Astron 400 Midterm Review

October 21, 2014

The mid-term exam will be an in-class, closed-book exam of 1:10 duration. The exam will cover chapters 1–4 in Phillips.

Topics: you should have at least general familiarity with these areas.

- The basics of nucleosynthesis: what are the starting and ending conditions
- Time-scales for the Sun: free-fall, Kelvin-Helmholtz
- Virial theorem, stability and the adiabatic index
- Star formation and Jeans mass/density
- Main sequence, HR diagrams
- Getting energy out of the Sun, random walks, optical depth
- Stellar scalings, turning derivatives into approximations
- Stellar energy sources, basics of fusion
- Ideal gas law, hydrostatic equilibrium
- Density of states, Fermi-Dirac, Bose-Einstein, Maxwell-Boltzmann distributions
- Quantum concentrations, chemical potential
- Degenerate gases, non-relativistic and relativistic
- Relativistic energy/momentum relation
- Blackbodies, light, radiation pressure
- Saha equation, equilibrium reactions
- Heat transfer, conduction vs. radiation vs. convection

- Critical condition for convection
- White dwarf cooling
- Fusion in stars, barrier penetration (classical)
- Quantum tunneling
- Fusion cross section, Gamow peak
- Hydrogen burning, pp vs. CNO, Solar neutrino problem
- Helium burning, more advanced burning, fusion timescales

Formulas:

Virial Theorem $E_{\rm kin} = -\frac{1}{2}E_{\rm pot}$; $E_{\rm tot} = E_{\rm kin} + E_{\rm pot} = \frac{1}{2}E_{\rm pot}$, [where $E_{\rm pot,binary} = -\frac{Gm_1m_2}{a}$ and $E_{\rm pot,star} \approx -\frac{GM^2}{R}$]

Jeans mass $M_J = 3k_BTR/2G\mu m_{\rm H}$; $\rho_J = (3/4\pi M^2)(3k_BT/2G\mu m_{\rm H})^3$

Generalized Ideal Gas number of states $g(p) = g_s(V/h^3)4\pi p^2$

Generalized Ideal Gas occupancy of states $f(\epsilon) = (e^{(\epsilon-\mu)/k_BT} \pm 1)^{-1}$

Generalized Ideal Gas $P = (1/3V)\int_0^\infty dp\, pv_p f(\epsilon)g(p)$

Quantum Concentration $n_{Q,NR} = (2\pi mk_BT/h^2)^{3/2}$, $n_{Q,UR} = 8\pi(k_BT/hc)^3$

Number density $n = \rho/\bar{m}$

Chemical Potential $\mu = mc^2 - k_BT\ln(g_sn_Q/n)$

Fermi momentum $p_F = (3n/8\pi)^{1/3}h$

Fermi pressure $P = K_{NR}n^{5/3}$ or $K_{UR}n^{4/3}$, with $K_{NR} = (h^2/5m)(3/8\pi)^{2/3}$ and $K_{UR} = (hc/4)(3/8\pi)^{1/3}$

Ideal Gas $P = nk_BT = \frac{\rho}{\bar{m}}k_BT$; typical KE per particle is $\frac{3}{2}k_BT$; energy density $u = \frac{3}{2}nk_BT = \frac{3}{2}P$

Saha Equation (example) $n({\rm H}^+)/n({\rm H}) \approx (n_{Q,e}/n_e)e^{-E/k_BT}$

Degenerate Gas $\Delta x\Delta p \sim \hbar$; $E_{\rm F} = \frac{1}{2}\frac{p_{\rm F}^2}{m_e} \propto n_e^{2/3}$; $P \propto n_eE_{\rm F} \propto n_e^{5/3} \propto (\rho/\bar{m})^{5/3} \rightarrow R \propto M^{-1/3}$ [non-relativistic]

Photon Propagation $l_{\rm mfp} = \frac{1}{n\sigma} = \frac{1}{\kappa\rho}$; $t_{\rm randomwalk} = \frac{R}{l_{\rm mfp}}\frac{R}{c}$

Blackbody $L = 4\pi R^2\sigma T_{\rm eff}^4$, $F = \sigma T_{\rm eff}^4$; $\lambda_{\rm peak} = 0.29\,{\rm cm}/T$, $u = aT^4$, $P = (a/3)T^4$.

Light $c = \lambda\nu$, $E = h\nu = hc/\lambda$, $p = E/c$, energy density $u = aT^4$, pressure $P = (a/3)T^4$

Hydrostatic Equilibrium $\frac{dP}{dr} = \rho\frac{GM}{r^2} = -g\rho \rightarrow P \propto M^2/R^4$

Stars $T_c \propto M/R$, $\rho_c \propto M/R^3$, $P_c \propto M^2/R^4$

Timescales $\tau_{\rm free-fall} \sim \sqrt{1/G\rho}$; $\tau_{\rm Kelvin-Helmholtz} \sim \frac{GM^2/R}{L}$

Hydrogen Fusion $E = \Delta mc^2 \approx 0.7\%c^2$

Hydrogen Atom $E_n = -13.6\,{\rm eV}/n^2$

Opacity Electron scattering $\kappa = 0.02(1 + X_{\mathrm{H}})\,\mathrm{m}^2\,\mathrm{kg}^{-1}$, Kramer's law $\kappa \propto \rho T^{-3.5}$

Radiative Heat Flux $dT/dr = (3\rho\kappa/4acT^3)(L/4\pi r^2)$

Convective Heat Flux $dT/dr = (\gamma - 1)/\gamma\,(T/P)dP/dr$

Probability of Barrier Penetration $\approx e^{-\sqrt{E_G/E}}$

Gamow energy $E_G = (\pi\alpha Z_A Z_B)^2 2m_r c^2$ with $m_r = m_A m_B/(m_A + m_B)$

Fusion Rate $R_{AB} \propto n_A n_B (k_B T)^{-3/2} \int dE\, S(E) \exp\left(-E/k_B T - \sqrt{E_G/E}\right)$

Away from resonance $R_{AB} \sim n_A n_B (E_G/4k_B T)^{2/3} e^{-3(E_G/4k_B T)^{1/3}}$
$R_{AB} \propto T^a$ with $a = (E_G/4k_B T)^{1/3}$

Hydrogen burning $4p \to^4 \mathrm{He} + 2e^+ + 2\nu_e$, $\epsilon_{pp} \propto X_{\mathrm{H}}^2 \rho^2 T^4$