approximately. Newton found 979 ft./sec. But, as we shall see, all the determinations give a value of Uo in the neighbourhood of 33,000 cm./sec., or about 1080 ft./sec. This discrepancy was not ex­plained till 1816, when Laplace *(Ann. dechimie,* 1816, vol. iii.) pointed out that the compressions and extensions in sound waves in air alternate so rapidly that there is no time for the temperature inequalities produced by them to spread. That is to say, instead of using Boyle’s law, which supposes that the pressure changes so exceedingly slowly that conduction keeps the temperature constant, we must use the adiabatic relation *p=kpy,* whence

*dp∣dp = ykpr1 = yp∕p,* and U = √(7p∕p) [Laplace’s formula]. (8)

If we take 7 = 1∙4 we obtain approximately for the velocity in dry air at 0° C,

Uo=33,150 cm./sec., which is closely in accordance with observation. Indeed Sir G. G. Stokes *(Math, and Phys. Papers,* iii. 142) showed that a very small departure from the adiabatic condition would lead to a stifling of the sound quite out of accord with observation.

If we put p = kp(1+at) in (8) we get the velocity in a gas at

Ui = √[γ⅛(i+α∕)∣.

At oo Cw we have Uo=√ (γ⅛), and hence

Ui = U0√(l+α∕)

= U0(ι÷o∙ooι84∕) (for small values of *t).* (9)

The velocity then should be independent of the barometric pressure, a result confirmed by observation.

For two different gases with the same value of 7, but with densi­ties at the same pressure and temperature respectively pL and p≡, we should have

U1∕Ua = √(p2∕p1), (10)

another result confirmed by observation.

*Alteration of Form of the Waves when Pressure Changes are Con­siderable.—*When the value of *dyfdx* is not very small E is no longer constant, but is rather greater in compression and rather less in extension than 7P. This can be seen by considering that the relation between *p* and p is given by a curve and not by a straight line. The consequence is that the compression travels rather faster, and the extension rather slower, than at the speed found above.

We may get some idea of the effect by supposing that for a short time the change in form is negligible. In the momentum equation (4) we may now omit X and it becomes

ω+p(U- tt)i = p0U2.

Let us seek a more exact value for ω. If when P changes to P⅛ω volume V changes to V—*v* then (P⅛ω)(V-v)v = PV>r, whence ω=P (τ7+~~τ~~~~^^Ι~~pf) =τγ (l +⅛w) ·

We have U—m = U(i — w∕U) =U(ι- v∕V), since *ufU = -dy∣dx≈v∣V.* Also since p(V-v)=poV, or p = po∕(ι-ι√V), then p(U-u)2 = Vp0U1(ι -v∕V).

Substituting in the momentum equation, we obtain 7V (1÷'5⅛'⅛) +\*,°u2 (1~γ∙) =p°u\*> whence u≡=⅞ (1+2⅜i⅜) ‘

If U = √(γP∕po) is the velocity for small disturbances, we may put Uo for U in the small term on the right, and we have v-u∙(.+⅛⅛)

or U = Uo+⅛(γ÷ι>. (íi)

This investigation is obviously not exact, for it assumes that the form is unchanged, ï.e. that the momentum issuing from A (fig. 9) is equal to that entering at B, an assumption no longer tenable when the form changes. But for very small times the assumption may perhaps be made, and the result at least shows the way in which the velocity is affected by the addition of a small term depending on and changing sign with *u.* It implies that the different parts of a wave move on at different rates, so that its form must change. As we obtained the result on the supposition of unchanged form, we can of course only apply it for such short lengths and such short times that the part dealt with does not appreciably alter. We see at once that, where w=o, the velocity has its “ normal ” value, while where *u* is positive the velocity is in excess, and where *u* is negative the velocity is in defect of the normal value. If, then, *a* (fig. 10) represents the displacement curve of a train of waves, *b* will represent the pressure excess and particle velocity, and from (11) we see that while the nodal conditions of *b,* with ω=o and *u*=o, travel with velocity √(E∕p), the crests exceed that velocity by ¼(y+1)*u* and the hollows fall short of it by ¼(y+1)*u*, with the result that the fronts of the pressure waves become steeper and steeper, and the train *b* changes into something like c. If the steepness gets very great our investigation ceases to apply, and neither experiment nor theory has yet shown what happens. Probably there is a breakdown of the wave somewhat like the breaking of a water-wave when the crest gains on the next trough. In ordinary sound-waves the effect of the particle velocity in affecting the velocity of transmission must be very small.

Experiments, referred to later, have been made to find the amplitude of swing of the air particles in organ pipes. Thus Mach found an amplitude 0∙2 cm. when the issuing waves were 250 cm. long. The amplitude in the pipe was certainly much greater than in the issuing waves. Let us take the latter as 0.1 mm. in the waves—a very extreme value. The maximum particle velocity is *2πna* (where *n* is the frequency and *a* the amplitude), or 2τrαU∕λ. This gives maximum w=about 8 cm.∕sec., which would not seriously change the form of the wave in a few wave­lengths. Meanwhile the waves are spreading out and the value of *u* is falling in inverse proportion to the distance from the source, so that very soon its effect must become negligible.

In loud sounds, such as a peal of thunder from a near flash, or the report of a gun, the effect may be considerable, and the rumble of the thunder and the prolonged boom of the gun may perhaps be in part due to the breakdown of the wave when the crest of maximum pressure has moved up to the front, though it is probably due in part also to echo from the surfaces of heterogeneous masses of air. But there is no doubt that with very loud explosive sounds the normal velocity is quite considerably exceeded. Thus Regnault in his classical experiments (described below) found that the velocity of the\* report of a pistol carried through a pipe diminished with the intensity, and his results have been confirmed by J. Violle and T. Vautier (see below). W. W. Jacques *(Phil. Mag.,* 1879, 7, p. 219) investigated the transmission of a report from a cannon in different directions; he found that it rose to a maximum of 1267 ft./sec. at 70, to 90 ft. in the rear and then fell off.

A very curious observation is recorded by the Rev. G. Fisher in an appendix to Captain Parry’s *Journal of a Second Voyage to the Arctic Regions.* In describing experiments on the velocity of sound he states that “ on one day and one day only, February 9, 1822, the officer’s word of command 'fire ’ was several times heard distinctly both by Captain Parry and myself about one beat of the chronometer [nearly half a second] *after* the report of the gun.” This is hardly to be explained by equation (11), for at the very front of the disturbance *u*=o and the velocity should be normal.

*The Energy in a Wave Train.,*—The energy in a train of waves carried forward with the waves is partly strain or potential energy due to change of volume of the air, partly kinetic energy due to the motion of the air as the waves pass. We shall show that if we sum these up for a whole wave the potential energy is equal to the kinetic energy.

The kinetic energy per cubic centimetre is ½*pu*2, where *p* is the density and *u* is the velocity of disturbance due to the passage of the wave. If V is the undisturbed volume of a small portion of the air at the undisturbed pressure P, and if it becomes V—*v* when the pressure increases to P+, the average pressure during the change may be taken as P-∏ωt since the pressure excess for a small change is proportional to the change. Hence the work done on the air is (P+iω)v, and the work done per cubic centi­metre is (P⅛⅜ω)ι√V. The term Pü/V added up for a complete wave vanishes, for P/V is constant and ∑v=o, since on the whole the compression equals the extension. We have then only to con­sider the term ⅛ωt>∕v.

But ι√V ≈tt∕U from equation (2) -⅝

and ω =E«/U from equation (3)

Then ⅞ωv∕V =JEw2∕U2 = ipu2 from equation (6)

Then în the whole wave the potential energy equals the kinetic energy and the total energy in a complete wave in a column 1 sq. cm. cross-section is W=J^° *puidx.*