We may find here the value of this when we have a train of waves in which the displacement is represented by a sine curve of amplitude *a,* viz. *y = a* sin U∕). For a discussion of this type of wave,

see below.

We have n==^= cos y^(x-Uí),

and *putdx =* αθcos2^-(x- Ut)dx

= 2pτr2U2α2∕λ (12)

The energy per cubic centimetre on the average is

2p1r2U2α2∕λ2 (13)

and the. energy passing per second through 1 sq. cm. perpendicular to the line of propagation is

2pτr2U¼2∕λ2 (14)

*The Pressure of Sound Waves,—*Sound waves, like light waves, exercise a small pressure against any surface upon which they im­pinge. The existence of this pressure has been demonstrated experimentally by W. Altberg *{Ann. der Physik,* 1903, 11, p. 405). A small circular disk at one end of a torsion arm formed part of a solid wall, but was free to move through a hole in the wall slightly larger than the disk. When intense sound waves impinged on the wall, the disk moved back through the hole, and by an amount showing a pressure of the order given by the following investigation.—

Suppose that a train of waves is incident normally on the surface S (fig. n), and that they are absorbed there without reflection. Let ABCD be a column of air I sq. cm. cross-section. The pressure on CD is equal, to the λ C momentum which it receives per

a∣ " second. On the whole the air

S within ABCD neither gains nor b, — loses momentum, so that on the

u whole it receives as much through AB as it gives up to CD. Fig. ii. If P is the undisturbed pressure

and P⅛ω the pressure at AB, the momentum entering through AB per second isy^1(P+ω+pu2)d∕. ButJ^Pd∕ = P is the normal pressure, and as we only wish to find the excess we may leave this out of account.

The excess pressure on CD is therefore *f^{ω* + pu2)d∕. But the values of ω+pw2 which occur successively during the second at AB exist simultaneously at the beginning of the second\* over the distance U behind AB. Or if the conditions along this distance U could be maintained constant, and we could travel back along it uniformly in one second, we should meet all the conditions actually arriving at AB and at the same intervals. If then <⅛ is an element of the path, putting *dt≈dξ∣∖J,* we have the average excess of pressure

P =J^tø÷pu2)«// = ⅛jy (ω+pu2)<⅛.

Here *dζ* is an actual length in the disturbance. We have *ω* and *u* expressed in terms of the original length *dx* and the displacement *dy* so that we must put *dξ=dx+dy = {i+dy∕dx)dx,* and (ι+½)dχ∙

We have already found that if V changes to V — *v* Mv⅛M⅛1≡l since t√V= *-dy∣dx.*

We also have pM2 = p9u2∕(ι+<Zy∕dx). Substituting these values and neglecting powers of *dy∣dx* above the second we get HJ>∣-M⅛)>

Butyy^⅛dx=o since the sum of the displacements = 0. Then putting (dy∕dx)2= (u∕U)2, we have

= J(γ÷ι) average energy per cubic centimetre, (15)

a result first published by Lord Rayleigh *{Phil. Mag.,* 1905, 10, p⅛ train of waves is reflected, the value of *p* at AB will be the sum of the values for the two trains, and will, on the average, be doubled. The pressure on CD will therefore be doubled. But the energy will also be doubled, so that (15) still gives the average excess of pressure.

*Experimental Determinations of the Velocity of Sound.*

An obvious method of determining the velocity of sound in air consists in starting some sound, say by firing a gun, and stationing an observer at some measured distance from the gun. The observer measures by a clock or chronometer the time elapsing between the receipt of the flash, which passes practically instantaneously, and the receipt of the report. The distance divided by the time gives the velocity of the sound. The velocity thus obtained will be affected by the wind. For instance, William Derham *{Phil. Trans.,* 1708) made a series of observations, noting the time taken by the report of a cannon fired on Blackheath to travel across the Thames to Upminster Church in Essex, 12 j m. away. He found that the time varied between 55⅛ seconds when the wind was blowing most strongly with the sound, to 63 seconds when ⅛ was most strongly against the sound. The value for still air he estimated at 1142 ft. per second. He made no correction for temperature or humidity. But when the wind is steady its effect may be eliminated by tt reciprocal” observations, that is, by observations of the time of passage of sound in each direction over the measured distance.

Let D be the distance, U the velocity of sound in still air, and *w* the velocity of the wind, supposed for simplicity to blow directly from one station to the other. Let Ti and T2 be the observed times of passage in the two directions. We have U+w=D∕Tι and U — w== D∕T2. Adding and dividing by 2

u=τ⅛+⅛)∙

If Tι and T2 are nearly equal, and if T=J(Tι+T2), this is very nearly U = D∕T.

The reciprocal method was adopted in 1738 by a commission of the French Academy *{Mémoires de Vacadémie des sciences,* (1738). Cannons were fired at half-hour intervals, alternately at Montmartre and Montlhéry, 17 or 18 m. apart. There were also two intermediate stations at which observations were made. The times were measured by pendulum clocks. The result obtained at a temperature about 6o C. was, when converted to metres, U = 337 metres/second.

The theoretical investigation given above shows that if U is the velocity in air at l·Q C. then the velocity Uo at oo C. in the same air is independent of the barometric pressure and that Uo = U∕(i+o-∞i84∕), whence Uo = 332 met./sec. »

In 1822 a commission of the Bureau des Longitudes made a series of experiments between Montlhéry and Villejuif, 11 m. apart. Cannons were fired at the two stations at intervals of five minutes. Chronometers were used for timing, and the result at 15∙90.C. was U = 34o∙9 met.∕sec., whence U0=33o∙6 met./sec. (F. J. D. Arago, *Connaissance des temps,* 1825).

When the measurement of a time interval depends on an observer, his “ personal equation ” comes in to affect the estimation of the quantity. This is the interval between the arrival of an event and his perception that it has arrived, or it may be the interval between arrival and his record of the arrival. This personal equation is different for different observers. It may differ even by a considerable fraction of a second. It is different, too, for different senses with the same observer, and different even for the same sense when the external stimuli differ in intensity. When the interval between a flash and a report is measured, the personal equations for the two arrivals are, in all probability, different, that for the flash being most likely less than that for the sound. In a long series of experi­ments carried out by V. Regnault in the years 1862 to 1866 on the velocity of sound in open air, in air in pipes and in various other gases in pipes, he sought to eliminate personal equation by dispensing with the human element in the observations, using electric receivers as observers. A short account of these experiments is given in P7Æ *Mag.,* 1868, 35, p. 161 and the full account, which serves as an excellent example of the extra­ordinary care and ingenuity of Regnault's work, is given in the *Mémoires de l'académie des sciences,* 1868, xxxvii. On page 459 of the *Mémoire* will be found a list of previous careful experiments on the velocity of sound.

In the open-air experiments the receiver consisted of a large