posed object-glass can be tested as regards its optical conditions by “tracing a ray,” *i.e.,* calculating the point at which, after refrac­tion through the two lenses, the ray so traced will cut their common axis. For the analytical solution of this problem it curved surface, whieh practically is nearly spherical. But the actual differences between the curves which may be required in certain conditions for producing a perfect lens differ so slightly from true spherical surfaces that it is impossible by any previously designed mechanical process to predict whether the resulting figure will be that of a sphere or some other curve very nearly that of a sphere. The mathematician, therefore, who discusses the subject is compelled to adopt spherical curves as the basis of his calcula­tion. On this assumption we may then trace a ray rigidly through any supposed object-glass as follows. Let A, B, A', B' be respectively the points where the refracted ray produced would intersect the optical axis after refraction at the first, second, third, and fourth refracting surfaces respectively ; also let *a* be the first angle of in­cidence, *μ* and *μ'* the refractive indexes for the crown and flint lens respectively for a ray of the wave-length whose course is to be traced, *r* and *s* the first and second radii for the crown lens, r' and *s'* the first and second radii for the flint lens, *a*, *β,* *a*', *β'*, *a',* and *b'* auxiliary angles, *d* the thickness of the crown lens, *d'* the thickness of the flint lens, Δ the distance between the second and third surfaces. Then for the intersect after refraction at the first surface

sin α = 1/μ sin*a ;*

(A) = *a* - a ; A = r*∙*sin*a*/sin(A) + *r* ;

for the intersect after refraction at the second surface

sin*b* = [(A+*s* - *d*)/*s*]sin(A);

sin*β = μ∙*sin*b* ;

(B) = (A) + *β - b ;* B *= s*sin*β/*sin(B) - *s* ;

for the intersect after refraction at the third surface

sin*a*' = -(B - *r*' - Δ)[(sinB)/*r*'] ;

sin*a*' = 1/*μ'* sin*a*' ;

(A') = (B) + *a*' - *a*' ; A' = *r* - *r*' *∙* sin*a*/sin(A') *;*

for the intersect after refraction at the fourth surface

sin *b' =* - (A' + *s' - d'*)sin(A')/*s*' ;

sin *β' = μ'* sin*b*';

(B')=(A') + *b*'- *β' ; B' = -s* - *s*' *∙* sin*β' /*sin(B')

The computation is very much simplified when we consider the angle of incidence to be very small—*i*.*e*., the point of incidence very near the optical axis, viz.,

*r \_μ-1 . μs , 1*

A *μ ’ B~A-d+μ~1',*

r' - r' , M, ~ 1 . *s' \_ μ's' , ,* A' √(B-Δ)+ *μ' ’ B'~Ar^ + μ '1\**

By means of these formulæ we can compute B' (the point where a ray, entering the first surface of the object-glass, will intersect the optical axis) for any angle of incidence=*a*, when for a ray of that wave-length the indexes of refraction are known for the glass of which the lenses are composed, if the radii of curvature of the lenses are also known. The most perfect object-glass would be one in which the value of B' is the same for two rays of the two selected wave-lengths, through whatever portion of the object-glass they may pass. This, however, is a condition which cannot be mathematically satisfied with spherical surfaces. It is of course possible to find values of the four unknown quantities *r, s, r',* and *s'* such that four conditions shall be satisfied. The ordinary approxi­mate method is to find such values of the radii that B' is the same for rays of two different wave-lengths when the incident rays are near the axis, and for mean rays which enter near the margin of the lens ; but of course this solution is indeterminate, and only becomes rigid when two radii are assumed. Thus, for any crown lens of any radii of curvature it is possible to find a flint lens to satisfy these conditions. The rigid solution becomes one of suc­cessive approximation to such four conditions as the computer may is necessary to assume that the adjacent surfaces of the sup­posed infinitely numerous prisms form together some continuous consider most desirable. Herschel advocates satisfying the terms depending on the second power of the aberration, Klügel that the refrac­tions of the rays should be as small as possible ; or we may make it a condition that the second and third surfaces shall have the same radius, so that the surfaces may be cemented together. The fourth condition is of course the desired focal length. But for all practical purposes it is sufficient to have placed the reader in a position to test the optical condi­tions of any combinations that may be proposed, and to refer him to the works mentioned in the subjoined note@@1 ; for, in fact the construction of object-glasses on paper is of far higher interest as a mathematical exercise than as a practical matter. By a slight departure from the spherical figure — a departure so minute that there are no mechanical means sufficiently delicate to measure it with certainty—the optician may fail to realize true spherical surfaces, and thus on the one hand miss the fine definition which his calculation led him to expect, or on the other hand convert an object-glass which with spherical curves would have large spherical aberration into one perfectly corrected in this respect. Having, therefore, for particular kinds of glass ascertained a good general form of object-glass, it becomes only necessary for the optician to perform an approximate calculation of the curva­tures requisite to produce correction of the chromatic aberration, and to trust to the process of final figuring for correction of the final spherical and chromatic aberration. It fortunately happens that in the rigid equations the terms which express the thick­ness and distance apart of the lenses involve only the focal distances of central rays, and have but a small influence on the ratios of the aberrations of the lenses ; and, further, they affect chiefly the focal length of the lens, and have a very small influence on the chromatic aberration. Thus in the preliminary computa­tion the optician may neglect the thickness of the lenses and employ the simple approximate formulæ given under Optics, vol. xvii. p. 804—

gM 1 *bμ'* 1 μ-Γ∕%'-Γ∕ ’ \_i\_i 1 F~∕+∕"

*, δμ , δμ, , .*

where ancι 7√ ~~l^ι~~ are t'ιe dispersive powers of the two kinds of

glass for the two rays which he desires to unite, *f* and *f*' the cor­responding focal lengths of the two lenses, and F the focal length of the combination. The focal lengths of the two lenses which secure the conditions of achromatism having been thus computed, the radii of curvature may be computed for either lens by the usual formula (see Light, vol. xiv. p. 593)— b<-\*>(H)∙

In the last expression, where *r* and *s* correspond to the radii of curvature, the optician has an infinite range of choice. He will of course select such a proportion of *r* to *s* as experience or more elaborate calculation has shown to be favourable. In the form of object-glass recommended by Sir John Herschel, as fulfilling the most favourable conditions for correction of a spherical aberration for parallel as well as nearly parallel rays, the required curvatures for the exterior surfaces of the crown and the flint lens were found to vary very slightly for a considerable range of the ratio of the dispersive powers of the crown and the flint glass. Assuming *μ* (the mean index of refraction) to be 1∙542 for crown glass and V585 for flint glass, Herschel proved that, if the radii in question are taken to be 6∙72 for the crown lens and 14∙20 for the flint lens (supposing the focal length of the desired combination to be 10), we have only to compute the radii of the second and third surfaces

@@@1 Euler, *Dioptrica,* St Petersburg, 1767-71 ; Clairaut, *Mém. de l'Acad. Scien.,* 1757 ; D’Alembert, *0pusc*., vol. iii.; Lagrange, *Miscel. Taurin.,* iii. 2, p. 152, and *Mem. Acad. Berl.,* 1778 ; Schmidt, *Lehrbuch der analytischen Optik* ; Santini, *Teorica degli Strumenti Ottici* ; Klügel, in Gilbert’s *Ann. d. Physik,* xxxiv., 1810, pp. 265-275 and 276-291 ; Herschel, *Phil. Trans. Roy. Soc.,* 1821, pp. 222-267; Littrow, *Mem. R.A.S.* (London), vol. iii. pp. 235-255 ; Robinson, *Mechanical Philosophy,* art. “Telescope,” vol. iii. pp. 403-514; Gauss, “Ueber die achro­matischen Doppel-Objective," in Lindenau’s *Zeitschr.,* iv., 1817, pp. 345-351, and Gilbert’s *Ann. d. Physik,* lix. pp. 188-195 ; Gauss, in Louville’s *Journal,* 1856, i. pp. 9-43; Steinheil *Astron. Nach.,* xlviii., 1851, col. 225-228, liii., 1860, col. 305-306, and 1861, coh 269-270 ; A. Steinheil, *Ueber Berechnung optischer Con- structionen* ; Carl Steinheil, *Repertorium,* iii., 1867, pp. 430-440, and *München Akad. Sitz.,* 1867, ii. pp. 284-297 ; Steinheil (Carl A. and H. A.), *Göttingsche Nachrichten,* 1865, pp. 131-143, 211-214.