ship acting vertically downwards through its centre of gravity and the resultant pressure of the water on the immersed hull.

If the ship be supposed removed and the cavity thus formed filled with water, then, since this volume of water is in equilibrium under the same system of fluid pres- sures, the resultant of these pressures must be equal and opposite to the weight of the water in the cavity and will therefore act vertically upwards through the centre of gravity of this portion of water. Defining the weight of water displaced by the ship as the *displacement,* and its centre of gravity as the *centre of buoyancy,* it is seen that the fundamental conditions for the equilibrium of a ship in still water are (*a*) that the weight of the ship must be equal to the displacement, and (*b*) that its centres of gravity and buoyancy must be in the same vertical line.

A floating ship is always subject to various external forces disturbing it from its position of equilibrium, and it is necessary to investigate the stability of such a position, *i.e.* to determine whether the ship, after receiving a small disturbance, will tend to return to its former position, in which case its equilibrium is termed *stable,* or whether, on the other hand, it will tend to move still farther from the original position, when the equilibrium is termed *unstable*. The intermediate case, when the ship tends to remain in its new position, is a third state of equilibrium, which is termed *neutral.*

Of the modes of disturbance possible, it is evident that a bodily movement of the ship in a horizontal direction or a rotation about a vertical axis will not affect the conditions of equilibrium; the equilibrium is also stable for vertical displacements of a ship. The remaining movements, viz. rotations about a horizontal axis, can be resolved into rotations in which the displacement is unaltered, and vertical displacements, the effect of the latter being considered separately. Of the various horizontal axes about which a ship can rotate two are of particular importance, viz. (1) an axis parallel to the longitudinal plane of symmetry, (2) an axis at right angles to this plane, both axes being so chosen that the displace­ment remains constant; the stability of a ship with reference to rotations about these axes is known as the *transverse stability* and the *longitudinal stability* respectively. In the following account the consideration of stability is confined at first to these two cases; the general case of rotation about any horizontal axis whatever being dealt with later.

Let fig. I represent a transverse section of a ship, WL being its water line when upright, and W'L' its water line when inclined to a small angle 0 as shown.

Assuming that the displacement is unaltered, if G be the position of the ship’s centre of gravity and B, B' the positions of its centre of buoyancy in the upright and inclined positions respectively, the forces acting on the ship con­sist of its weight W vertically downwards through G and the re­sultant water pressure equal to W acting verti­cally upwards through B'. These constitute a couple of moment WxGZ where Z is the foot of the perpendicular from G on to the vertical through B'; the direction of the couple as drawn in the figure is such as would cause the ship to return to its original position, *i.e.* the equi­librium is stable for the inclination shown.

If M be the intersection of the vertical through B' with the original vertical, the moment of the restoring couple is equal to W ×GM sin 0*,* and GM sin *θ* is termed the *righting lever.*

If, by moving weights on board, G be moved to a different position on the original vertical through B, the original position of the ship will remain one of equilibrium, but the moment of stability at the angle of inclination *θ* will vary with GM. If G be brought to the position G' above M the moment W ×G'Z' will tend to turn the ship away from the original position. It follows that the condition that the original position of equilibrium shall be stable for the given inclination is that the centre of gravity shall be below the intersection of the verticals through the upright and inclined centre of buoyancy; and the moment of stability is proportional to the distance between these two points.

When the inclination 0 is made smaller the point M approaches a definite position, which, in the limit when *θ* is indefinitely small, is termed the *metacentre.*

In ships of ordinary form it is found that for 10 to 15 degrees of inclination, the intersection of the verticals through the centres of buoyancy B and B’ remains sensibly at the metacentre M;. and therefore within these limits the moment of stability is approximately equal to W ×GM sin *θ.*

Since the angle on either side of the vertical within which a ship rolls in calm or moderate weather does not usually exceed the limit above stated, the stability and to a great extent the behaviour of a vessel in these circumstances are governed by the distance GM which is known as the *metacentric* height. The position of G can be calculated when the weights and positions of the component parts of the ship are known. This calculation is made for a new ship when the design is sufficiently advanced to enable these com­ponent weights and their positions to be de­termined with reason­able accuracy; in the initial stages of the design ah approximation to the vertical position of G is made by comparison with previous vessels.

The position of the centre of gravity of a ship is entirely inde- pendent of the form or draught of water, except so far as they affect the amount and distribution of the component weights of the ship. The position of the metacentre, on the other hand, depends only on the geometrical properties of the immersed part of the ship; and it is determined as follows:

Let WL, W’L' (fig. 2) be the traces of the upright and inclined water planes of a ship on the transverse plane; B, B, the corre- sponding position of the centre of buoyancy; 0 the angle of in­clination supposed indefinitely small in the limit, and S the intersection of WL and W'L'; join BB'.

By supposition the displacement is unchanged, and the volumes WAL, W'AL' are equal ; on subtracting W'AL it is seen that the two wedges WSW', LSL' are also equal. If *dx* represent an element of length at right angles.to the plane of the figure, *yu y‰* the half­breadths one on each side at any point in the original water line, so that WS = yb SL≡syjt the areas WSW', LSL' differ from ⅜yι2.0, *iyit.θ* by indefinitely small amounts, neglecting which the volumes of WSW', LSL' are equal to *fìy&dx and f⅛yilθdx.*

Since these are equal we have *⅛ Çy∖idx = ⅛Çyιtdx* or

*Le.* the moments of the two portions of the water plane about their line of intersection passing through S are equal. This line is also the axis of rotation, which therefore passes through the centre of gravity of the water plane. For vessels of the usual shape, having a middle line plane of symmetry and floating initially up­right, for small inclinations consecutive water planes intersect on the middle line.

Again if *gi, gι* are the centres of gravity of the wedges WSW', LSL7, and *v* the volume of either wedge, the moment of transference of the wedges v×gιg2 is equal to the moment of transference of the whole immersed volume V×BB' where V is the volume of displace­ment.

But υ×gιS = moment of wedge WSW' about *S≈⅛fyι\*.θ.dx,* and ti×Sg1 = moment of wedge LSL’ about S = J∕y√.0.dx. Adding, ¾∕(y1a÷y√)0∙dx = v×g1g2 = V×BB'. But BB' = BM.0 to the same order of accuracy, and *lf(yιi+y22) ∙dx* is the moment of inertia of the water plane about the axis of rotation; denoting the latter by I, it follows that BM = 1/V; *Le.* the height of the metacentre above the centre of buoyancy is equal to the moment of inertia of the water plane about the axis of rotation divided by the volume of displacement. These quantities, and also the position of the centre of buoyancy can be obtained by the approximate methods, of quadrature usual in ship calculations, and from them the position of the metacentre can be found.

If the ship is wholly immersed, or if the inertia of the water plane is negligible as in a submarine when diving, BM = 0, and the condi- tion for stability is that G should be below B ; the righting lever at any angle of inclination is then equal to BG sin 0.

During the process of design the position of the centre of gravity