly new bearings may be introduced without disturb­ing previous work, and observations from a large number of stations may be readily utilized, whereas, when calcula­tion is resorted to, observations in excess of the minimum number required are frequently rejected because of the labour of computing them.@@1

III. Levelling.

Levelling is the art of determining the relative heights of points on the surface of the ground as referred to a hypothetical surface which cuts the direction of gravity everywhere at right angles. When a line of instrumental levels is commenced at the sea-level, a series of heights is determined corresponding to what would be found by perpendicular measurements upwards from the surface of water communicating freely with the sea in underground channels ; thus the line traced indicates a hypothetical prolongation of the surface of the sea inland, which is everywhere conformable to the earth’s curvature.

The trigonometrical determination of the relative heights of points at known distances apart, by the measurements of their mutual vertical angles,—as already described in section I.—is a method of levelling. But the method to which the term “ levelling ” is always applied is that of the direct determination of the differences of height from the readings of the lines at which graduated staves, held vertically over the points, are cut by the horizontal plane which passes through the eye of the observer. Each method has its own advantages. The former is less accu­rate, but best suited for the requirements of a general geographical survey, to obtain the heights of all the more prominent objects on the surface of the ground, whether accessible or not. The latter may be conducted with extreme precision, and is specially valuable for the deter­mination of the relative levels, however minute, of easily accessible points, however numerous, which succeed each other at short intervals apart ; thus it is very generally undertaken *paid passu* with geographical surveys, to furnish lines of level for ready reference as a check on the accuracy of the trigonometrical heights. In levelling with staves the measurements are always taken from the horizontal plane which passes through the eye of the observer ; but the line of levels which it is the object of the operations to trace is a curved line, everywhere conforming to the normal curvature of the earth’s surface, and deviating more and more from the plane of reference as the distance from the station of observation increases. Thus, either a correction for curvature (see footnote, page 705) must be applied to every staff reading, or the instrument must be set up at equal distances from the staves ; the curvature correction, being the same for each staff, will then be eliminated from the difference of the readings, which will thus give the true difference of level of the points on which the staves are set up.

Levelling is an essentially simple operation ; but, as it has to be repeated very frequently in executing a long line of levels—say seven times on an average in every mile—it must be conducted with every precaution against errors of various kinds, instrumental and personal, some accidental and tending to cancel each other, others systematic and cumulative. Instrumental errors arise when the visual axis of the telescope is not perpendicular to the axis of rotation, and when the focusing tube does not move truly parallel to the visual axis on a change of focus. The first error is eliminated, and the second avoided, by placing the instrument at equal dis­tances from the staves ; and, as this procedure has also the advan­tage of eliminating the corrections for both curvature and refraction, it should invariably be adopted. Errors of staff readings should be guarded against by having the staves graduated on both faces, but differently figured, so that the observer may not be biassed to repeat an error of the first reading in the second. The staves of the Indian survey have one face painted white with black divisions—feet, tenths, and hundredths—from 0 to 10, the other black with white divisions from 5·55 to 15·55. Deflexion from horizontality may either be measured and allowed for by taking the readings of the ends of the bubble of the spirit-level and applying corresponding cor­rections to the staff readings, or be eliminated by setting the bubble to the same position on its scale at the reading of the second staff as at that of the first, both being equidistant from the observer.

Certain errors are liable to recur in a constant order and to accumulate to a considerable magnitude, though they may be too minute to attract notice at any single station, as when the work is carried on under a uniformly sinking or rising refraction—from morning to midday or from midday to evening—or when the instru­ment takes some time to settle down on its bearings after being set up for observation. They may be eliminated (i. ) by alternating the order of observation of the staves, taking the back staff first at one station and the forward first at the next ; (ii. ) by working in a circuit, or returning over the same line back to the origin ; (iii. ) by dividing a line into sections and reversing the direction of operation in alternate sections. Cumulative error, not eliminable by working in a circuit, may be caused when there is much northing or southing in the direction of the line, for then the sun’s light will often fall endwise on the bubble of the level, illuminating the outer edge of the rim at the nearer end and the inner edge at the further end, and so biassing the observer to take scale readings of edges which are not equidistant from the centre of the bubble ; this introduces a tendency to raise the south or depress the north

@@@1 For fuller details and an application to spherical surfaces, see *Account of the Graphic Method of the Ordnance Survey,* by J. O’Farrell, London, 1886.