If at any instant

*λ a a dθκ*

θΛ=-θc-θBand∙^≈-si-∙^

it follows that

(720a „ *(Pile <Pθa dtl - dp dp'*

whence the above three relations hold at the successive instants and consequently for all time. Hence the rolling of C differs from that of B in having the free oscillations of A in still water superposed upon it. If, therefore, it is possible to obtain any one motion in the swell, any other motion due to a different phase relation between ship and wave slope can be at once determined. A convenient motion in the swell to form a basis for obtaining other motions is the forced oscillation proper to the swell, *i.e.* the particular oscillation that is recurrent in the period of the swell. The amplitude of roll at any instant is therefore the sum of the amplitudes due to the forced oscillation and to an arbitrary free oscillation in still water. If the latter component be regarded as perfectly arbitrary there is no limit to the angle of roll obtained by postulating suitable initial conditions; to determine the practical limitation of rolling, however, it may reasonably be assumed that at or near the commencement of the motion there will be a brief period of no roll, and that the maximum angle of roll obtained will occur at no great interval of time after this period. At the instant when there is no roll, the forced and free oscillations are equal in magnitude and opposite in phase, and the period of maximum (termed the “ criterion ”) ampli­tude θc will occur as soon as the two components are in phase;

T1 the time interval between the two conditions is *n*T, where *n= =fc*~~ψ~~~~-~~~~1~~~~qγ~~ It is assumed also that during the above interval—(1) the effect of the swell was sensibly the same as that of a simple harmonic wave, A being the amplitude of the forced oscillation (and of the initial free oscillation); (2) the extinction equation of the free oscillation \_^2=tσθ4-⅛θ\* can be replaced by the simple form — j-==Eθ, where E≈α+⅛θc approximately; this has been implied by the absence of terms containing *(jføj 2* in the differential equation above. The amplitude of the free oscillation during the maximum roll is, from equation (8) *Ae~~nt\*t* whence

θc=A(1+<rn∙).

Also, from equation (9), the forced oscillation is given by \*-\*√(∙-15÷)O∙≡)\*

From these equations Θi can be determined if T, T1 *a*, *b* and Θc are given; conversely if θc is known, Θ1 can be tentatively obtained.

The following table gives the criterion «angle (Θc) and the angle of steady roll (A) for the “ Revenge,” both without and with bilge keels, obtained on the above-mentioned assumptions:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Maximum waνe-Sloρc, 3 Degrees.** | | | | | | | | |
|  |  | | **T**  **Ti"1'2∙ n-5.** | | **τ**  **Ti"11\* n->ιo.** | | **T**  **TΓ,∙°∙ *n* ∙oo.** | |
| **Criterion Angle.** | **Angle of**  **Steady Roll.** | **Criterion Angle.** | **Angle of**  **Steady Rott.** | **Criterion Angle.** | **Angle of**  **Steady Roll.** | **Criterion Angle.** | **Angle of**  **Steady RolI.** |
| "Revenge’’ (deep draught), with no bilge keels  “Revenge’’ (deep draught), with bilge keels | **deg.**  **8∙25**  **6∙6** | **deg.**  **4∙35**  **4’24** | **deg.**  **I2∙25**  **8∙6** | **dcg.**  **6∙8**  **6.4** | **deg.**  **2I∙2**  **ιx∙55** | **(leg.**  **13\*9**  **ιo∙8** | **deg.**  **41∙I**  **14∙85** | **deg.**  **4I∙1**  **14∙8j** |

Among the conclusions reached by Mr R. E. Froude in the case of a ship rolling in a uniform swell were;

However non-uniform initially, the rolling ultimately falls into the uniform forced oscillation; it does so the sooner, *caeteris paribus,* the higher the resistance, and with the fewer “cycles ” or alterations of amplitude of rolling, the more nearly syn- chronous the swell with the ship. The amplitude of the ultimate uniform rolling is an approximate mean of the alternate maxima and minima of the precedent non-uniform rolling. If the rolling starts from zero, the maximum ampli­tude falls short of twice the ultimate uniform amplitude, the more so the higher the resistance and the more synchronous the swell; and in a synchronous swell the maximum amplitude cannot exceed the ultimate uniform amplitude, unless it does so initially.

In two papers by Captain and Professor Kriloff of St Petersburg, read before the I.N.A. in 1896 and 1898, the whole motion of the ship, including pitching and rolling, is dealt with; every variation which can reasonably be conceived is taken into account in these papers.

Of the various appliances adopted to reduce rolling, the most important and successful are bilge keels. Some reference has already been made to the influence they exert on the rolling of ships, as illustrated by H.M.S. “ Revenge,” in which there was one bilge keel on each side, 200 ft. in length and 3 ft. in depth, tapered at the extreme ends. The great value of bilge keels in diminishing rolling was pointed out by Froude and demonstrated by him in 1872 by experiment with the "Perseus” and the “ Greyhound,” which were alike in every essential respect, except that the former was not provided with bilge keels and the latter was. The general conclusion was that the rolling of the “ Greyhound,” was only about one-half that of the “ Perseus.”

Bilge keels were usual in warships until, in the design of the “ Royal Sovereign ” class, it was decided not to fit them, owing to the large dimensions of the vessels and the difficulties in certain circumstances of docking them if provided with bilge keels. Ultimately one of the class, the “ Repulse,” had them fitted for purposes of comparison, and the effect on her rolling was so marked that it was resolved to fit them to all the ships of the class. Before fitting them on the “ Revenge,” a careful programme was drawn up of experiments to be made before and after the bilge keels were fitted; and on carrying out this programme some valuable results were obtained. The experiments were made at Spithead in smooth water, the general effect of the bilge keels was to reduce the rolling to one-third of its former amount. When, instead of having no motion in the line ahead, the ship had a speed of 12 knots, an even greater reduction in the rolling was observed. Their effect on other qualities of ships is on the whole beneficial, and in general little, if any, reduction in speed has resulted from their use. The experience of Great Britain with regard to bilge keels has been repeated in America. Bilge keels were omitted for the same reasons as they were in the “ Royal Sovereign ’’class; they were afterwards fitted in the U.S.S. “ Oregon,” experimental investigation being made both without and with them, and the general conclusion arrived at was that the rolling was diminished by two-thirds by the adoption of the bilge keels.

A method for reducing rolling of ships in a sea-way by the use of water-chambers was devised by the writer in 1874 in connexion with the design of the “ Inflexible,” which was expected to be a bad roller. It consists in fitting one or more tanks across the ship of such shape that when filled to a suitable height with water the motion of the water from side to side as the vessel rolls is such as to retard the rolling. Let fig. 30 represent a series of transverse sections of a ship fitted with a water-chamber, in various positions in rolling from port to starboard; and suppose the water to move so as to be most effective in quelling rolling. Let G represent the centre of gravity of the ship including the water in the chamber, g the centre of gravity of the water in the chamber, and B the centre of buoyancy of the ship; and let the arrows over the sections indicate the direction in which the ship is rolling at the instant considered. In position No. 1 suppose the ship to have reached the extreme heel to port and to be on the point of commencing the return roll, then *g* should have reached the middle line on its way down towards the port side and the righting couple will be that due to the angle of heel, supposing the water to be a fixed weight amidships. In the position No. 2 the ship has performed part of the roll back towards the upright; the water will have moved farther down the incline, so that *g* will be some distance from the middle line on the port side as shown, and therefore *G* will also have moved out from the middle line on the port side; hence the righting couple will be less than what would correspond to the angle of heel if the water were a fixed weight amidships. In position No. 3 the ship has just reached the upright and will be moving with the maximum angular velocity; the water will have moved still farther down the incline, and *g* will be at a greater distance from the middle line on the port side, and therefore G will have moved farther out from the middle line, whereas B will have returned to the middle line; so that the weight of the ship and the upward pressure of the water will form a couple tending to retard the ship’s rotation, although she is for the moment in the upright position. In the position No. 4 the ship is heeling over to starboard and the centre of gravity of the water is returning towards the middle line; but it and G are still on the port side, and the righting couple is therefore greater than that corresponding to the angle of heel of the ship and a fixed centre of gravity amidships. In the position No. 5 the ship has momentarily