represents the forced oscillations imposed on the ship by the passage of the series of waves during the time *t;* and the first and second terms,

Ci. sinψf-f-C2. cos ψt,

are the same as the free oscillations of the ship in still water.

Equation (5) indicates, therefore, that the ship performs oscillations as in still water, but has superposed on these a series of oscillations, governed by the wave-slope and the relation existing between the period of the ship and that of the wave. The equation shows that there will be innumerable phases, and of these three are worthy of notice.

(*a*) In the case in which the shin’s period T is equal to the semi- period T1 of the wave, equation (5) becomes indeterminate. The correct solution to equation (4) is then—

0 = Ci siny∕-f-C2cosψ∕-^ψΘ∕ cosψf. . . (6)

It is seen that at each successive wave crest and hollow the range of the oscillation is increased, so that the ship under these conditions would inevitably capsize but for the effect of the resistances and the departure from synchronism at large angles of roll.

T

(*b*) When ψ- = O, in which case the ship is assumed to be quick in her movements, or the period of the wave is infinitely long as compared with that of the ship, the equation (5) becomes—

0==Θι sin τS√, t 1

that is to say, the ship will behave very much as a thin flat board does on the surface of a wave, her masts being always perpendicular to the surface.

(c) If we choose the initial conditions in equation (5) so that the coefficients C1 and C2 are zero, then the equation will become—

*Θ≈Θ.*—íyjsin^-Z.

I\_T?

Since *0ι,* the slope of the wave, is equal to Θι sin ψ√, the ratio of the ship’s angle to the vertical to the angle that the normal to the wave-slope makes with the vertical, or *0∕0ι,*

= —-τp = constant.

I\_T?

That is to say, the ship forsakes her own period and takes up “forced” oscillations in the period of the wave. Under these conditions the ship’s masts will lean towards the wave-crest if T is greater than T1, and from the wave-crest if T is less than T1.

Froude in his first paper further showed how the successive angles of a ship’s rolling may be exhibited graphically, and he touched on the influence of resistance in reducing rolling. The following is the summary he gave in 1862 of the conclusions he had reached:

“ (i.) All ships having the same 'periodic time,’ or period of natural roll, when artificially put in motion in still water, will go through the same series of movements when subjected to the same series of waves, whether this stability in still water (one of the conditions which govern the periodic time) be due to breadth of beam, or to deeply stowed ballast, or to any such peculiarity of form as is in practical use.

“ This statement would be almost rigorously true if the oscillations were performed in a non-resisting medium, or if the surface- friction and keel-resistance, by which the medium operates to destroy motion, were of the same equivalent value for all the ships thus compared. It requires, however, to be modified in reference to the circumstance that of two ships having the same periodic time in still water, the comparative forms may be such that the one shall experience such resistance in a higher proportionate degree than the other, and the necessary modification may be expressed in terms of their relative behaviour when set in motion in still water. The vessel which is the more rapidly brought to rest by resistance in still water will in the greater degree resist the accumulations of angle imposed on her by consecutive wave-impulses, and will the more fall short of the maximum angle which both would alike attain if oscillating in a non-resisting medium.

“ (ii.) The condition which develops the largest angles of rolling is equality in the periodic times of the ship and of the waves; and this is true alike for all ships, whether their scale of resistance, as above referred to, be large or small.

“ (iii.) That ship will fare the best which, *caeteris paribus,* has the slowest periodic time.

“ (a) The waves which have a periodic time as slow as hers will have a greater length from crest to crest than those of quicker period ; and, on the whole, long waves are relatively less steep than short ones. Now it is the steepness of the waves in a wave-series, not their height simply, which governs the rate at which angles of rolling will accumulate in a given ship when exposed to it.

“ (*b*) Of two ships one of which has periodic time rather slower than the waves in a given ratio, the quicker ship will accumulate the larger angles.

“ (c) It will require a heavier or a more continued gale to rear waves which have the lengthened period.

“ (*d*) When the gale has continued so long that the largest waves have outgrown the period of the ship, she will not thereby have been released from the operation of waves having her own period, since the larger waves carry on their surface smaller waves of every intermediate period (this, at least, I believe to be the case).

“ (*e*) When the gale has ceased and the sea is going down, the slower the period of the ship the sooner she will be released from waves of as slow a period.

“ (iv.) There are two, and only two, methods of giving a slow period to a ship:

“ (*a*) By increasing her ‘moment of inertia,’ as by removing her weights as far as possible from her centre of gravity; an arrangement which for the most part can only be accomplished to a limited extent.

“ (*b*) By diminishing her stability under canvas. This can always be accomplished in the construction of a ship, and generally in her stowage, to any degree consistent with her performance of her regular duties, by simply raising her weights. Were we to raise these so high as to render her incapable of standing up against the action of the wind on her sails, the steepest waves would pass under her with- out putting her in motion.

“Thus the enormous weights carried by the armour-plated ships, extended laterally to the greatest possible distance from the centre of gravity, and raised high above it, serve in both respects to moderate, not to enhance, this tendency to roll; and when it is said that with the weights thus placed, and once put in motion, a ship 'must roll deep (deep, though easy),' it should be remembered that those very relations of force and momentum, which show how difficult it must be to check her motion when once it has been impressed on her, show also that it must be equally difficult to impart that motion to her in the first instance. The difficulty of starting her has a priority in point of time over the difficulty of stopping her, and prevents it from being felt by limiting the motion which would have called it into play.

“ (v.) The conditions which govern pitching may be noticed here, though they have not been discussed in the paper.

“ Were it possible, by concentrating her weights or by extending her plane of flotation, to give to the ship a period indefinitely quick for both longitudinal and transverse oscillations, as compared with that of such waves as are large enough to put her in motion, she would acquire no cumulative oscillation, but would float always conformably to the mean surface of the wave which passes under her.

“ But this condition, which is so unapproachable in practice in reference to transverse oscillations that the attempt to approach it will but develop the evils pointed out in (iii.), is of necessity so closely approached in practice in reference to’ longitudinal oscillations, that those evils can only be escaped by approaching it as closely as is possible. The plunging of a ship whose weights are extended far fore and aft is but an incipient development of those phases of oscillation which have their proper development in trans- verse motion only. The best that can be desired in reference to longitudinal motion is that the ship’s period, for longitudinal oscillation, shall be as quick as possible, and her position always as conform able as possible to the mean surface of the passing waves.

“ I have insisted here, more prominently than in the body of the paper, on the circumstance that a total loss of stability, using that word in the ordinary sense of power of carrying sail, implies the possession of absolute stability, as regards rolling motion due to wave-impulse, because it has been pointed out to me that the attention of readers should be more strongly directed to it, not indeed as representing a practically available possibility, but as serving best to force the mind, by contact with an extreme conclusion immediately deducible from the theory, to appreciate its funda- mental principles. And the proposition thus certainly furnishes a crucial test of whether the principles have been appreciated or not, and it supplies also a ready means of testing the theory by a crucial experiment. I must, in addition, express my own confident belief that any one who will try the experiment fairly will find the pro­position so fully verified that he will feel obliged to admit that the theory which leads to so paradoxical yet true a conclusion deserves at least a careful study. But the more practically useful aspect of the theory is that which presents to view the varying phases of cumulative oscillation which a ship tends to undergo when exposed to various types of wave-series; the phases depending on the relation which her natural period of rolling, when set in motion in still water, bears to the period of wave-recurrence, and on the maximum steep- ness of each individual wave of the series—phases, in fact, which she would actually undergo but for the effect of surface-friction and keel-resistance ; the nature and value of which conditions, as well as the nature and necessity of experiments for their determination, have been pretty fully dealt with in the body of the paper.

“ I will here only add a synoptical statement of the principal features of those phases, given in a rather more complete form than in that part of the paper which referred to them, though they are pretty fully exhibited by the diagrams.

“ By a ‘complete phase’ is meant that series of oscillations which the ship undergoes counting from the time when, for a moment, she is stationary and upright in a similar position, and is about to