sides and hypothenuse are all rational integers are frequently termed Pythagorean triangles, as, for example, the triangles 3, 4, 5 and 5, 12, 13. Schulze, *Sammlung* (1778), contains a table of such triangles subject to the condition tan ½*ω*>1/25 (*ω* being one of the acute angles). About 100 triangles are given, but some occur twice. Large tables of right-angled rational triangles were given by Bretschneider, in *Grunert’s Archiv,* vol. i. p. 96 (1841), and by Sang, *Edinburgh Transactions,* vol. xxiii. p. 727 (1864). In these the triangles are arranged according to hypothenuses and extend to 1201, 1200, 49, and 1105, 1073, 264 respectively. Whitworth, in a paper read before the Lit. and Phil. Society of Liverpool in 1875, carried his list as far as 2465, 2337, 784. See also Rath, “Die rationalen Dreiecke,” in *Grunert's Archiv,* vol. lvi. p. 188 (1874). Sang’s paper also contains a table of triangles having an angle equal to 120° and their sides integers.

*Powers of* π*.—*Paucker, in *Grunert’s Archiv,* vol. i. p. 10, gives π-1 and π½ to 140 places, and π-2*,* π-½*,* π⅓, π⅔ to about 50 places ; and in Maynard’s list of constants (see “Constants,” above) *π2* is given to 31 places. The first twelve powers of π and π-1 to 22 or more places were printed by Glaisher, *Proc. Lond. Math. Soc.,* vol. viii. p. 140, and the first hundred multiples of *π* and π-1 to 12 places by Kulik, *Tafel der Quadrat-und Kubik-Zahlen* (Leipsic, 1848).

*The Series* 1-*n* + 2-n + 3-n + &c.—Let *Sn,sn,σn* denote respectively the sums of the series l-*n* + 2-*n* + 3-*n*+ &c., 1-n - 2-n +3-n-&c., l-*n* + 3-*n* + 5-*n* + &c. Legendre (*Traité des Fonctions Elliptiques,* vol. ii. p. 432) has computed *Sn* to 16 places from *n*=l to 35, and Glaisher *{Proc. Lond. Math. Soc.,* vol. iv. p. 48) has deduced sn and *σn* for the same arguments and to the same number of places. The latter has also given *Sn, sn, σn* for *n = 2,* 4, 6, ... 12 to 22 or more places *{Proc. Lond. Math. Soc.,* vol. viii. p. 140), and the values of ∑n, where Σn=2-*n* + 3-*n* + 5-n +&c. (prime numbers only in­volved), for n=2, 4, 6, ... 36 to 15 places (*Compte Rendu de l' Assoc. Française* for 1878, p. 172).

*Tables of ex and e-x, or Hyperbolic Antilogarithms.—*The largest tables are the following. Gudermann, *Theorie der potenzial- oder cyklisch-hyperbolischen Functionen* (Berlin, 1833), which consists of papers reprinted from vols. viii. and ix. of *Crelle's Journal,* and gives log10 sinh *x,* log10 cosh *x,* and log10 tanh *x* from *x*=2 to 5 at intervals of ∙001 to 9 places and from *x*=5 to 12 at intervals of ∙01 to 10 places. Since sinh *x=½(ex-e-x)* and cosh *x=½*(*ex + e-x),* the values of *ex* and *e-x* are deducible at once by addition and sub­traction. Newman, in *Camb. Phil. Trans.,* vol. xiii. pp. 145-241, gives values of *e-x* from *x=*0 to 15∙349 at intervals of ∙001 to 12 places, from *x*=15∙350 to 17∙298 at intervals of ∙002, and from *x*=17∙300 to 27∙635 at intervals of ∙005, to 14 places. Glaisher, in *Camb. Phil. Trans.,* vol. xiii. pp. 243-272, gives four tables of *ex, e-x,* log10 *ex,* log10 *e-x,* their ranges being from *x*=∙001 to ∙1 at in­tervals of ∙001, from ∙01 to 2 at intervals of ∙01, from Ί to 10 at intervals of ∙1, from 1 to 500 at intervals of unity. Vega, *Tabules* (1797 and later edd.), has log10e*x* to 7 places and *ex* to 7 figures from *x=* ∙01 to 10 at intervals of ∙01. Köhler’s *Handbuch* contains a small table of *ex.* In Schulze’s *Sammlung* (1778) *ex* is given for ce=l, 2, 3,... 24 to 28 or 29 figures and for *x* = 25, 30, and 60 to 32 or 33 figures ; this table is printed in Glaisher’s paper *{loc. cit.).* In Salomon’s *Tafeln* (1827) the values of *en, e∙n, e ∙0n, e∙00n,..*. e∙000000*n*, where *n* has the values 1, 2,... 9, are given to 12 places. Bret­schneider, in *Grunert’s Archiv,* iit p. 33, worked out *ex* and *e-x* and also sin *x* and cos *x* for *x*=l, 2,... 10 to 20 places.

*Factorials.—*The values of log10 (n!), where n! denotes 1.2.3... *n*, from *n* = 1 to 1200 to 18 places, are given by Degen, *Tabularum Enneas* (Copenhagen, 1824), and reprinted, to 6 places, at the end of De Morgan’s article “ Probabilities ” in the *Encyclopædia Metro­politana.* Shortrede, *Tables* (1849, vol. i. ), gives log *{n* !) to *n=*1000 to 5 places, and for the arguments ending in 0 to 8 places. Degen also gives the complements of the logarithms. The first 20 figures of the values of *n* × *n*! and the values of log10 are computed

by Glaisher as far as *n*=71 in the *Phil. Trans.* for 1870 (p. 370), and the values of — to 28 significant figures as far as *n*=50 in *Camb. Phil. Trans.,* vol. xiii. p. 246.

*Bernoullian Numbers.—*The first fifteen Bernoullian numbers were given by Euler, *Inst. Calc. Diff.,* part ii. ch. v. Sixteen more were calculated by Rothe, and the first thirty-one were published by Ohm in *Crelle's Journal,* vol. xx. p. 11. Prof. J. C. Adams has calculated the next thirty-one, and a table of the first sixty- two was published by him in the *Brit. Assoc. Report* for 1877 and in *Crelle’s Journal,* vol. lxxxv. p. 269. The first nine figures of the values of the first 250 Bernoullian numbers, and their Briggian logarithms to 10 places, have been printed by Glaisher, *Camb. Phil. Trans.,* vol. xii. p. 384.

*Tables of log tan* (¼*π + ½φ*)*.—*Gudermann, *Theorie der potenzial- oder cyklisch-hyperbolischen Functionen* (Berlin, 1833), gives (in 100 pages) log tan(¼π

+ ½*φ*) for every centesimal minute of the quadrant to 7 places. Another table contains the values of this function, also at intervals of a minute, from 88° to 100° (centesimal) to 11 places. Legendre, *Traité des Fonctions Elliptiques* (vol. ii. p. 256), gives the same function for every half degree (sexagesimal) of the quadrant to 12 places.

*Tiee Gamma Function.—*Legendre’s great table appeared in vol. ii. of his *Exercices de Calcul Intégral* (1816), p. 85, and in vol. ii. of his *Traité des Fonctions Elliptiques* (1826), p. 489. Log10 Γ(*x*) is given from *x*=l to 2 at intervals of ∙001 to 12 places, with differ­ences to the third order. This table is reprinted in full in Schlö­milch, *Analytische Studien* (1848), p. 183 ; an abridgment in which the arguments differ by ∙01 occurs in De Morgan, *Diff. and Int. Calc.,* p. 587. The last figures of the values omitted are also sup­plied, so that the full table can be reproduced. A seven-place abridgment (without differences) is published in Bertrand, *Calcul Intégral* (1870), p. 285, and a six-figure abridgment in Williamson, *Integral Calculus* (1884), p. 169. In vol. i. of his *Exercices* (1811), Legendre had previously published a seven-place table of log10 *Γ*(*x*)*,* without differences.

*Tables connected with Elliptic Functions.—*Legendre calculated elaborate tables of the elliptic integrals in vol. ii. of *Traité des Fonctions Elliptiques* (1826). Denoting the modular angle by *θ,* the amplitude by *φ,* and the incomplete integral of the second kind by *E1* (*φ*) the tables are—(1) log10 *E* and log10*K* from θ = 0° to 90° at intervals of 0°∙1 to 12 or 14 places, with differences to the third order ; (2) *E1*(*φ*) and *F*(*φ*)*,* the modular angle being 45° from *φ* = 0° to 90° at intervals of 0°∙5 to 12 places, with differences to the fifth order ; (3) *E1* (45°) and *F* (45°) from *θ* = 0° to 90° at intervals of 1°, with differences to the sixth order, also *E* and *K* for the same argu­ments, all to 12 places; (4) *E1*(*φ*) and *F*(*φ*) for every degree of both the amplitude and the argument to 9 or 10 places. The first three tables had been published previously in vol. iii. of the *Exer­cices de Calcul Intégral* (1816).

*Tables involving q.—*Verhulst, *Traité des Fonctions Elliptiques* (Brussels, 1841), contains a table of log10log10 for argument θat intervals of 0°∙l to 12 or 14 places. Jacobi, in *Crelle's Journal,* vol. xxvi. p. 93, gives log10 *q* from *θ* = 0° to 90° at intervals of 0°∙l to 5 places. Meissel, *Sammlung mathematischer Tafeln,* i. (Iser­lohn, 1860), consists of a table of log10 *q* at intervals of 1' from *θ* = 0° to 90° to 8 places. Glaisher, in *Month. Not. Roy. Ast. Soc.,* vol. xxxvii. p. 372 (1877), gives log10 *q* to 10 places and *q* to 9 places for every degree. In Bertrand, *Calcul Intégral* (1870), a table of log10 *q* from *θ* = 0° to 90° at intervals of 5' to 5 places is accompanied by √2K/*π* 1

— and log10log10 - and by abridgments of Legendre’s tables of the elliptic integrals. Schlömilch, *Vorlesungen der höheren Analysis* (Brunswick, 1879), p. 448, gives a small table of log10 *q* for every degree to 5 places.

*Legendrian Coefficients.—*The values of *Pn*(*x*) for *n* = l, 2, 3, ... 7 from *x* = 0 to 1 at intervals of Ό1 are given by Glaisher, in *Brit. Assoc. Rep.* for 1879, pp. 54-57. The functions tabulated are *P1*(*x*)*=x,* P2(*x*) = *½*(*3x2-1*), *P3*(*x*) = *½*(*5x3* - 3*x*)*,* P4(*x*) = ⅛ (*35x4* - *30x2* + 3), *P5*(*x*) *=* *⅛(63x5*- 70*x*3 + 15*x*), *P*6(*x*) = *1/16(231x6* - 315*x*4 + 105*x*2 - 5), *P7(x)=1/16(429x7*- 693*x*5 + 315*x*3-35*x*). The functions occur in connexion with the theory of interpolation, the attraction of spheroids, and other physical theories.

*Bessel's Functions.—*Bessel’s original table appeared at the end of his memoir “Untersuchung des planetarischen Theils der Störungen, welche aus der Bewegung der Sonne entstehen ” (in *Abh. d. Berl. Akad.,* 1824 ; reprinted in vol. i. of his *Abhand­lungen,*p. 84). It gives *J0*(*x*) and *J*1(*x*) from *x*=0 to 3∙2 at intervals of ∙01. More extensive tables were calculated by Hansen in “ Ermit­telung der absoluten Störungen in Ellipsen von beliebiger Excen- tricität und Neigung” (in *Schriften der Sternwarte Seeberg,* part i., Gotha, 1843). They include an extension of Bessel’s original table to *x*= 20, besides smaller tables of *Jn*(*x*) for certain values of *n* as far as *n* = 28, all to 7 places. Hansen’s table was reproduced by Schlömilch, in *Zeitschr. für Math.,* vol. ii. p. 158, and by Lommel, *Studien über die Bessel’schen Functionen* (Leipsic, 1868), p. 127. Hansen’s notation is slightly different from Bessel’s ; the change amounts to halving each argument. Schlömilch gives the table in Hansen’s form ; Lommel expresses it in Bessel’s.

*Sine, Cosine, Exponential, and Logarithm Integrals.—*The fιιnc- *,. 1 ., . , 1 Γx*since , *fx* cosec , *Γx e? j*

tions so named are the integrals ∕ *dx, ∕ —— dx, ∕ — dx,*

*∕\* dx . J ί» ∙c J ∞ x J -∞ x*

logx, are denoted by the functional signs Si *x,* Ci *x,* Ei *x,*

li *x* respectively. Soldner, *Théorie et Tables d'une Nouvelle Fonction Transcendante* (Munich, 1809), gave the values of li *x* from *x*=0 to 1 at intervals of ∙1 to 7 places, and thence at various intervals to 1220 to 5 or more places. This table is reprinted in De Morgan’s *Diff. and Int. Calc.,* p. 662. Bretschneider, in *Grunert's Archiv,* vol. iii. p. 33, calculated Ei (±*x*), Si *x*, Ci *x* for *x*=l, 2,... 10 to 20 places, and subsequently (in Schlömilch’s *Zeitschrift,* vol. vi.) worked out the values of the same functions from *x*=0 to 1 at intervals of •01 and from 1 to 7∙5 at intervals of Ί to 10 places. Two tracts by L. Stenberg, *Tabules Logarithmi Integralis* (Malmö, part i. 1861