of such tables is to give log (*a*±*b*) by only one entry when log *a* and log *b* are given (see Logarithms, vol. xiv. p. 777). Let

*A=logx*, *B*=log(l + 1/*x*), *C*=log (l+*x*).

Leaving out the specimen table in Leonelli’s *Théorie des Logarithmes Additionnels et Déductifs* (Bordeaux, 1803), the principal tables are the following. Gauss, in Zach’s *Monatliche Correspondenz* (1812), giving *B* and *C* for argument *A* from 0 to 2 at intervals of ∙001, thence to 3∙40 at intervals of ∙01, and to 5 at intervals of ∙1, all to 5 places. This table is reprinted in Gauss’s *Werke,* vol. iii. p. 244. Matthiessen, *Tafel zur bequemem Berechnung* (Altona, 1818), giving *B* and *C* to 7 places for argument *A* from 0 to 2 at intervals of ∙0001, thence to 3 at intervals of ∙001, to 4 at intervals of ∙01, and to 5 at intervals of ∙1 ; the table is not conveniently arranged. Peter Gray, *Tables and Formulæ* (London, 1849, and “Addendum,” 1870), giving *C for* argument *A* from 0 to 2 at intervals of ∙0001 to 6 places, with proportional parts to hundredths, and log (1 - *x)* for argument *A* from 3 to I at intervals of ∙001 and from 1 to 1∙9 at intervals of ∙0001, to 6 places, with proportional parts. Zech, *Tafeln der Additions- und Subtractions - Logarithmen* (Leipsic, 1849), giving *B* for argument *A* from 0 to 2 at intervals of ∙0001, thence to 4 at intervals of ∙001 and to 6 at intervals of ∙01 ; also *C* for argument *A* from 0 to ∙0003 at intervals of ∙0000001, thence to ∙05 at intervals of ∙000001 and to ∙303 at intervals of •00001, all to 7 places, with proportional parts. These tables are reprinted from Hülsse’s edition of Vega (1849) ; the 1840 edition of Hülsse’s Vega contained a reprint of Gauss’s original table. Wittstein, *Logarithmes de Gauss à Sept Décimales* (Hanover, 1866), giving *B* for argument *A* from 3 to 4 at intervals of Ί, from 4 to 6 at intervals of Ό1, from 6 to 8 at intervals of ∙001, from 8 to 10 at intervals of ∙0001, also from 0 to 4 at the same intervals. In this handsome work the arrangement is similar to that in a seven-figure logarithmic table. Gauss’s original five-place table was reprinted in Pasquich, *Tabulæ* (Leipsic, 1817) ; Köhler, *Jerome de la Lande's Tafeln* (Leipsic, 1832), and *Handbuch* (Leipsic, 1848) ; and Galbraith and Haughton, *Manual* (London, 1860). Houël, *Tables de Logarithmes* (1871), also gives a small five-place table of Gaussian logarithms, the addition and subtraction logarithms being separated as in Zech. Modified Gaussian logarithms are given by J. H. T. Muller, *Vierstellige Logarithmen* (Gotha, 1844), viz., a four-place table of *B* and - log [1-(1/*x*)] from *A = 0* to ∙03 at intervals of ∙0001, thence to ∙23 at intervals of ∙001, to 2 at intervals of ∙01, and to 4 at intervals of Ί ; and by Shortrede, *Logarithmic Tables* (vol. i., 1849), viz., a five-place table of *B* and log (1 +*x*) from *A = 5* to 3 at intervals of Ί, from *A= 3* to 2∙7 at intervals of ∙01, to 1∙3 at intervals of ∙001, to 3 at intervals of ∙01, and to 5 at intervals of ∙1. Filipowski’s *Antilogarithms* (1849) contains Gaussian logarithms arranged in a new way. The principal table gives log (*x*+l) as tabular result for log *x* as argument from 8 to 14 at intervals of ∙001 to 5 places. Weiden- bach, *Tafel um den Logarithmen* (Copenhagen, 1829), gives log[(*x*+1)/(*x*-1)] for argument *A* from ∙382 to 2∙002 at intervals of ∙001, to

3∙6 at intervals of ∙01, and to 5∙5 at intervals of Ί, to 5 places.

*Logistic and Proportional Logarithms.—*In most collections of tables of logarithms a five-place table of logistic logarithms for every second to 1o is given. Logistic tables give log 3600 - log *x* at intervals of a second, *x* being expressed in degrees, minutes, and seconds; Schulze (1778) and Vega (1797) have them to *x*=3600" and Callet and Hutton to *x=*5280". Proportional logarithms for every second to 3° *(i.e.,* log 10,800-log*x*) form part of nearly all collections of tables relating to navigation, generally to 4 places, sometimes to 5. Bagay, *Tables* (1829), gives a five-place table, but such are not often to be found in collections of mathematical tables. The same remark applies to tables of proportional loga­rithms for every minute to 24“, which give to 4 or 5 places the values of log 1440 - log *x.* The object of a proportional or logistic table, or a table of log *a -* log *x,* is to facilitate the calculation of propor­tions in which the third term is *a.*

*Interpolation Tables.—*All tables of proportional parts may be regarded as interpolation tables. Bremiker, *Tafel der Proportional­theile* (Berlin, 1843), gives proportional parts to hundredths of all numbers from 70 to 699. Schrön, *Logarithms,* contains an inter­polation table giving the first hundred multiples of all numbers from 40 to 410. Tables of the values of binomial theorem coef­ficients, which are required when second and higher orders of differ­ences are used, are described below. Woolhouse, *On Interpolation, Summation, and the Adjustment of Numerical Tables* (London, 1865), contains nine pages of interpolation tables. The book con­sists of papers extracted from vols. xi. and xii. of the *Assurance Magazine.*

*Dual Logarithms.—*This term is used by Mr Oliver Byrne in his *Dual Arithmetic, Young Dual Arithmetician, Tables of Dual Logarithms,* &c. (London, 1863-67). A dual number of the ascend­ing branch is a continued product of powers of 1∙1, 1∙01, l∙001, &c., taken in order, the powers only being expressed; thus ↓6,9,7,8 denotes (1∙1)6 (1∙01)9(1∙001)7(1∙0001)8, the numbers following the ↓ being called dual digits. A dual number which has all but the last digit zeros is called a dual logarithm ; the author uses dual logarithms in which there are seven ciphers between the ↓ and the logarithms. A dual number of the descending branch is a con­tinued product of powers of ∙9, ∙99, &c. : for instance, (∙9)3(∙99)2 is denoted by ’3 ’2 ↑. The *Tables,* which occupy 112 pages, give dual numbers and logarithms, both of the ascending and descend­ing branches, and the corresponding natural numbers. The author claimed that his tables were superior to those of common logarithms.

*Constants.—*In nearly all tables of logarithms there is a page de­voted to certain frequently used constants and their logarithms, such as π, 1/π, π2, √*π.* A specially good collection is printed in Templeton’s *Millwright's and Engineer's Pocket Companion* (cor­rected by S. Maynard, London, 1871), which gives 58 constants involving π and their logarithms, generally to 30 places, and 13 others that may be properly called mathematical. A good list of constants involving π is given in Salomon (1827). A paper by Paucker in *Grunert’s Archiv* (vol. i. p. 9) has a number of con­stants involving *π* given to a great many places, and Gauss’s memoir on the lemniscate function ( *Werke,* vol. iii.) has *e-π, e-1/4*π, *e-2/4π,* &c., calculated to about 50 places. The quantity *π* has been worked out to 707 places (Shanks, *Proc. Roy. Soc.,* vol. xxi. p. 319) and Euler’s constant to 263 places (Adams, *Proc. Roy. Soc.,* vol. xxvii. p. 88). The value of the modulus *M,* calculated by Prof. Adams, is given in Logarithms, vol. xiv. p. 779. This value is correct to 263 places ; but the calculation has since been carried to 272 places (see Adams, *Proc. Roy. Soc.,* vol. xlii. p. 22, 1887).

*Tables for the Solution of the Irreducible Case in Cubic Equations.—* Lambert, *Supplementa* (1798), gives ± *(x* - *x*3) from *x*= ∙001 to 1∙155 at intervals of ∙001 to 7 places, and Barlow (1814) gives *x*3-*x* from *x*=l to 1∙1549 at intervals of ∙0001 to 8 places.

*Binomial Theorem Coefficients.—*The values of

[*x*(*x*-1)/(1 . 2)] , [*x*(*x*-1)(*x*-2)/(1 . 2. 3)] , . . . [*x*(*x*-1). . .(*x*-5)]/[1 . 2 . . . 6] ,

from *x=* ∙01 to *x=* 1 at intervals of ∙01 to 7 places, are serviceable for use in interpolation by second and higher orders of differences. The table quoted above occurs in Schulze (1778), Barlow (1814), Vega (1797 and succeeding editions), Hantschl (1827), and Köhler (1848). Rouse, *Doctrine of Chances* (London, no date), gives on a folding sheet *(a + b)n* for *n=l,* 2,... 20. Lambert, *Supplementa* (1798), has the coefficients of the first 16 terms in (l+ *x*)1/2 and (1 - *x)1/2,* their accurate values being given as decimals. Vega (1797)

has a page of tables giving 1/(2∙4), (1∙3)/(2∙4∙6), . . . 1/(2∙3), . . .and similar­

quantities to 10 places, with their logarithms to 7 places, and a page of this kind occurs in other collections. Köhler (1848) gives the values of 40 such quantities.

*Figurate Numbers.—*Lambert, *Supplementa,* gives *x,* [*x*(*x*+1)]/[1∙2], . . . [x(x+1). . .(x+11)]/[1.2. . . 12] from x=l to 30.

*Trigonometrical Quadratic Surds.—*The surd values of the sines of every third degree of the quadrant are given in some tables of logarithms; *e.g.,* in Hutton’s (p. xxxix., ed. 1855), we find sin 3o = ⅛{√(5 + √5) + √15/2+ √5/2 - √(15 + 3√5) - √3/2 - √½} ; and the numerical values of the surds √(5 + √5), √(15/2), &c., are given to 10 places. These values were extended to 20 places by Peter Gray, *Messenger of Math.,* vol. vi. (1877), p. 105.

*Circulating Decimals.—*Goodwyn’s tables have been described above, p. 8. Several others have been published giving the num­bers of digits in the periods of the reciprocals of primes : Burck­hardt, *Tables des Diviseurs du Premier Million* (Paris, 1814-17), gave one for all primes up to 2,543 and for 22 primes exceeding that limit. Desmarest, *Théorie des Nombres* (Paris, 1852), included all primes up to 10,000. Reuschle, *Mathematische Abhandlung, enthaltend neue zahlentheoretische Tabellen* (1856), contains a simi­lar table to 15,000. This Shanks extended to 60,000 ; the portion from 1 to 30,000 is printed in the *Proc. Roy. Soc.,* vol. xxii. p. 200, and the remainder is preserved in the archives of the society *(Id.,* xxiii. p. 260 and xxiv. p. 392). The number of digits in the decimal period of - is the same as the exponent to which 10 be­longs for modulus *p,* so that, whenever the period has *p-1* digits, 10 is a primitive root of *p.* Tables of primes having a given number, *n,* of digits in their periods, *i.e.,* tables of the resolutions of 10*n*- 1 into factors and, as far as known, into prime factors, have been given by Loof (in *Grunert’s Archiv,* vol. xvi. p. 54 ; reprinted in *Nouv. Annales,* vol. xiv. p. 115) and by Shanks *(Proc. Roy. Soc.,* vol. xxii. p. 381). The former extends to *n* = 60 and the latter to n = 100, but there are gaps in both. Reuschle’s tract also contains resolutions of 10*n*- 1. For further references on circulating deci­mals, see *Proc. Camb. Phil. Soc.,* vol. iii. p. 185 (1878).

*Pythagorean Triangles.—*Right-angled triangles in which the