

Name: _____

Date: _____

Section: _____

Astron 104 Laboratory #4a

A More Rounded View of the Earth

Discovery 0.1

More than 2000 years ago, Eratosthenes (276–195 BCE) performed an experiment that showed that the Earth was much larger than previously believed. If the Earth was a sphere, its circumference C (the distance around) and radius R can be related through:

$$C = 2\pi R \tag{1}$$

where $\pi = 3.14\dots$. In order to compute the size of the Earth, Eratosthenes employed the following logic: If the Earth were flat, sticks placed vertically at various places on the Earth would all simultaneously point directly toward the Sun at noon, and none would cast a shadow. If, however, the Earth was round (a sphere), when the Sun was directly overhead at one location it could not be simultaneously overhead at another location to the North or South.

Eratosthenes worked at the Great Library at Alexandria, Egypt. He read that in Syene, Egypt, at noon on June 21, obelisks cast no shadows and sunlight fell directly down a well. This indicated that the Sun was directly overhead. At noon on the same date, he observed that in Alexandria (located directly north of Syene) the shadow of a pillar indicated that the Sun was about 7.2° south of the zenith (see Figure 1).

1. In Figure 2, you can see the overall situation. Use Figure 2 along with the geometry of parallel lines to determine what angular distance (how many degrees), measured from the center of the Earth, separate Alexandria and Syene [**10 pts**].

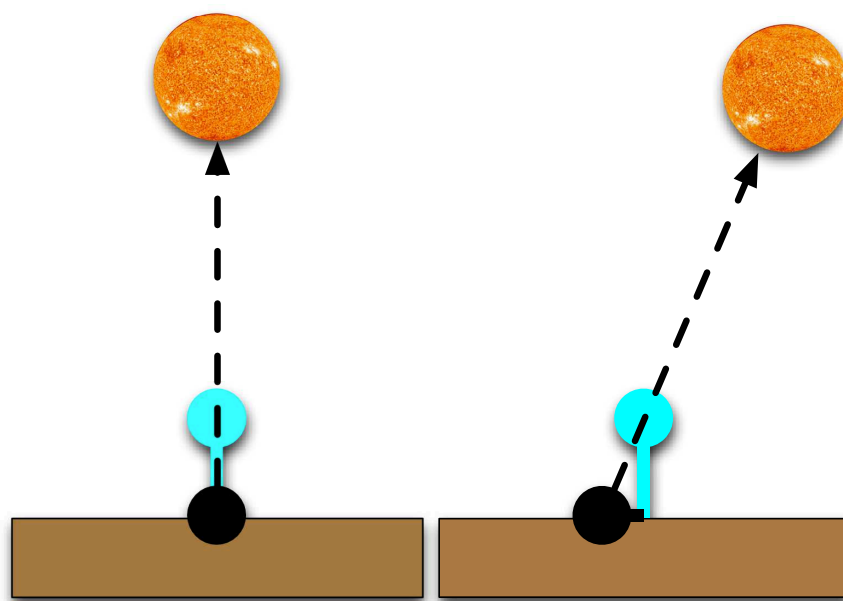


Figure 1: On the left is the situation in Syene on June 21: the Sun is directly overhead and casts no shadow. On the right is the situation in Alexandria, where the Sun is 7.2° south of the zenith.

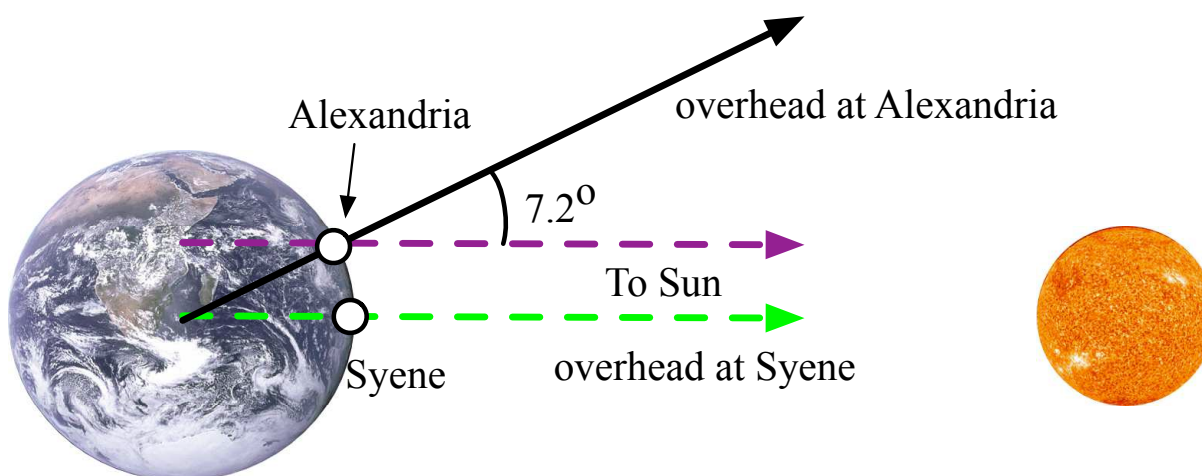


Figure 2: The overall geometry. Lines going to the Sun are parallel.

5. How close is your answer to the actual circumference of the Earth (40,008 km) [**10 pts**]? Calculate the relative error, which is given by:

$$\text{Relative Error}(\%) = 100 \times \frac{\text{Observed value} - \text{Actual value}}{\text{Actual value}}$$

6. Please explain why this result is good, bad, or indifferent? [**5 pts**]
7. In 2011, some UWM students recreated this experiment with help from students at Mississippi State University in Starkville, MS (1,070 km south of Milwaukee). They found that the shadow in Starkville was 9.8° smaller than the shadow in Milwaukee. Use the same procedure that you used above to determine how well the UWM students did in determining the circumference of the Earth. [**15 pts**]

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Astron 104 Laboratory #4b
The Mass of the Earth
More Precisely 1.1, Section 6.1

Although the size of the Earth has been known since ancient times, an accurate measurement of the mass of the Earth was possible only relatively recently. We will investigate two methods to calculate the mass of the Earth.

1. A crude estimate of the mass of the Earth can be made by estimating the density of the Earth. Density, d , measures how much mass is in a volume of space. The density of an object of mass M and volume V is:

$$d = \frac{M}{V}$$

The volume of a sphere of radius R is:

$$V = \frac{4}{3}\pi R^3$$

To calculate the volume, we can use the radius of the Earth that Eratosthenes found, which was about 6,366 km = 6.36×10^6 m.

Compute the volume of the Earth in m^3 [**10 points**]:

The typical density of a rock on the Earth's crust is $d_{\text{rock}} = 2700 \text{ kg/m}^3$. If we assume that the overall density of the Earth is the same as rock, compute the mass of the Earth using this value and the volume you found above. Call this number M_1 [**10 points**]:

2. We can estimate the mass of the Earth by measuring the amount of gravitation the Earth's mass produces. To do so, we need (1) measurements of the Earth's radius (6.36×10^6 m) and of the gravitational acceleration at its surface, g , which measures how quickly objects accelerate when they fall (use $g = 10 \text{ m/s}^2$); and (2) a formula for the Earth's gravitational acceleration g given the mass and radius. We use Isaac Newton's work. He was the first person to describe how gravity operated, and he showed how a body's mass and radius determined its gravitational acceleration. He found that the acceleration at the surface of the Earth could be expressed as:

$$g = \frac{GM}{R^2}$$

where G is Newton's constant, M is the mass of the Earth, and R is the radius of the Earth. Use this equation to solve for the mass of the Earth (call it M_2 to distinguish it from the result we got before; you don't need to plug in values but leave it in terms of G , g , and R) [**10 points**]:

3. In 1798 Henry Cavendish made an accurate measurement of G , getting $6.1 \times 10^{-11} \text{ m}^3/\text{kg}/\text{s}^2$. Using his value of G and the expression for mass, Cavendish was able to compute a better estimate of the mass of the Earth. Following this procedure, compute the mass of the Earth M_2 [**10 points**].

4. Why is this result M_2 different from the previous result M_1 ? Hint: what assumptions were required to get M_1 ? [**10 points**]

5. Using the new mass of the Earth M_2 compute the average density of the Earth. [**10 points**]

6. Is the average density of the Earth the same as the density of rock? What does that tell you about the composition of the Earth? [**10 points**]