number of “ side" and other geometrical equations of condition, which entered irregularly and caused great entanglement. Equa­tions 9 and 10 of the illustration are of a simple form because they have a single geometrical condition to maintain, the triangular, which is not only expressed by the simple and symmetrical equation *x*+*y*+*z*=0, but—what is of much greater importance—recurs in a regular order of sequence that materially facilitates the general solution. Thus, though the calculations must in all cases be very numerous and laborious, rules can be formulated under which they can be well controlled at every stage and eventually brought to a successful issue. The other geometrical conditions of networks are expressed by equations which are not merely of a more complex form but have no regular order of sequence, for the networks pre­sent a variety of forms; thus their introduction would cause much entanglement and complication, and greatly increase the labour of the calculations and the chances of failure. Wherever, therefore, any compound figure occurred, only so much of it as was required to form a chain of single triangles was employed. The figure having previously been made consistent, it was immaterial what part was employed, but the selection was usually made so as to introduce the fewest triangles. The triangulation for final simultaneous reduction was thus made to consist of chains of single triangles only; but all the included angles were “ fixed ” simultaneously. The excluded angles of compound figures were subsequently har­monized with the fixed angles, which was readily done for each figure *per se.*

This departure from rigorous accuracy was not of material im­portance, for the angles of the compound figures excluded from the simultaneous reduction had already, in the course of the several independent figural adjustments, been made to exert their full in­fluence on the included angles. The figural adjustments had, how­ever, introduced new relations between the angles of different figures, causing their weights to increase *caeteris paribus* with the number of geometrical conditions satisfied in each instance. Thus, suppose w to be the average weight of the *t* observed angles of any figure, and *n* the number of geometrical conditions presented for satisfaction; then the average weight of the angles after adjustment may betaken as *w.t*/*t*-*n,* the factor thus being 1∙5 for a triangle,

1∙8 for a hexagon, 2 for a quadrilateral, 2∙5 for the network around the Sironj base-line, &c.

In framing the normal equations between the indeterminate factors λ for the final simultaneous reduction, it would have greatly added to the labour of the subsequent calculations if a separate weight had been given to each angle, as was done in the primary figural reductions; this was obviously unnecessary, for theoretical requirements would now be amply satisfied by giving equal weights to all the angles of each independent figure. The mean weight that was finally adopted for the angles of each group was therefore taken as

*wo.t*/*t*-*n,*

*p* being the modulus.

The second of the two processes for applying the method of minimum squares having been adopted, the values of the errors *y* and z of the angles appertaining to any, the *p*th, triangle were finally expressed by the following equations, which are derived from (10) by substituting *u* for the reciprocal final mean weight as above determined :—

*yp =* ⅛,[(2⅛ *— ap —* <⅛)λ]

u ' (11)

¾> ^^ ~[(2cP *aP ~* ⅛)λ]

ö -\*

The following table gives the number of equations of condition and unknown quantities—the angular errors—in the five great sections of the triangulation, which were respectively included in the simultaneous general reductions and relegated to the sub­sequent adjustments of each figure *per se:—*

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Section. | | Simultaneous. | | | External Figural. | | | | | |
| Equations. | | Angular errors. | Equations. | | | | Angular errors. | No. of figures. |
| Circuit and base­line. | Tri­angular. | Tri­angular. | Central. | Side. | Toto- partial. |
| 1. | N.W. Quad. . | 23 | 550 | 1650 | 267 | 104 | 152 | 6 | 761 | 110 |
| 2. | S.E. Quad. | 15 | 277 | 831 | 164 | 64 | 92 | 2 | 476 | 68 |
| 3∙ | N.E. Quad. | 49 | 573 | 1719 | 112 | 56 | 69 | 0 | 341 | 50 |
| 4. Trigon. | | 22 | 303 | 909 | 192 | 79 | 101 | 2 | 547 | 77 |
| 5∙ | S.W. Quad. . | 24 | 172 | 516 | 83 | 32 | 52 | 1 | 237 | 40 |

The corrections to the angles were generally minute, rarely ex­ceeding the theoretical probable errors of the angles, and therefore applicable without taking any liberties with the facts of observa­tion.

Azimuth observations in connexion with the principal triangula­tion were determined by measuring the horizontal angle between a referring mark and a circumpolar star, shortly before and after elongation, and usually at both elongations in order to eliminate the error of the star's place. System­atic changes of “ face ” and of the zero settings of the azimuthal circle were made as in the measurement of the principal angles,; but the repetitions on each zero, were more numerous; the azimuthal levels were read and corrections applied to the star observations for dislevelment. the triangulation was not adjusted, in the course of the final simultaneous reduction, to the astronomi­cally determined azimuths, because they are liable to be vitiated by local attractions; but the azimuths observed at about fifty stations around the primary azimuthal station, which was adopted as the origin of the geodetic calculations, were referred to that station, through the triangulation, for comparison with the primary azimuth. A table was prepared of the differences (observed at the origin— computed from a distance) between the primary and the geodetic azimuths; the differences were assumed to be, mainly due to the local deflexions of the plumb-line and only partially to error in the triangulation, and each was multiplied by the factor

tangent of latitude of origin, tangent of latitude of comparing station in order that the effect of the local attraction on the azimuth ob­served at the distant station—which varies with the latitude and is = the deflexion in the prime vertical × the tangent of the latitude —might be converted to what it would have been had the station been situated in the same latitude as the origin. Each deduction was given a weight, *w,* inversely proportional to the number of triangles connecting the station with the origin, and the most prob­able value of the error of the observed azimuth at the origin was taken as

*x=*[(observed—computed) *p**w*]/[*w*] (12);

the value of *x* thus obtained was — 1 · 1*''*.

, The formulae employed in the reduction of the azimuth observa­tions were as follows. In the spherical triangle *PZS,* in which *P* is the pole, *Z* the zenith and *S* the star, the co-latitude *PZ* and the polar distance *PS* are known, and, as the angle at *S* is a right angle at the elongation, the hour angle and the azimuth at that time are found from the equations

*cosP* = tan*PS*cot*PZ*, cosZ = cos*PS*sin*P*.

The interval, *δP,* between the time of any observation and that of the elongation being known, the corresponding azimuthal angle, δZ, between the two positions of the star at the times of observa­tion and elongation is given rigorously by the following expression —tan δZ

2sin2⅜δP ( < cotP5sinPZsinP[ 1 +tan2PScosδP +sec2PSeotPsinδP J which is expressed as follows for logarithmic computation—

*t,rr τn* tan Z cos2 *PS δz ~ ι-n+~l t δP δP*

where *m = 2* sin2— cosec ιr, n = 2 sin2PS sin2—, and

*l* = cot *P* sin *δΡ*; *l, m,* and *n* are tabulated.

Let *A* and *B* (fig. 4) be any two points the normals at which meet at *C,* cutting the sea-level at *p* and *q;* take *Dq = Ap,* then *BD* is the difference of height; draw thc tangents *Aa* and *Bb* at *A* and *B,* then *aAB* is the depression of *B* at *A* and *bBA* that of *A* at *B;* join *AD,* then *BD* is determined from the triangle *ABD.* The triangulation gives the distance between *A* and *B* at the sea-level, whence *pq=c*; thus, putting *Ap,* the height of *A* above the sea-level, = *H,* and *pC=r,*

*ΛD=c^+1j-^* (14).

Putting *Da* and *Db* for the actual depres­sions at *A* and *B, S* for the angle at *A,*usually called the “ subtended angle,” and *h* for *BD—*

*S =* 1/2*(Db-Da)* (15),

and *h = AD s\*n* (16).

cos *Db v ,*

The angle at *C* being =*Db+Da, S* may be expressed in terms of a single vertical angle and *C* when observations have been taken at only one of the two points. *Ct* the “contained arc,”= *cρ+v*/2*ρv*cosec 1*''* in seconds. Putting *D'a* and *D'b* for the observed vertical angles, and *φa, φb* for the amounts by which they are affected by refraction, *Da = D''a+φa* and *Db=D'b*+*φb*; *φa* and *φb* may differ in amount, but as they