we know that in the 8th century B.c. there were obser­vatories in most of the large cities in the valley of the Euphrates, and that professional astronomers regularly took observations of the heavens, copies of which were sent to the king of Assyria; and from a cuneiform inscrip­tion found in the palace of Sennacherib at Nineveh, the text of which is given by George Smith,@@1 we learn that at that time the epochs of eclipses of both sun and moon were predicted as possible—probably by means of the cycle of 223 lunations or Chaldæan Saros—and that observations were made accordingly.

The wonderful fame of Thales amongst the ancients must have been in great part due to this achievement, which seems, moreover, to have been one of the chief causes that excited amongst the Hellenes the love of science which ever afterwards characterized them. Thales seems not to have left any writings behind him, though as to this there appears to be some doubt (see Diog. Laer., i. 23). Many anecdotes, amusing rather than instructive, are related of him, which have been handed down by Diogenes Laertius and other writers. From some of them it would appear that he was engaged in trade, which is indeed expressly stated by Plutarch (*Solon,* c. 2). It is probable that in the pursuit of commerce he was led to visit Egypt. Of. the fact that Thales visited Egypt, and there became acquainted with geometry, there is abun­dant evidence. Hieronymus of Rhodes (ap. Diog. Laer., i. 27) says, “ he never had any teacher except during the time when he went to Egypt and associated with the priests.” @@2

But the characteristic feature of the work of Thales was that to the knowledge thus acquired he added the capital creation of the geometry of lines, which was essentially abstract in its character. The only geometry known to the Egyptian priests was that of surfaces, together with a sketch of that of solids, a geometry consisting of some simple quadratures and elementary cubatures, which they had obtained empirically. Thales, on the other hand, intro­duced *abstract* geometry, the object of which is to establish precise relations between the different parts of a figure, so that some of them could be found by means of others in a manner strictly rigorous. This was a phenomenon quite new in the world, and due, in fact, to the abstract spirit of the Greeks.

The following discoveries in geometry are attributed to Thales :— (1) the circle is bisected by its diameter (Procl., *op. cit.,* p. 157) ; (2) the angles at the base of an isosceles triangle are equal (Id., p. 250) ; (3) when two straight lines cut each other the vertically opposite angles are equal (Id., p. 299) ; (4) the angle in a semicircle is a right angle ;@@3 (5) the theorem Euclid i. 26 is referred to Thales by Eudemus (Procl., *op. cit.,* p. 352). Two applications of geometry to the solution of practical problems are also attributed to him :— (1) the determination of the distance of a ship at sea, for which he made use of the last theorem ; (2) the determination of the height of a pyramid by means of the length of its shadow : according to Hieronymus of Rhodes (Diog. Laert., i. 27) and Pliny (*N*. *H*., xxxvi. 12), the shadow was measured at the hour of the day when a man’s shadow is the same length as himself. Plutarch, however, states the method in a form requiring the knowledge of Euclid vi. 4, but without the restriction as to the hour of the day (*Sept. Sap. Conviυ.,* 2). Further, we learn from Diogenes Laertius (i. 25) that he perfected the things relating to the scalene triangle and the theory of lines. Proclus, too, in his summary of the history of geometry before Euclid, which he probably derived from Eudemus of Rhodes, says that Thales, having visited Egypt, first brought the knowledge of geometry into Greece, that he discovered many

things himself, and communicated the beginnings of many to his successors, some of which he attempted in a more abstract manner *(κaθoλικώτeρov)* and some in a more intuitional or sensible manner *(aίσθητικώτερov*) *(op. cit.,* p. 65).

From these indications it is no doubt difficult to determine what Thales brought from Egypt and what was due to his own inven­tion. This difficulty has, however, been lessened since the transla­tion and publication of the papyrus Rhind by Eisenlohr;@@4 and it is now generally admitted that, in the distinction made in the last passage quoted above from Proclus, reference is made to the two forms of his work,—*aίσθηnκώreρov* pointing to what he derived from Egypt or arrived at in an Egyptian manner, while *κaθoλικώτεpov* indicates the discoveries which he made in accordance with the Greek spirit. To the former belong the theorems (1), (2), and (3), and to the latter especially the theorem (4), and also, probably, his solution of the two practical problems. We infer, then, [1] that Thales must have known the theorem that the sum of the three angles of a triangle are equal to two right angles. This inference is made from (4) taken along with (2). No doubt we are informed by Proclus, on the authority of Eudemus, that the theorem Euclid i. 32 was first proved in a general way by the Pytha­goreans (see Pythagoras, vol. xx. p. 140) ; but, on the other hand, we learn from Geminus that the ancient geometers observed the equality to two right angles in each kind of triangle—in the equilateral first, then in the isosceles, and lastly in the scalene (Apoll., *Conica,* ed. Halleius, p. 9), and it is plain that the geometers older than the Pythagoreans can be no other than Thales and his school. The theorem, then, seems to have been arrived at by induc­tion, and may have been suggested by the contemplation of floors or walls covered with tiles of the form of equilateral triangles, or squares, or hexagons. [2] We see also in the theorem (4) the first trace of the important conception of geometrical loci, which we, therefore, attribute to Thales. It is worth noticing that it was in this manner that this remarkable property of the circle, with which, in fact, abstract geometry was inaugurated, presented itself to the imagination of Dante :—

“ O se del mezzo cerchio far si puote

Triangol sì, ch’un retto non avesse."—*Par.,* c. xiii. 101.

[3] Thales discovered the theorem that the sides of equiangular triangles are proportional. The knowledge of this theorem is dis­tinctly attributed to Thales by Plutarch, and it was probably made use of also in his determination of the distance of a ship at sea.

Let us now consider the importance of the work of Thales.

1. In a scientific point of view : (*a*) we see, in the first place, that by his two theorems he founded the geometry of lines, which has ever since remained the principal part of geometry ; (*b*) he may, in the second place, be fairly considered to have laid the founda­tion of algebra, for his first theorem establishes an equation in the true sense of the word, while the second institutes a proportion.@@5
2. In a philosophic point of view : we see that in these two theorems of Thales the first type of a natural law, *i.e.,* the ex­pression of a fixed dependence between different quantities, or, in another form, the disentanglement of constancy in the midst of variety—has decisively arisen.@@6 III. Lastly, in a practical point of view : Thales furnished the first example of an application of theoretical geometry to practice,@@7 and laid the foundation of an important branch of the same—the measurement of heights and distances. For the further progress of geometry see Pythagoras.

As to the astronomical knowledge of Thales we have the follow­ing notices :—(1 ) besides the prediction of the solar eclipse, Eu­demus attributes to him the discovery that the circuit of the sun between the solstices is not always uniform;@@8 (2) he called the last day of the month the thirtieth (Diog. Laert., i. 24); (3) he divided the year into 365 days (Id., i. 27); (4) he determined the dia­meter of the sun to be the 720th part of the zodiac ;@@9 (5) he appears to have pointed out the constellation of the Lesser Bear to his countrymen, and instructed them to steer by it [as nearer the pole] instead of the Great Bear (Callimachus ap. Diog. Laert., i. 23 ; *cf.* Aratus, *Phaenomena,* v. 36 *sq.*)*.* Other discoveries in astronomy are attributed to Thales, but on authorities which are not trust­worthy. He did not know, for example, that “ the earth is spher­ical,” as is erroneously stated by Plutarch *(Placita,* iii. 10); on the contrary, he conceived it to be a flat disk, and in this supposition he was followed by most of his successors in the Ionian schools, including Anaxagoras. The doctrine of the sphericity of the earth,

@@@1 *Assyrian Discoveries,* p. 409.

@@@2 *Cf.* Pamphila and the spurious letter from Thales to Pherecydes, ap. Diog. Laer. ; Proclus, *In primum Euclidis Elementorum Librum Commentarii,* ed. Friedlein, p. 65 ; Pliny, *H. N.,* xxxvi. 12 ; Iam­blichus, *In Vit. Pythag.,* 12; Plutarch, *Sept. Sap. Conviv.,* 2, *De Iside,* 10, and *Plac.,* i. 3, 1.

@@@3 This is unquestionably the meaning of the statement of Pamphila (temp. Nero), ap. Diog. Laert., i. 24, that he was the first person to describe a right-angled triangle in a circle.

@@@4 *Ein mathematisches Handbuch der alten Aegypter,* Leipsic, 1877. @@@5 Auguste Comte, *Système de Politique Positive,* iii. pp. 297, 300.

@@@6 P. Laffitte, *Les Grands Types de l' Humanité,* vol. ii. p. 292.

@@@7 *Ibid.,* p. 294.

@@@8 Theonis Smyrnæi Platonici *Liber de Astronomia,* ed. Th. H. Mar­tin, p. 324, Paris, 1849. *Cf.* Diog. Laert., i. 24.

@@@9 This is the received interpretation of the passage in Diogenes Laertius, i. 24 (see Wolf, *Gesch. der Astron.,* p. 169), where *σεληvaίoυ* is probably a scribe’s error for *ζωδιακοΰ. Cf.* Apuleius, *Florida,* iv. 18, who attributes to Thales, then old, the discovery : “ quotiens sol mag­nitudine sua circulum quem permeat metiatur.”