By impressing, as above, a suitable velocity on the whole system of ship and water, the problem is reduced to one of steady motion in a stream flowing past a stationary ship. The stream tubes, originally of uniform width, become broader on approaching the bow of the ship, and attain their greatest breadth close to the stem. Proceeding aft, the tubes contract, and near amidships they become smaller than they were originally ; an enlargement in the tubes again takes place near the stern. The changes in size and velocity in the stream tubes lead to corresponding alterations of pressure in accordance with the energy equation, which alterations appear as elevations and depressions of the surface forming what is termed the statical wave system. If this were a permanent system, no resistance to the motion of the ship would be caused thereby. The surface disturbance, however, is subject to the dynamical laws underlying the propagation of waves; in consequence the wave formation differs from the “ statical wave,” the crest lagging astern of the “ statical ” wave crest, and the ship) being followed by a train of waves whose lengths are appropriate to the speed attained. The energy within the wave system travels backward relative to the ship at one-half its speed; the resistance experienced by the ship is due to the sternward drain of the wave energy which requires work to be done on the ship to replace that absorbed by the waves.

The form of the wave system is not susceptible of complete mathematical investigation ; but the circumstances are approximately realized and the conditions considerably simplified when the actions of the bow and stern of the vessel are each replaced by the mathematical conception of a “ pressure point.” This consists of an infinitely large pressure applied over an indefinitely small region of the water surface ; it is assumed to move forward in place of the ship through still water, or, equally, to be stationary in a uniform stream. The resulting wave system has been investigated by Lord Kelvin and others. It is found to consist of a local disturbance surrounding the pressure point and depending on the pressure distribution combined with a series of waves which are confined within two straight lines drawn backwards through the pressure point and making angles of about 20° (tan~1J^≈) with the line of motion. The waves within this region extend indefinitely astern with crests crossing the line of motion perpendicularly. The crest lines are slightly curved, convex to the pressure point, and at the bounding lines form cusps whose tangents are inclined to the line of flow at an angle of about 36° (tan^1^=) . The crest lines afterwards curve forward towards the pressure point. The distance apart of the transverse wave crests is equal to the length *l* of wave appropriate to the speed *v,* as expressed in the formula ν2 = g∕∕2x. These results are of interest since they are in agreement in many respects with those of actual observation for ships and models. In fig. 32, reproduced from a paper in the *LN.A.* 1877, read by Froude, is shown the bow-wave system obtained from a model, which is also illustrative of that produced by ships of all types. It appears therefore that two types of waves accompany a ship—(1) diverging waves having sharply defined crests placed in echelon, the foremost wave alone extending to the ship; (2) transverse waves limited in breadth by the diverging crests and reaching the sides of the vessel through­out its length. These compare with the crest lines obtained in the above hydrodynamical investigation ; the transverse and diverging waves correspond to the different portions of the crest lines which are separated by the cusps.

Since the bow diverging waves are not in contact with the ship except at the bow, the energy spent in their maintenance travels away from the ship and is lost. A diverging wave system of similar form but of smaller dimensions attends the passage of the stern; and the resistance due to the diverging systems of waves is therefore the sum of its components at the bow and stern, following a regular although unknown law, increasing with the speed, and depending considerably on the shape of the bow and stern.

On the other hand the interference between the transverse bow and stern wave systems produces a stern wave in contact with the ship; the resistance due to the resultant transverse wave system depends therefore on the phase relation between the waves of the component systems. The effect of interference on the wave resistance was investigated by Froude *(Trans, I.N.A.* 1877) by means of experiments on a series of models having the same entrance and run, but in which the length of parallel middle body was varied. At constant speed curves of residuary resistance on a length base con­sisted of humps and hollows, whose spacing was constant and approximately equal to the wave length appropriate to the speed ; the amplitude of the fluctuation diminished as the length increased. For a given length the residuary resistance in general increased at a high power of the speed ; but it was also subject to a series of fluctuations whose magnitude and spacing increased with the speed. The results of these experiments were fully analysed in 1881 by Mr R. E. Froude, who showed that a reduction in the resistance occurred when the trough of the bow wave coincided with the crest of the component stern wave, the resultant wave system being of relatively small dimen- sions. Conversely, the resistance was abnormally increased when the crests of the bow and stern systems coincided. The fluctuation in the resistance thereby obtained was smaller when the length of middle body became greater, owing to the greater degradation of the bow wave system at the stern through viscosity and lateral spreading. For very considerable lengths of middle body, the height of the bow wave system at the stem was insufficient to produce interference or affect the resistance.

The speed in knots (V) of a wave is related to the length in feet (*l*) by the formula V2 = 1∙8*l*. If L' be the distance apart of the com­ponent bow and stern waves (which is generally rather greater than the length of the ship), relatively small resistance would be antici­pated when V2 is approximately equal to 3∙6 L' or any odd sub­multiple of 3∙6 L'; on the other hand when V2 was not greatly different from 1∙8 L', or any submultiple of 1∙8 L', abnormal wave resistance would be developed. This result is to a great extent con­firmed by experience with ships of all classes; for economical pro­pulsion at a speed V, the length L of a ship should be generally equal to or slightly less than V2, corresponding to the “ favourable ”

V2

value of about 1∙2 of the ratio ∙p ; torpedo-boat destroyers and similar vessels of extremely high speed constitute an exception, the V2

value of the ratio -j- being then frequently as great as 4, which ap- V2 proximately coincides with the highest “favourable” value of ∣y.

The foregoing description of the resistance experienced by ships through wave making makes it evident that the conditions under­lying wave resistance are too complex to enable its amount to be directly estimated as is possible in the case of frictional resistance. Experiments also show that there is no simple law connecting wave resistance with size, form or speed. The effect of size alone, *i.e.* the scale of the experiment, can, however, be eliminated by means of the “ principle of similitude ” enunciated by Newton, which is applicable with certain limitations to all dynamical systems. The extension of this principle forms the foundation of all methods employed practically for estimating the residuary resistance and horse power of ships. The principle states that in two geometrically and mechanically similar systems, whose linear dimensions vary as the squares of the velocities of the corresponding particles, and whose forces vary as their masses, the motions of the two systems will be similar. A proof of this theorem follows at once from the equations of motion for any particle. The law of comparison, which is the application (originally made by Froude) of the principle of similitude to the resistance of ships, is enunciated as follows:

“It the linear dimensions of a ship be *n* times those of its model, and the resistances of the latter be R1 R2, R3, . . . at speeds V1, V2, V3, . . . , then the resistances of the ship at the ‘corresponding speeds’ V1 √n, V2 √n, V3 √n, ... will be R1n3, R2n3, R3n3,... and therefore the effective horse powers at corresponding speeds are increased in the ratio n’: 1.”

It is necessary to ensure that the conditions underlying the principle of similitude are satisfied by all the components of resistance, when the law of comparison is employed for the purpose of obtaining the ratio between the total resistances of two ships at corresponding speeds. Residuary resistance, consisting of that caused by wave making, eddies, and air resistance, is attributable to normal pressures on various surfaces caused by changes of velocity in the water or air. It appears from Bernoulli’s energy equation that the pressures per unit area are proportional to the square of the velocity, *i.e.* at corre­sponding speeds, to the linear dimensions. The total pressures are therefore proportional to the cube of the linear dimensions, *Le.* to the masses, thus complying with the primary condition regarding the force ratios. Frictional resistance, which varies with the length of surface and as the 1∙83 power of the speed, does not satisfy this condition. In the application of the law of comparison to ships and