nator not exceeding 100. A posthumous table of Gauss’s, entitled “Tafel zur Verwandlung gemeiner Brüche mit Nennern aus dem ersten Tausend in Decimalbrüche,” occurs in vol. ii. pp. 412-434 of his *Gesammelte. Werke* (Göttingen, 1863), and resembles Goodwyn’s *Table of Circles.* On this subject see a paper “On Circulating Decimals, with special reference to Henry Goodwyn’s *Table of Circles* and *Tabular Series of Decimal Quotients,"* in *Camb. Phil. Proc.,* vol. iii. (1878), pp. 185-206, where is also given a table of the periods of fractions corresponding to denominators prime to 10 from 1 to 1024 obtained by counting from Goodwyn’s table. See also the section on “ Circulating Decimals,” p. 13 below.

*Sexagesimal and Sexcentenary Tables. —*Originally all calculations were sexagesimal ; and the relics of the system still exist in the division of the degree into 60 minutes and the minute into 60 seconds. To facilitate interpolation, therefore, in trigonometrical and other tables the following large sexagesimal tables were con­structed. John Bernoulli, *A Sexcentenary Table* (London, 1779), gives at once the fourth term of any proportion of which the first term is 600" and each of the other two is less than 600" ; the table is of double entry, aud may be more fully described as giving the value of correct to tenths of a second, *x* and *y* each con- 600

taining a number of seconds less than 600. Michael Taylor, *A Sexa­gesimal Table* (London, 1780), exhibits at sight the fourth term of any proportion where the first term is 60 minutes, the second any number of minutes less than 60, and the third any number of minutes and seconds under 60 minutes ; there is also another table in which the third term is any absolute number under 1000. Not much use seems to have been made of these tables, both of which were published by the Commissioners of Longitude. Small tables for the conversion of sexagesimals into centesimals and *vice versa* are given in a few collections, such as Hülsse’s edition of Vega.

*Trigonometrical Tables (Natural).—*Peter Apian published in 1533 a table of sines with the radius divided decimally. The first complete canon giving all the six ratios of the sides of a right-angled triangle is due to Rheticus (1551), who also introduced the semi- quadrantal arrangement. Rheticus’s canon was calculated for every ten minutes to 7 places, and Vieta extended it to every minute (1579). In 1554 Reinhold published a table of tangents to every minute. The first complete canon published in England was by Blunedvile (1594), although a table of sines had appeared four years earlier. Regiomontanus called his table of tangents (or rather cotangents) *tabula fœcunda* on account of its great use ; and till the introduction of the word “ tangent ” by Finck (*Geometriæ Rotundi Libri XIV.,* Basel, 1583) a table of tangents was called a *tabula fœcunda* or *canon fœcundus.* Besides “ tangent,” Finck also introduced the word “ secant,” the table of secants having pre­viously been called *tabula benefica* by Maurolycus (1558) and *tabula fœcundissima* by Vieta.

By far the greatest computer of pure trigonometrical tables is George Joachim Rheticus, whose work has never been superseded. His celebrated ten-decimal canon, the *Opus Palatinum,* was pub­lished by Valentine Otho at Neustadt in 1596, and in 1613 his fifteen-decimal table of sines by Pitiscus at Frankfort under the title *Thesaurus Mathematicus.* The *Opus Palatinum* contains a complete ten-decimal trigonometrical canon for every ten seconds of the quadrant, semiquadrantally arranged, with differences for all the tabular results throughout. Sines, cosines, and secants are given on the left-hand pages in columns headed respectively ‘•Perpendiculum,” “Basis,” “Hypotenusa,” and on the right-hand appear tangents, cosecants, and cotangents in columns headed respectively “Perpendiculum,” “Hypotenusa,” “Basis.” At his death Rheticus left the canon nearly complete, and the trigonometry was finished and the whole edited by Valentine Otho ; it was named in honour of the elector palatine Frederick IV., who bore the ex­pense of publication. The *Thesaurus* of 1613 gives natural sines for every ten seconds throughout the quadrant, to 15 places, semi­quadrantally arranged, with first, second, and third differences. Natural sines are also given for every second from 0° to 1° and from 89° to 90°, to 15 places, with first and second differences. The rescue of the manuscript of this work by Pitiscus forms a striking episode in the history of mathematical tables. The alterations and emendations in the earlier part of the corrected edition of the *Opus Palatinum* were made by Pitiscus, who had his suspicions that Rheticus had himself calculated a ten-second table of sines to 15 decimal places ; but it could not be found. Eventually the lost canon was discovered amongst the papers of Rheticus, which had passed from Otho to James Christmann on the death of the former. Amongst these Pitiscus found (1) the ten-second table of sines to 15 places, with first, second, and third differences (printed in the *Thesaurus)* ; (2) sines for every second of the first and last degrees of the quadrant, also to 15 places, with first and second differences ; (3) the commencement of a canon of tangents and secants, to the same number of decimal places, for every ten seconds, with first and second differences ; (4) a complete minute canon of sines, tangents, and secants, also to 15 decimal places. These tables taken in connexion with the *Opus Palatinum* give an idea of the enormous labours undertaken by Rheticus ; his tables not only remain to this day the ultimate authorities but formed the data whereby Vlacq calculated his logarithmic canon. Pitiscus says that for twelve years Rheticus constantly had computers at work.

A history of trigonometrical tables by Hutton was prefixed to all the early editions of his *Tables of Logarithms,* and forms Tract xix. of his *Mathematical Tracts,* vol. i. pp. 278-306,1812. A good deal of bibliographical information about the *Opus Palatinum* and earlier trigonometrical tables is given in De Morgan’s article “Tables” in the *English Cyclopaedia.* The invention of logarithms the year after the publication of Rheticus’s volume by Pitiscus changed all the methods of calculation ; and it is worthy of note that Napier’s original table of 1614 was a logarithmic canon of sines and not a table of the logarithms of numbers. The logarithmic canon at once superseded the natural canon ; and since Pitiscus’s time no really extensive table of pure trigonometrical functions has appeared. In recent years the employment of the arithmometer of Thomas de Colmar has revived the use of tables of natural trigonometrical functions, it being found convenient for some purposes to employ an arithmometer and a natural canon instead of a logarithmic canon. Junge’s *Tafel der wirklichen Länge der Sinus und Cosinus* (Leipsic, 1864) was published with this object. It gives natural sines and cosines for every ten seconds of the quadrant to 6 places. F. Μ. Clouth, *Tables pour le Calcul des Coordonnées Goniométriques* (Mainz, n.d.), gives natural sines and cosines (to 6 places) and their first nine multiples (to 4 places) for every centesimal minute of the quadrant. Tables of natural func­tions occur in many collections, the natural and logarithmic values being sometimes given on opposite pages, sometimes side by side on the same page.

The following works contain tables of trigonometrical functions other than sines, cosines, and tangents. Pasquich, *Tabulæ Log- arithmico-Trigonometricæ* (Leipsic, 1817), contains a table of sin2«, cos2*x*, tan2*x*, cot2*x* from *x*=1° to 45° at intervals of 1' to 5 places. Andrew, *Astronomical and Nautical Tables* (London, 1805), con­tains a table of “squares of natural semichords,” *i.e.,* of sin2z from *x*=0° to 120° at intervals of 10" to 7 places. This table has recently been greatly extended by Major-General Hannyngton in his *Haversines, Natural and Logarithmic, used in computing Lunar Distances for the Nautical Almanac* (London, 1876). The name “ haversine,” now frequently used in works upon navigation, is an abbreviation of “ half versed sine ” ; viz., the haversine of *x* is equal to 1/2(l-cos*x*), that is, to sin2(1/2)*x*. The table gives logarithmic haversines for every 15" from 0° to 180°, and natural haversines for every 10" from 0° to 180°, to 7 places, except near the beginning, where the logarithms are given to only 5 or 6 places. The work itself occupies 327 folio pages, and was suggested by Andrew’s, a copy of which by chance fell into Hannyngton’s hands. Han­nyngton recomputed the whole of it by a partly mechanical method, a combination of two arithmometers being employed. A table of haversines is useful for the solution of spherical triangles when two sides and the included angle are given, and in many other problems in spherical trigonometry. Andrew’s original table seems to have attracted very little notice. Hannyngton’s was printed, on the recommendation of the superintendent of the *Nautical Almanac* office, at the public cost. Before the cal­culation of Hannyngton’s table Farley’s *Natural Versed Sines* (London, 1856) was used in the *Nautical Almanac* office in com­puting lunar distances. This fine table contains natural versed sines from 0° to 125° at intervals of 10" to 7 places, with proportional parts, and log versed sines from 0° to 135° at intervals of 15" to 7 places. The arguments are also given in time. The manuscript was used in the office for twenty-five years before it was printed. Traverse tables, which occur in most collections of navigation tables, contain multiples of sines and cosines.

*Common or Briggian Logarithms of Numbers and Trigono­metrical Ratios.—*For an account of the invention and history of logarithms, see Logarithms (vol. xiv. p. 773) and Napier. The following are the fundamental works which contain the results of the original calculations of logarithms of numbers and trigono­metrical ratios :—Briggs, *Arithmetica Logarithmica* (London, 1624), logarithms of numbers from 1 to 20,000 and from 90,000 to 100,000 to 14 places, with interscript differences ; Vlacq, *Arithmetica Log­arithmica* (Gouda, 1628, also an English edition, London, 1631, the tables being the same), ten-figure logarithms of numbers from 1 to 100,000, with differences, also log sines, tangents, and secants for every minute of the quadrant to 10 places, with interscript differences ; Vlacq, *Trigonometria Artificialis* (Gouda, 1633), log sines and tangents to every ten seconds of the quadrant to 10 places, with differences, and ten-figure logarithms of numbers up to 20,000, with differences ; Briggs, *Trigonometria Britannica* (London, 1633), natural sines to 15 places, tangents and secants to 10 places, log sines to 14 places, and tangents to 10 places, at intervals of a hundredth of a degree from 0° to 45°, with interscript differences for all the functions. In 1794 Vega re­printed at Leipsic Vlacq’s two works in a single folio volume, *Thesaurus Logarithmorum Completus.* The arrangement of the