we discuss methods of determining the frequencies of sources, prove conclusively that for a given note the frequency is the same whatever the source of that note, and that the ratio of the frequencies of two notes forming a given musical interval is the same in whatever part of the musical range the two notes are situated. Here it is sufficient to say that the frequencies of a note, its major third, its fifth and its octave, are in the ratios of 4 : 5 : 6 : 8.

The *quality* or *timbre* of sound, *i.e.* that which differentiates a note sounded on one instrument from the same note on another instrument, depends neither on amplitude nor on frequency or wave-length. We can only conclude that it depends on wave form, a conclusion fully borne out by investigation. The dis­placement curve of the waves from a tuning-fork on its resonance box, or from the human voice sounding *oo,* are nearly smooth and symmetrical, as in fig. 7*a*. That for the air waves from a violin are probably nearly as in fig. 7*b*.

*Calculation of the Velocity of Sound Waves in Air.—*The velocity with which waves of longitudinal disturbance travel in air or in any other fluid can be calculated from the resistance to com­pression and extension and the density of the fluid. It is con­venient to give this calculation before proceeding to describe the experimental determination of the velocity in air, in other gases and in water, since the calculation serves to some extent as a guide in conducting and interpreting the observations.

The waves from a source surrounded by a uniform medium at rest spread out as spheres with the source as centre. If we take one of these spheres a distance from the source very great as compared with a single wave-length, and draw a radius to a point on the sphere, then for some little way round that point the sphere may be regarded as a plane perpendicular to the radius or the line of propagation. Every particle in the plane will have the same displacement and the same velocity, and these will be perpendicular to the plane and parallel to the line of propagation. The waves for some little distance on each side of the plane will be practically of the same size. In fact, we may neglect the divergence, and may regard them as “ plane waves.”

We shall investigate the velocity of such plane waves by a method which is only a slight modification of a method given by W. J. Μ. Rankine (*Phil. Trans.,* 1870, p. 277).

Whatever the form of a wave, we could always force it to travel on with that form unchanged, and with any velocity we chose, if we could apply any "external ” force we liked to each particle, in addition to the “ internal ” force called into play by the com­pressions or extensions. For instance, if we have a wave with displacement curve of form ABC (fig. 8), and we require it to travel on in time *dt* to A'B'C', where AA' = Udt the displacement of the particle originally at M must change from PM to P'M or by PP'. This change can always be effected if we can apply whatever force may be needed to produce it.

We shall investigate the external force needed to make a train of plane waves travel on unchanged in form with velocity U.

We shall regard the external force as applied in the form of a pressure X per square centimetre parallel to the line of propagation and varied from point to point as required in order to make the dis­turbance travel on unchanged in form with the specified velocity U. In addition there will be the internal force due to the change in volume, and consequent change in pressure, from point to point.

Suppose that the whole of the medium is moved *backwards* in space along the line of propagation so that the undisturbed portions travel with the velocity U. The disturbance, or the train of waves, is. then fixed in space, though fresh matter continually enters the disturbed region at one end, undergoes the disturbance, and then leaves it at the other end.

Let A (fig. 9) be a point fixed in space in the disturbed region, B a fixed point where the medium is not yet disturbed, the medium moving through A and B from right to left. Since the condition of the medium between A and B remains constant, even though the matter is continually changing, the momentum possessed by the matter between A and B is constant. Therefore the momentum entering through a square centimetre at B per second is equal to the momentum leaving through a square centimetre at A. Now the transfer of momentum across a surface occurs in two ways, firstly by the carriage of moving matter through the surface, and secondly by the force acting between the matter on one side of the surface and the matter on the other side. U cubic centimetres move in per second at B, and if the density is po the mass moving in through a square centimetre is po U. But it has velocity U, and therefore momentum poU2 is carried in. In addition there is a pressure between the layers of the medium, and if this pressure in the undisturbed parts of the medium is P, momentum P per second is being transferred from right to left across each square centimetre. Hence the matter moving in is receiving on this account P per second from the matter to the right of it. The total momentum moving in at B is therefore P+poU2. Now consider the momentum leaving at A. If the velocity of a particle at A relative to the undisturbed parts is *u* from left to right, the velocity of the matter moving out at A is. U — *u,* and the momentum carried out by the moving matter is p(U—*u*)2. But the matter to the right of A is also receiving momentum from the matter to the left of it at the rate indicated by the force across A. Let the excess of pressure due to change of volume be ω, so that the total " internal ” pressure is P+ω. There is also the “ external ” applied pressure X, and the total momentum flowing out per second is

X+P÷ω÷p(U-w)2.

Equating this to the momentum entering at B and subtracting P from each

, X+ω-∣-p(U-tt)2 = poU2. (4)

If *y* is the displacement at A, and if E is the elasticity, substituting for ώ and *u* from (2) and (3) we get

x-e½+"u2G+⅛),=^u,∙

Rut since the volume *dx* with density po has become volume *dx +dy* with density p

P(l+È) =p0∙

Then · X-Eg+poU=(ι+g)=poU∖ .

or X = (E-p0U2)dy∕dx. (5)

If then we apply a pressure X given by (5) at every point, and move the medium with any uniform velocity I , the disturbance remains fixed in space. Or if we now keep the undisturbed parts of the medium fixed, the disturbance travels on with velocity U if we apply the pressure X at.every point of the disturbance.

If the velocity U is so chosen that.E — pt>U2=o, then X = o, or the wave travels on through the action of the internal forces only, unchanged in form and w íth velocity

U = √(E∕p). (6)

The pressure X is introduced in order to show that a wave can be propagated unchanged in form. If w,e omitted it we should have to assume this, and equation (6) would give us the velocity of propagation if the assumption were justified. But a priori we are hardly justified in assuming that, waves can be propagated at all, and certainly not justified in assuming that they go on unchanged by the action of the internal forces alone.. If, however, we put on external forces of the required type X it is obvious that any wave can be propagated with any velocity, and our investigation shows that vhen U has the value in (6) then and. only then X is zero everywhere, and the wave will be propagated with that velocity when once set going.

It may be noted that the elasticity E is only constant for small volume changes or for small values of *dyjdx.*

Since by definition E = *-v{dp∣dv) = p{dp∣dp)* equation (6) becomes

U = √(⅛√⅛). . (7)

The value U = √(E∕p) was first virtually obtained by Newton *{Principia,* bk. ii., § 8, props. 48-49). He supposed that in air Boyle’s law holds in the extensions and compressions, or that *p.= kp,* whence *dp∣dp\*=k=p∣p.* His Value of the velocity in air is therefore

U = √ *(p∣p)* (Newton’s formula).

At the standard pressure of 76 cm. of mercury or 1,014,000 dynes ∕ sq. cm., tne density of dry air at oo C. being taken as 0∙001293, we get for the velocity in dry air at oo C.

Up≡28,000 cm.sec. (about 920 ft./sec.)