Name: ____________

Date: ____________

Section: ____________

Astron 104 Laboratory #8
Gravity and Black Holes
Chapter 14

Introduction

Gravity is a familiar force—when you drop a pencil, it falls to the floor. A ball on a slope rolls downhill. Rainwater runs down the storm drain. All these effects depend on the gravity of Earth. Despite the dominant role it plays in our lives, Earth's gravity is still relatively weak. More massive Jupiter has stronger gravity than Earth, and the Sun's gravity is stronger still. The most extreme gravity in the Universe belongs to black holes. In this lab, you'll investigate some of the exotic phenomena that result from the enormous gravity in the vicinity of a black hole.

Learning Objectives

At the completion of this lab, you should be able to:

1. Define escape speed and differentiate it from orbital speed
2. Define the event horizon of a black hole in relation to escape speed
3. Describe the effects of gravitational redshift on light
4. Explain how gravitational redshift leads to time dilation near the event horizon
5. Describe the gravitational effect of a black hole on light rays
6. Describe the brightening of background sources from gravitational lensing

Escape Speed [5 pts each, 40 pts total]

1. Imagine standing outside with an apple in your hand. Toss the apple lightly straight up above your hear and catch it as it returns to your hand. Describe how the speed of the apple changes during its "flight."

2. This time you throw the apple straight up as hard as you can, and again deftly catch it as it returns. How does ths flight of the apple this time compare to the lighter toss? What is the same about its flight and what is different: speed? height? time of flight?

3. Which apple spends more time in flight, the one tossed at low speed or the one at high speed?

4. If you throw (or launch!) the apple fast enough, the time required for it to slow to a stop becomes infinite—at this speed (or faster) it's not coming back! We call this the **escape speed**. At the surface of the Earth, escape speed is 11 km/s (that's 25,000 mph!). At this speed, how long would it take to travel from Los Angeles to New York (4,000 km)?

5. The speed required for an apple (or a rock or a spaceship, etc) to escape the gravitational dominance of a massive object is:

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{R}}$$

where G is the gravitational constant in Newton's law of universal gravitation, M is the mass of the object, and R is the distance from the object's center of mass.

Identify the variables in the equation and rewrite it as a proportionality relation. (For example, if $A = 4\pi r^2$, then $A \propto r^2$ has only the variables left. The symbol $\propto$ means "proportional to.").

6. Imagine an alien planet that has the same radius as Earth, but is 100 times more massive than Earth. What is the ratio of the escape speeds for this alien planet and Earth?

7. How would the escape speed from the surface of an object (e.g., Earth) change if we simply compress its mass into a smaller volume (i.e., a smaller radius R)?

8. Look at your proportionality relation for the escape speed. Describe two ways you could increase the escape speed of a planet to be faster than the speed of light by changing only one variable at a time.

Black Holes: Abandon All Hope, Ye Who Enter Here! [5 pts each, 15 pts total]

One consequence of Einstein's theory of relativity is that no object can travel faster than the speed of light. The speed of light, c, is thus a "cosmic speed limit." If the escape speed from an object exceeds the speed of light, **nothing** can escape. Gravity wins! We call this object a **black hole**.

1. Imagine a rocket heading towards a massive star. How does the speed required for the rocket to escape from the star depend on how far the rocket is from the star? Use your proportionality relation to support your answer.

2. The radius R when $v_{\text{esc}} = c$ has a special name: the **event horizon**. If a spacecraft were to cross inside the event horizon, could it reverse course and fly away? Explain.

3. If a space probe was sent across the event horizon of a black hole, could it send us back radio reports of its observations? Explain your reasoning.

Time Dilation [5 pts each, 50 pts total]

Black holes are the ultimate victory of gravity, effectively pinching off the space inside the event horizon from the rest of the Universe. The effects of this extreme gravity are governed by Einstein's theory of relativeity, which predicts some amazingly strange phenomena.

1. If you toss a ball up in the air, the Earth's gravity pulls it downward. How does the kinetic energy (the energy of motion) of the ball change while it is moving upward?

2. Say we stand on the surface of a very massive star (somehow!) and shine a green laser upwards. Relativity says that gravity affects light as well as matter. But relativity requires that light always travels at the same speed: 3×10^8 m/s. How can the photons from the laser somehow lose energy while still traveling at the speed of light?

3. Light losing energy as it moves away from a massive object is called a **gravitational redshift.** Why is it a "redshift?"

4. Now let's consider an unusual type of clock, one that ticks every time the light wave from a laser beam has a "peak." To help visualize this sort of clock, sketch a light wave and write "tick" next to each peak.

5. Now let's send our laser clock down towards a black hole (outside the event horizon somewhere, so that the light can still get out). Since black holes are dangerous, we'll send astronaut Dr. A to hold the laser and we'll watch through a telescope from a safe distance away.

As Dr. A gets closer to the event horizon, what happens to the energy of the laser light as it climbs out to you? What happens to the wavelength of the light?

6. Think about the rate of the ticks of Dr. A's laser clock compared to an identical laser clock in your hand far away from the event horizon. If we measure time with these laser clocks, how does time pass for astronaut Dr. A compared to how time passes for you?

7. Right at the event horizon, relativity says that the light must use all of its energy to get out of the gravitational pull of the black hole. What happens to the wavelength of the laser clock photons when the clock reaches the event horizon?

8. Based on this thought experiment, how does time pass at the event horizon as seen from far away?

9. Will (doomed!) astronaut Dr. A notice this time **dilation** (the opposite of **contraction**) as he falls towards the black hole? (The laser clock is right there in his hand!)

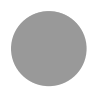
10. What would he say about the rate of time passing on our clock? Explain your reasoning.

Gravitational Lensing [5 pts each, 45 pts total]

Gravity from a compact, massive object (like a black hole) has another unexpected effect on light: the path of a light ray bends as a result of gravity.

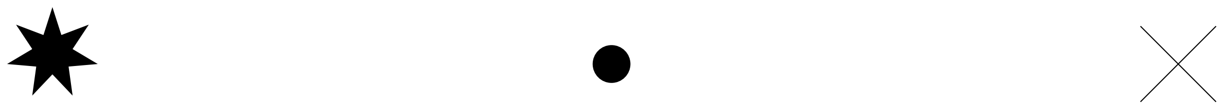
1. In empty space, light travels in a perfectly straight line. Draw a laser source and a beam of light.

2. The presence of a massive object causes a beam of light to bend slightly toward the object. The more massive the object, the more the light bends. Redraw your laser source and laser beam, this time bending the path slightly as the beam passes the massive object shown below.



3. A star emits light in all directions. When we observe the star from a large distance, we observe only the portion of the light that comes in our directin. Draw some beams of light coming from the star, and in particular indicate any beams that can be seen by the distant observer illustrated by the ×.

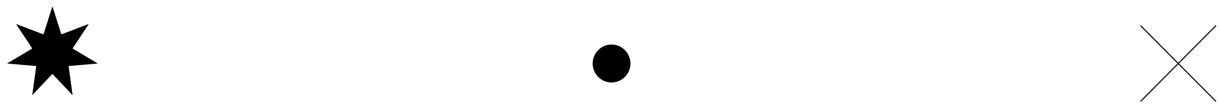
4. Now let's reconsider the same situation, but let a dense, compact object like a black hole come between the star and the observer. Again draw light beams emitted by the star, specifically including any that can reach the observer at the $\times$. This time, be sure to account for the bending of the light in the vicinity of the black hole.



5. Since light carries energy, we can consider the beams you drew as energy delivered by the star in a certain direction. Compare the beams you drew in each case above. When the black hole passes between the star and the observer, does the number of beams reaching the observer increase, decrease, or remain the same?

6. When the black hole passes between the star and the observer, what would the observer measure for the brightness of the star? Would it get brighter, dimmer, or remain the same? Explain your reasoning.

7. When we see an object, we interpret its location by looking back along the beam of light. Our intuition is based on the notion that light travels in straight lines. In the diagram below, redraw only the beams that reach the observer. Then draw **straight** lines **outward** along the beams from the **observer's** position. Remember, an observer interprets these lines as straight!



8. Where does the observer determine the source of these beams to be, relative to the actual position of the star?

9. Our model on the page is two dimensional, but of course there are beams moving in all directions in three-dimensional space. Imagine the beams coming from the star, passing the black hole, and reaching the observer. Tracing back along these beams with straight lines, what shape would the star appear in the sky as seen by the observer? Explain. (This is known as **gravitational lensing**, and is really observed!)