cannot be separately ascertained they are always assumed to be equal; the hypothesis is sufficiently exact for practical purposes when both verticals have been measured under similar atmospheric conditions. The refractions being taken equal, the observed verticals are substituted for the true in (15) to find *S*, and the difference of height is calculated by (16); the third term within the brackets of (14) is usually omitted. The mean value of the refraction is deduced from the formula

*φ* = *1/2*{*C-D'a+D'b*)} (17)∙

An approximate value is thus obtained from the observations between the pairs of reciprocating stations in each district, and the corresponding mean “coefficient of refraction,” *φ÷C,* is computed for the district, and is employed when heights have to be deter­mined from observations at a single station only. When either of the vertical angles is an elevation—*E* must be substituted for *D* in the above expressions.@@1

2. Levelling

Levelling is the art of determining the relative heights of points on the surface of the ground as referred to a hypothetical surface which cuts the direction of gravity everywhere at right angles. When a line of instrumental levels is begun at the sea-level, a series of heights is determined corresponding to what would be found by perpendicular measurements upwards from the surface of water communicating freely with the sea in underground channels; thus the line traced indicates a hypothetical prolonga­tion of the surface of the sea inland, which is everywhere conformable to the earth’s curvature.

The trigonometrical determination of the relative heights of points at known distances apart, by the measurements of their mutual vertical angles—is a method of levelling. But the method to which the term “ levelling ” is always applied is that of the direct determination of the differences of height from the readings of the lines at which graduated staves, held vertically over the points, are cut by the horizontal plane which passes through the eye of the observer. Each method has its own advantages. The former is less accurate, but best suited for thc requirements of a general geographical survey, to obtain the heights of all the more prominent objects on the surface of the ground, whether accessible or not. The latter may be conducted with extreme precision, and is specially valuable for the deter­mination of the relative levels, however minute, of easily accessible points, however numerous, which succeed each other at short intervals apart; thus it is very generally undertaken *pari passu* with geographical surveys to furnish lines of level for ready reference as a check on the accuracy of the trigonometrical heights. In levelling with staves the measurements are always taken from the horizontal plane which passes through the eye of the observer; but the line of levels which it is the object of the operations to trace is a curved line, everywhere conforming to the normal curvature of the earth’s surface, and deviating more and more from the plane of reference as the distance from the station of observation increases. Thus, either a correction for curvature must be applied to every staff reading, or the instru­ment must be set up at equal distances from the staves; the curvature correction, being the same for each staff, will then be eliminated from the difference of the readings, which will thus give the true difference of level of the points on which the staves are set up.

Levelling has to be repeated frequently in executing a long line of levels—say seven times on an average in every mile—and must be conducted with precaution against various errors. Instru­mental errors arise when the visual axis of the telescope is not perpendicular to the axis of rotation, and when the focusing tube does not move truly parallel to the visual axis on a change of focus. The first error is eliminated, and the second avoided, by placing the instrument at equal distances from the staves; and as this procedure has also the advantage of eliminating the corrections for both curvature and refraction, it should invariably be adopted.

Errors of staff readings should be guarded against by having the staves graduated on both faces, but differently figured, so that the observer may not be biased to repeat an error of the first reading in the second. The staves of the Indian survey have one face painted white with black divisions—feet, tenths and hundredths —from o to 10, the other black with white divisions from 5∙55 to 15∙55. Deflexion from horizontality may either be measured and allowed for by taking the readings of the ends of the bubble of the spirit-level and applying corresponding corrections to the staff readings, or be eliminated by setting the bubble to the same position on its scale at the reading of the second staff as at that of the first, both being equidistant from the observer.

Certain errors are liable to recur in a constant order and to accumulate to a considerable magnitude, though they may be too minute to attract notice at any single station, as when the work is carried on under a uniformly sinking or rising refraction—from morning to midday or from midday to evening—or when the instru­ment takes some time to settle down on its bearings after being set up for observation. They may be eliminated (i.) by alternating the order of observation of the staves, taking the back staff first at one station and the forward first at the next; (ii.) by working in a circuit, or returning over the same line back to the origin; (iii.) by dividing a line into sections and reversing the direction of operation in alternate sections. Cumulative error, not eliminable by working in a circuit, may be caused when there is much northing or southing in the direction of the line, for then the sun’s light will often fall endwise on the bubble of the level, illuminating the outer edge of the rim at the nearer end and the inner edge at the farther end, and so biasing the observer to take scale readings of edges which are not equidistant from the centre of the bubble; this introduces a tendency to raise the south or depress the north ends of lines of level in the northern hemisphere. On long lines, the employment of a second observer, working independently over the same ground as the first, station by station, is very desirable. The great lines are usually carried over the main roads of the country, a number of “ bench marks " being fixed for future reference. In the ordnance survey of Great Britain lines have been carried across from coast to coast in such a manner that the level of any common crossing point may be found by several independent lines. Of these points there are 166 in England, Scotland and Wales; the dis­crepancies met with at them were adjusted simultaneously by the method of minimum squares.

The sea-level is the natural datum plane for levelling opera­tions, more particularly in countries bordering on the ocean. The earliest surveys of coasts were made for the use

of navigators and, as it was considered very important that the charts should everywhere show the minimum depth of water which a vessel would meet with, low water of spring­tides was adopted as the datum. But this does not answer the requirements of a land survey, because the tidal range between extreme high and low water differs greatly at different points on coast-lines. Thus the generally adopted datum plane for land surveys is the mean sea-level, which, if not absolutely uniform all the world over, is much more nearly so than low water. Tidal observations have been taken at nearly fifty points on the coasts of Great Britain, which were connected by levelling operations; the local levels of mean sea were found to differ by larger magnitudes than could fairly be attributed to errors in the lines of level, having a range of 12 to 15 in. above or below the mean of all at points on the open coast, and more in tidal rivers.@@2 But the general mean of the coast stations for England and Wales was practically identical with that for Scotland. The observations, however, were seldom of longer duration than a fortnight, which is insufficient for an exact determination of even the short period components of the tides, and ignores the annual and semi­annual components, which occasionally attain considerable mag­nitudes. The mean sea-levels at Port Said in the Mediterranean and at Suez in the Red Sea have been found to be identical, and a similar identity is said to exist in the levels of the Atlantic and the Pacific oceans on the opposite coasts of the Isthmus of Panama. This is in favour of a uniform level all the world over; but, on the other hand, lines of level carried across the continent of Europe make the mean sea-level of the Mediterranean at Marseilles and Trieste from 2 to 5 ft. below that of the North Sea and the Atlantic at Amsterdam and Brest—a result which

@@@1 In topographical and levelling operations it is sometimes con­venient to apply small corrections to observations of the height for curvature and refraction simultaneously. Putting *d* for the distance, *r* for the earth's radius, and *κ* for the coefficient of refraction, and expressing the distance and radius in miles and the correction *to* height in feet, then correction for curvature = 2/3*d*2 correction for refraction = -4/3*κd*2; correction for both

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@@@2 In tidal estuaries and rivers the mean water-level rises above the mean sea-level as the distance from the open coast-line increases; for instance, in the Hooghly river, passing Calcutta, there is a rise of 10 in. in 42 m. between Sagar (Saugor) Island at the mouth of the river and Diamond Harbour, and a further rise of 20 in. in 43 m. between Diamond Harbour and Kidderpur.