may be treated very easily if the motions are all in the line joining source and receiver. Let S (fig. 20) be the source at a given instant, and let its frequency of vibration, or the number of waves it sends out per second, be n. Let S' be its position one second later, its velocity being *u.* Let R be the, receiver at a given instant, R' its position a second later, its velocity being *v.* Let the velocity of the air from S to R be *w,* and let U be the velocity of sound in still air.

If all were still, the *n* waves emitted by S in one second would spread over a length U. But through the wind velocity the first wave is carried to a distance U + *w* from S, while through the motion of the source the last wave is a distance *u* from S. Then the *n* waves occupy a space U ÷ *ïv* — w. Now turning to the receiver, let us consider what length is occupied by the waves which pass him in one second. If he were at rest, it would be the waves in length U + w, for the wave passing him at the beginning of a second would be so far distant at the end of the second. But through his motion *v* in the second, he receives only the waves in distance U + w — î>. Since there are *n* waves in distance U + w - z∕ the number he actu­ally receives is. w(U ÷ *w* — v)∕(U ÷ *w — ü).* If the velocities of source and receiver are equal then the frequency is not affected by their motion or by the wind. But if their velocities are different, the frequency of the waves received is affected both by these velocities and by that of the wind.

The change in. pitch through motion of the source may be illustrated by putting a pitch-pipe in one end of a few feet of rubber tubing and blowing through the other end while the tubing is whirled round the head. An observer in the plane of the motion can easily hear a change in the pitch as the pitch-pipe moves to and from him.

*Musical Quality or Timbre.—*Though a musical note has definite pitch or frequency, notes of the same pitch emitted by different instruments, have quite different quality or timbre. The three characteristics of a longitudinal periodic disturbance are its ampli­tude, the length after which it repeats itself, and its form, which may be represented by the shape of the displacement curve. Now the amplitude evidently corresponds to the loudness, and the length of period corresponds to the pitch or frequency. Hence we must put down the quality or timbre as depending on the form.

The simplest, form of wave, so far as our sensation goes—that is, the one giving rise to a pure tone-yis, we have every reason to suppose, one in which the displacement is represented by a harmonic curve or a curve of sines, *y=a* sin *m(x-e).* If we put this in the form *y=a* sin y (x—e), we see that y=o, for x=e, e-∏λ, e+∣λ, e + ’λ, and so on, that .*y* is + from *x=e* to x=e+⅜λ, —from e+⅛λ to e+∣λ, and so on, and that it alternates between the values+α and— *a.*

The form of the curve is evidently as represented in fig. 21, and it may easily be drawn to exact scale from a table of sines.

In this curve ABCD are nodes. OA = e is termed the epoch, being the distance from O of the first ascending node. AC is the shortest distance after which the curve begins to repeat itself; this length λ is termed the wavelength. The maximum height of the curve HM =α is the amplitude. If we transfer O to A, e = o, and the curve may be represented by y=α sin yx.

If now the curve moves along unchanged in form in the direction ABC with uniform velocity U, the epoch e=OA at any time *I* will be U∕, so that the value of *y* may be represented as

y=α sin y(x-U∕). (16)

The velocity perpendicular to the axis of any point on the curve at a fixed distance *x* from O is

⅛2ττU(I 2ιrz ττ

r-cosτ(x-U0. (17)

The acceleration perpendicular to the axis is g^·= ~~~z~~~~⅞~~~~j a~~~~s~~i∏γ(¾~UQ

—⅛⅛ (.«

which is an equation characteristic of simple harmonic motion.

Γhe maximum velocity of a particle in the wave-train is the amplitude of *dy∣dt.* It is, therefore,

‰=2ffUα∕λ = 2ιτnα. (19)

The maximum pressure excess is the amplitude of ω = Ew∕U = (E∕U)⅛y∕dZ. It is therefore

ωm = (E∕U)27rUα∕λ *=2πnpUa.* (20)

We have already found the energy density in the train and the energy stream in equations (iβ) and (14),

The chief experimental basis for supposing that a train of longi­tudinal waves with displacement curve of this kind arouses the sensation of a pure tone is that the more, nearly a source is made to vibrate with a single simple harmonic motion, and therefore, presumably, the more nearly it sends out such a harmonic train, the more nearly does the note heard approximate to a single pure tone.

Any periodic curve may be resolved into sine or harmonic curves by Fourier’s theorem.

Suppose that any periodic sound disturbance, consist­ing of plane waves, is being propagated in the direction ABCD (fig. 22). Let it be represented by a displacement curve AHBKC. Its periodicity implies that after a certain distance the displacement curve exactly repeats itself. Let AC be the shortest distance after which the repetition occurs, so that CLDME is merely AHBKC moved on a distance AC. Then AC = λ is the wave-length or period of the curve. Let ABCD be drawn at such level that the areas above and below it are equal; then ABCD is the axis of the curve. Since the curve represents a longitudinal disturbance in air it is always continuous, at a finite distance from the axis, and with only one ordinate for each abscissa.

Fourier’s theorem asserts that such a curve may be built up by the superposition, or addition of ordinates, of a series of sine curves of wave-lengths λ, ⅛λ, ⅜λ, Jλ... if the amplitudes α, *bt c.*. .and the epochs *e, f, g....* are suitably adjusted, and the proof of the theorem gives rules for finding these quantities when the original curve is known. We may therefore put

y = α sin e)-J-Z> sin ^(x-∕)+c sin g)+&c. (21)

where the terms may be infinite in number, but always have wave­lengths submultiples of the original or fundamental wave-length λ. Only one such resolution of a given periodic curve is possible, and each of the constituents repeats itself not only after a distance equal to its own wave-length λ∕w, but evidently also after a distance equal to the fundamental wave-length λ. The successive terms of (21) are called the *harmonics of the first term.*

It follows from this that any periodic disturbance in air can be resolved into a definite series of simple harmonic disturbances of wave-lengths equal to the original wave-length and its successive submultiples, and each of these would separately give the sensa­tion of a pure tone. If the series were complete we should have terms which separately would correspond to the fundamental, its octave, its twelfth, its double octave, and so on. Now we can see that two notes of the same pitch, but of different quality, or different form of displacement curve, will, when thus analysed, break up into a series having the same harmonic wave-lengths; but they may differ as regards the members of the series present and their, ampli­tudes and epochs. We may regard quality, then, as determined by the members of the harmonic series present and their amplitudes and epochs. It may, however, be stated here that certain experiments of Helmholtz appear to show that the epoch of the harmonics has not much effect on the quality.

Fourier’s theorem can also be usefully applied to the disturbance of a source of sound under certain conditions. The nature of these conditions will be best realized by considering the case of a stretched string. It is shown below how the vibrations of a string may.be deduced from stationary waves. Let us here suppose that the string AB is displaced into the form AHB (fig. 23) and is then let go. Let

us imagine it to form half a wave-length of the extended train ZGAHBKC, on an indefinitely extended stretched string, the values of *y* at equal distances from A (or from B) being equal and opposite. Then, as we shall prove later, the vibrations of the string may be represented by the travelling of two trains in opposite directions each with velocity

∙√ tension ÷ mass per unit length

each half the height of the train represented in fig. 23. For the superposition of these trains will give a stationary wave between A