and B. Now we may resolve these trains by Fourier’s theorem into harmonics of wave-lengths λ, ⅜λ, ⅜λ, &c., where λ = 2AB and the conditions as to the values of *y* can be shown to require that the harmonics shall all have nodes, coinciding with the nodes of the fundamental curve. Since the velocity is the same for all disturb­ances they all travel at the same speed, and the two trains will always remain of the same form. If then we resolve AHBKC into harmonics by Fourier’s theorem, we may follow the motion of the separate harmonics, and their superposition will give the form of the string at any instant. Further, the same harmonics with the same amplitude will always be present.

We see, then, that the conditions for the application of Fourier’s theorem are equivalent to saying that all disturbances will travel along the system with the same velocity. In many vibrating systems this does not hold, and then Fourier’s theorem is no longer an appropriate resolution. But where it is appropriate, the disturb­ance sent out into the air contains the same harmonic series as the source.

The question now arises whether the sensation produced by a periodic disturbance can be analysed in correspondence with this geometrical analysis. Using the term u note ” for the sound produced by a periodic disturbance, there is no doubt that a well-trained ear can resolve a note into, pure tones of frequencies equal to those of the fundamental and its harmonics. If, for instance, a note is struck and held down on a piano, a little practice enables us to hear both the octave and the twelfth with the fundamental, especially if we have previously directed our attention to these tones by sounding them. But the har­monics are most readily heard if we fortify the ear by an air cavity with a natural period equal to that of the harmonic to be sought. The form used by Helmholtz is a glove of thin brass (fig. 24) with a large hole at one end of a diameter, at the other end of, which the brass is drawn out into a short, narrow tube that can be put close to the ear. But a card­board tube closed at one end, with the open end near the ear, will often suffice, and it may be tuned by more or less covering up the open end. If the harmonic corresponding to the resonator is present its tone swells out loudly.

This resonance is a particular example of the general principle that a vibrating system will be set in vibration by any periodic force applied to it, and ultimately in the period of the force, its own natural vibrations gradually dying down. Vibrations thus excited are termed *forced vibrations,* and their amplitude is greater the more nearly the period of the applied force approaches that of the system when vibrating freely. The mathematical investigation of forced vibra­tions (Rayleigh,.Soκwd, i. § 46) shows that, if there were nodissipation of energy, the vibration would increase indefinitely when the periods coincided. But there is always leakage of energy either through friction or through wave-emission, so that the vibration only increases up to the point at which the leakage of energy balances the energy put in by the applied force. Further, the greater the dissipation of energy the less is the prominence of the amplitude of vibration for exact coincidence over the amplitude when the periods are not quite the same, though it is still the greatest for coincidence.

The principle of forced vibration may be illustrated by a simple case. Suppose that a mass M is controlled by some sort of spring, so that moving freely it executes harmonic vibrations ζiven by Mx = — *μx,* where *µx* is the restoring force to the centre of vibration. Putting μ∕M=n2 the equation becomes x⅛n2x = o, whence x=A sin *nt,* and the period is *21r∣n.*

Now suppose that in addition to the internal force represented by *—µx,* an external harmonic force of period *21r∕p* is applied. Repre­senting it by — P sin *pt,* the equation of motion is now

χ+n2x+^ sin *pt=o. ■* (22)

Let us assume that the body makes vibrations in the new period *2ιrp,* and let us put x = B sin *pt;* substituting in (22) we have —∕>2B-J-n2B+P∕M =0, whence

b-P . \_J

ii-Νl p2-n2J and the “ forced ” oscillation due to —P sin *pt* is

P sin *pt* z

M ' p2-n,' ( 3)

If *p> n* the motion agrees in phase with that which the applied force alone would produce, obtained by putting n=o. If *p<n* the phases are opposite. If *p = n* the amplitude becomes infinite. This is the case of “ resonance.” The amplitude does not, of course, become infinite in practice. There is always loss of energy by dissi­pation in the vibrating machinery and by radiation into the medium, and the amplitude only increases until this loss is balanced by the gain from the work done by the applied force.

According to Helmholtz, the ear probably contains within it a series of resonators, with small intervals between the periods of the successive members, while the series extends over the whole range of audible pitch. We need not here enter into the question of the structure constituting these resonators. Each of them is supposed to have its own natural frequency, and to be set into vibration when the ear receives a train of waves of that frequency. The vibration in some way arouses the sensation of the corresponding tone. But the same resonator will be appreciably though less affected by waves of frequency differing -slightly from its own. Thus Helmholtz from certain observations *(Sound,* ii. § 388) thought that if the intensity of response by a given resonator in the ear to its own tone is taken as 1, then its response to an equally loud tone a semitone different may be taken as about ⅛. According to this theory, then, when a pure tone is received the auditory apparatus corresponding to that tone is most excited, but the apparatus on each side of it is also excited, though by a rapidly diminishing amount, as the interval increases. If the sensations corresponding ίο these neigh­bouring elements are thus aroused, we have no such perception as a pure tone, and what we regard as a pure tone is the mean of a group of sensations. The sensitiveness of the ear in judging of a given tone must then correspond to the accuracy with which it can judge of the mean.

*Measurements of Intensity of Sound or Loudness.—*Various- devices have been successfully employed for making sounds of determinate loudness in order to test the hearing of partially deaf people. But the converse, the measurement of the loudness of a sound not produced at our will, is by no means so easy. If we compare the problem with that of measuring the illumina­tion due to a source of light, we see at once how different it is. In sound sensation we have nothing corresponding to white light. A noise such as the roar due to traffic in a town may correspond physically in that it could probably be resolved into a nearly continuous series of wave-lengths, but psychically it is of no interest. We do not use such noise, but rather seek to avoid it. We certainly do not wish to measure its loudness, and even if we did it might be difficult to fix on any unit of noisiness. Probably we should be driven to a purely physical unit, the stream of energy proceeding in any direction, and if the noise were great enough we might measure it possibly by the pressure against a surface.

The intensity of the stream of energy passing per second through a square centimetre when a given pure tone is sounded is more definite and can be measured. There are two practical methods. In the one,.the energy of vibration of the source is measured, and the rate at which that energy decreases is observed. The amount radiated out in the form of sound waves is deduced, and hence the energy of the stream at any distance is known. In the other, the waves produce a measurable effect on a vibrating system of the same frequency, and the amplitude in the waves can be deduced.

The first may be illustrated by Lord Rayleigh’s experiments to determine the amplitude of vibration in waves only just audible *(Sound,* ii. § 384). He used two kinds of experiment, but it will be sufficient here to indicate the second. A fork of frequency 256 was used as the source. The energy of this fork with a çiven amplitude of vibration could be calculated from its dimensions and elasticity, and the amplitude was observed by measuring with a microscope the line into which the image of a starch grain on the prong was drawn by the vibration. The rate of loss of energy was calculated from the rate of dying down of the vibration. This rate of loss for each amplitude was determined (1) when the fork was vibrating alone, and (2) when a resonator was placed with its mouth under the free ends of the fork. The difference in loss in the two cases measured the energy given up to and sent out by the resonator as sound. The amplitude of the fork was observed when the sound just ceased to be audible at 27<4 metres away, and the rate of energy emission from the resonator was calculated to be 42·I ergs ∕ second. Assuming this energy to be propagated in hemispherical waves, it is easy to find the quantity per second going through 1 sq. cm. at the distance of the listener, and thence from the energy in a wave, found above, to determine the amplitude. The result was an amplitude of 1∙27×1cj7 cm. Other forks gave results not very different.

In a later series of experiments Lord Rayleigh *(Phil. Mag.,* 1907, 14, p. 596) found that the least energy stream required to excite sensation did not vary greatly between frequencies of 512 and 256,