that value it again becomes negative. In this case the stability is unstable at the upright position, and the ship will roll to an angle of 15° on either side where the equilibrium is stable. This peculiarity is not uncommon in merchant steamers at light draught. Ample stability at large angles and good range is provided in such cases by high freeboard; but, apart from any considerations of safety, water ballast is used to lower the centre of gravity to a sufficient extent to avoid excessive tenderness.

The properties of the loci of centres of buoyancy and of prometacentres were fully investigated by Dupin in 1822, including also the surfaces into which these curves develop when admit- ting inclinations about transverse and "skew ” axes. It has been shown that the tangent to the curve of buoyancy at any point is parallel to the corresponding water-line; and assuming that the ship is only free to turn in a plane perpendicular to the axis of inclination, the positions of equilibrium are found by drawing from the centre of gravity all possible normals to the buoyancy curve, or equally, all possible tangents to its evolute, the metacentric curve, since the condition to be satisfied is, that the centres of gravity and buoyancy shall lie in the same vertical. Again,

when the curve of statical stability crosses the axis, making an acute positive angle as at P in fig. 14, the values of GZ on either side of P are such as to tend to move the ship towards the position at P, and the equilibrium at P is stable. Similarly, when the curve crosses the axis "negatively,” as at the origin and Q, the equilibrium is unstable. Since the angle of intersection cannot be either positive or negative twice in succession, on considering rotation in one direction only, it follows that positions of stable and unstable equilibrium occur alternately and the total number of positions of equilibrium is even.

The radius of curvature of the curve of buoyancy is equal to I/V, and is always positive. The curve, therefore, has no re-entrant parts or cusps, is continuous and has no sudden changes in direction; parallel tangents (or normals) can be drawn through two points only (corresponding to inclinations separated by 180°), which property is shared by its evolute, the metacentric curve. On the other hand, the moment of inertia 1 varies continuously with the inclination, attaining maximum and minimum values alternately; and the metacentric curve, therefore, contains a series of cusps corresponding to the values of 1 when dl =≡o, which will generally occur at positions of symmetry *(e.g.* at 0° and 180°), near the angles at which the deck edge is immersed or emerged, and at about 90° and 270°.

The curves of buoyancy and flotation and the metacentric curve for H.M. troopship “ Serapis ” are shown with reference to the section of the ship in fig. 15, and on an enlarged scale for greater

clearness in fig. 16.@@1 It will be seen that the metacentric curve contains eight cusps, M1, M2, . . . M8. Assuming the ship to heel to starboard, M1 corresponds to the upright position, M2 to the immersion of the starboard topsides and emersion of the port bilge; M3 corresponds to 90° of heel, M4 to the complete immersion of the deck and the emersion of the starboard bilge. M5 corresponds to the bottom-up position and similarly for M6, M7 and M8. There are also 6 nodes, of which P and Q are on the middle line. By means of those curves, the effect of a rise or fall in the position of the ship’s centre of gravity can readily be traced. The positions of equilibrium correspond to the normals that can be drawn from G to the buoy- ancy curve, or equally to the tangents drawn to its evolute the meta­centric curve. For stable equilibrium G lies below M, *i.e.* generally between B and M; and for unstable equilibrium, similarly, B is between G and M. In the ship under consideration, G1 was the actual centre of gravity, and G1M1 corresponds to the upright position of stable equilibrium. As the vessel heels over, equilibrium (this time un­stable) is again reached at about 90°, and a third position (stable) is obtained when the vessel is bottom up, G1M5 being then the meta­centric height. A fourth (unstable) position is obtained at about 270°, after which the original position G1M1 is reached, the vessel having turned completely round. For this position of G1 therefore, there are four positions of equilibrium, two of which are stable and two unstable; and this is also true for all positions of G between M1 and M5.

If G lies at G4 between Ms and the point P, there are six positions of equilibrium, alternately stable and unstable. If G is below P as at G5, there are two positions of equilibrium of which the upright only is stable. A self-righting life-boat exactly corresponds to this condition, the vessel being capable of resting only in the original upright position. If G is above Q, on the other hand, as at G3, there are again only two positions of equilibrium, the vessel being unstable when upright. If G is at G2 there are again six positions of equilibrium ; the up­right position is unstable, but a stable position is reached at a certain angle on cither side. This phase is often realised in merchant ships when light, as already stated *(vide* fig. 14). When G is exactly upon one of the branches of the metacentric curve, the equilibrium is neutral; if it is at M1 the ship is stable for finite inclinations, and if at Q unstable; similarly for M5 (except that the neutral state is then reached at 180°) and for P.

ln all the above cases it will be observed that the positions of stable and unstable equilibrium arc equal in number and occur alternately. There are two exceptions:—

1. When the moment of inertia of the water plane changes abruptly so that the B curve receives a sudden change of curvature. This is possible with bodies of peculiar geometrical forms, and two positions of M then correspond to one position of the body; if G lies between them, the equilibrium is stable for inclinations in one direction and unstable for those in the opposite direction, and is then termed “ mixed.”

2. When the equilibrium is neutral, this condition may be regarded as the coincidence of two or more positions of equilibrium alternately stable and unstable. The ship may then be either stable, unstable or neutral for finite inclinations; in exceptional cases she may be stable in one direction and unstable in the other, resembling to some extent the condition of “ mixed equilibrium.”

Another curve whose properties were originally investigated by Dupin is the *curve of flotation* F1F2F3 . . . (fig. 15), which is the envelope of all the possible water-lines for the ship when inclined transversely at constant displacement. Since, as previously shown, consecutive water-planes intersect on a line passing through their

@@@1 The curves of buoyancy and flotation and the metacentric curve for various forms, including that of H.M.S. “Serapis,” were obtained by practical investigation by the writer in 1871. The results showed that Dupin's investigations, which were apparently purely theoretical, had not fully disclosed certain features of the curves, such as the cusps, &c.