in the printing. The errata thus discovered are given in the first million. Burckhardt gives but a very brief account of the method by which he constructed his table ; and the introduction to Dase’s millions merely consists of Gauss’s letter suggesting their con­struction. The Introduction to the *Fourth Million* (pp. 52) con­tains a full account of the method of construction and a history of factor tables, with a bibliography of writings on the subject. The Introduction (pp. 103) to the *Sixth Million* contains an enumeration of primes and a great number of tables relating to the distribution of primes in the whole nine millions, portions of which had been published in the *Cambridge Philosophical Proceedings* and else­where. The factor tables w’hich have been described greatly exceed in both extent and accuracy any others of the same kind, the largest of which only reaches 408,000. This is the limit of Felkel's *Tafel aller einfachen Factoren* (Vienna, 1776), a remark­able aud extremely rare book,@@1 nearly all the copies having been de­stroyed. Vega ( *Tabulæ,* 1797) gave a table showing all the divisors of numbers not divisible by 2, 3, or 5 up to 102,000, followed by a list of primes from 102,000 to 400,313. In the earlier editions of this work there are several errors in the list, but these are no doubt corrected in Hülsse’s edition (1840). These are the largest and most convenient tables after those of Chernac. Salomon (1827) gives a factor table to 102,011, Kohler (*Handbuch,* 1848) all divisors up to 21,524, and Houël ( *Tables de Logarithmes,* 1871) least divisors up to 10,841. Barlow *(Tables,* 1814) gives the complete resolution of every number up to 10,000 into its factors ; for example, corre­sponding to 4932 we have given 22.32.137. This table is unique so far as we know. The work also contains a list of primes up to 100,103. Both these tables are omitted in the stereotyped reprint of 1840. In *Rees's Cyclopaedia* (1819), article “Prime Numbers,” there is a list of primes to 217,219 arranged in decades. The *Fourth Million* (1879) contains a list of primes up to 30,341. On the first page of the *Second Million* Burckhardt gives the first nine multiples of the primes to 1423 ; and a smaller table of the same kind, extending only to 313, occurs in Lambert’s *Supplementa.*

*Multiplication Tables.—*A multiplication table is usually of double entry, the two arguments being the two factors ; when so arranged it is frequently called a Pythagorean table. The largest and most useful work is Crelle’s *Rechentafeln* (stereotyped, Bremiker’s edition, 1864), which gives in one volume all the products up to 1000 X 1000, so arranged that all the multiples of any one number appear on the same page. The original edition was published in 1820 and consisted of two thick octavo volumes. The second (stereotyped) edition is a convenient folio volume of 450 pages. Only one other multiplication table of the same extent has appeared, viz., Herwart von Hohenburgs *Tabulæ Arithmeticæ IIροσθαφαι- p^σeωs Universales* @@2 (Munich, 1610), on which see Napier, vol. xvii. p. 183. The invention of logarithms four years later afforded another means of performing multiplications, and Von Hohenburg’s work never became generally known. The three following tables are for the multiplication of a number by a single digit. (1) Crelle, *Erleichterungs-Tafel für jeden, der zu rechnen hat* (Berlin, 1836), a work extending to 1000 pages, gives the product of a number of seven figures by a single digit, by means of a double operation of entry. Each page is divided into two tables : for example, to multiply 9382477 by 7 we turn to page 825, and enter the right­hand table at line 77, column 7, where we find 77339 ; we then enter the left-hand table on the same page at line 93, column 7, and find 656, so that the product required is 65677339. (2)

Bretschneider, *Produktentafel* (Hamburg and Gotha, 1841), is some­what similar to Crelle’s table, but smaller, the number of figures in the multiplicand being five instead of seven. (3) In Laundy, *A Table of Products* (London, 1865), the product of any five-figure number by a single digit is given by a double arrangement. The extent of the table is the same as that of Bretschneider’s, as also is the principle, but the arrangement is different, Laundy’s table occupying only 10 pages and Bretschneider’s 99 pages. Among earlier works may be noticed Gruson, *Grosses Einmaleins von Eins bis Hunderttausend* (Berlin, 1799),—a table of products up to 9 × 10,000. The author’s intention was to extend it to 100,000, but we believe only the first part was published. In this book there is no condensation or double arrangement; the pages are very large, each containing 125 lines.

*Quarter-Squares.—*Multiplication may be performed by means of a table of single entry in the manner indicated by the formula— *ab* = 1/4(*a* + *b*)2-1/4(*a*-*b*)2.

Thus with a table of quarter-squares we can multiply together any two numbers by subtracting the quarter-square of their difference from the quarter-square of their sum. The largest table of quarter­squares is Laundy, *Table of Quarter-Squares of all Numbers up to 100,000* (London, 1856). Smaller works are Centnerschwer, *Neu­erfundene Multiplications- und Quadrat-Tafeln* (Berlin, 1825), which extends to 20,000, and Merpaut, *Tables Arithmonomiques*

(Vannes, 1832), which extends to 40,000. In Merpaut’s work the quarter-square is termed the “arithmone.” Ludolf, who published in 1690 a table of squares to 100,000 (see next paragraph), explains in his introduction how his table may be used to effect multiplications by means of the above formula ; but the earliest book on quarter­squares is Voisin, *Tables des Multiplications, ou logarithmes des nombres entiers depuis 1 jusqu'à 20,000* (Paris, 1817). By a log­arithm Voison means a quarter-square, *i.e.,* he calls *a* a root and 1/4*a*2 its logarithm. On the subject of quarter-squares, &c., see the paper (already referred to) in *Phil. Mag.,* November 1878.

*Squares, Cubes, &c.—*The most convenient table for general use, as well as the most extensive, is Barlow’s *Tables* (Useful Knowledge Society, London, from the stereotyped plates of 1840), which gives squares, cubes, square roots, cube roots, and reciprocals to 10,000. The largest table of squares and cubes is Kulik, *Tafeln der Quadrat- und Kubik-Zahlen* (Leipsic, 1848), which gives both as far as 100,000. Two early tables also give squares as far as 100,000, viz., Maginus, *Tabula Tetragonica* (Venice, 1592), and Ludolf, *Tetragonometria Tabularia* (Amsterdam, 1690). Hutton, *Tables of Products and Powers of Numbers* (London, 1781), gives squares up to 25,400, cubes to 10,000, and the first ten powers of the first hundred numbers. Barlow, *Mathematical Tables* (original edition, London, 1814), gives the first ten powers of the first hundred numbers. The first nine or ten powers are given in Vega, *Tabulæ* (1797), and in Hülsse’s edition of the same (1840), in Köhler, *Handbuch* (1848), and in other collections. Faà de Bruno, *Calcul des Erreurs* (Paris, 1869), and Miiller, *Vierstellige Logarithmen* (1844), give squares for use in connexion with the method of least squares. Small tables occur frequently in books intended for engineers and practical men. Drach *(Messenger of Math.,* vol. vii, 1878, p. 87) has given to 33 places the cube roots (and the cube roots of the squares) of primes up to 127. Small tables of powers of 2, 3, 5, 7 occur in various collections. In Vega’s *Tabulæ* (1797, and the subsequent editions, including Hülsse’s) the powers of 2, 3, 5 as far as the 45th, 36th, and 27th respectively are given ; they also occur in Kohler’s *Hand­buch* (1848). The first 25 powers of 2, 3, 5, 7 are given in Salomon, *Logarithmische Tafeln* (1827). Shanks, *Rectification of the Circle* (1853), gives powers of 2 up to 2721.

*Triangular Numbers.—*É. de Joncourt, *De Natura et Præclaro Usu Simplicissimae Speciei Numerorum Trigonalium* (The Hague, 1762), contains a table of triangular numbers up to 20,000 : viz., 1/2*n*(*n* + l) is given for all numbers from *n =* 1 to 20,000. The table occupies 224 pages.

*Reciprocals.*—Barlow’s *Tables* give reciprocals up to 10,000 to 9 or 10 places ; and they have been carried to ten times this extent by Oakes, *Table of the Reciprocals of Numbers from 1 to 100,000* (London, 1865). This gives seven figures of the reciprocal, and is arranged like a table of seven-figure logarithms, differences being added at the side of the page. The reciprocal of a number of five figures is therefore taken out at once, and two more figures may be interpolated for as in logarithms. Picarte, *La Division réduite à une Addition* (Paris, 1861), gives to ten significant figures the reciprocals of the numbers from 10,000 to 100,000, and also the first nine multiples of these reciprocals. Small tables of reciprocals are not common.

*Tables for the Expression of Vulgar Fractions as Decimals.—*Tables of this kind have been given by Wucherer, Goodwyn, and Gauss. Wucherer, *Beyträge zum allgemeinem Gebrauch der Decimalbrüche* (Carlsruhe, 1796), gives the decimal fractions (to 5 places) for all vulgar fractions whose numerator and denominator are each less than 50 and prime to one another, arranged according to denomi­nators. The most extensive and elaborate tables that have been published are contained in Henry Goodwyn’s *First Centenary of Tables of all Decimal Quotients* (London, 1816), *A Tabular Series of Decimal Quotients* (1823), and *A Table of the Circles arising from the Division of a Unit or any other Whole Number by all the Integers from 1 to 1024* (1823). The *Tabular Series* (1823), running to 153 pages, gives to 8 places the decimal corresponding to every vulgar fraction less than 99/991 whose numerator and denominator do not surpass 1000. The arguments are not arranged according to their numerators or denominators, but according to their magnitude, so that the tabular results exhibit a steady increase from ∙001 (=1/1000) to ∙09989909 ( = 99/991). The author intended the table to include all fractions whose numerator and denominator were each less than 1000, but no more was ever published. The *Table of Circles* (1823) gives all the periods of the circulating decimals that can arise from the division of any integer by another integer less than 1024. Thus for 13 we find ∙076923 and ∙1538465, which are the only periods in which the fraction *x*/13 can circulate. The table occupies 107

pages, some of the periods being of course very long *(e.g.,* for 1021 the period contains 1020 figures). The *First Centenary* (1816) gives the complete periods of the reciprocals of the numbers from 1 to 100. Goodwyn’s tables are very scarce, but as they are nearly unique of their kind they deserve special notice. A second edition of the *First Centenary* was issued in 1818 with the addition of some of the *Tabular Series,* the numerator not exceeding 50 and the denomi-

@@@1 For information about it, see a paper on “ Factor Tables,” in *Camb. Phil. Proc.,* vol. iii. (1878) pp. 99-138, or the Introduction to the *Fourth Million.*

@@@2 See a paper “ On Multiplication by a Table of Single Entry,” in *Phil. Mag.,* November 1878, for a notice of this book.