Astron 211 Final Review

Monday, Dec 21, 2015, 10a-12p

Exam Policy: You may only consult the sheet of notes/formulas than I hand out. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

Do not spend all your time on one problem. Try to look at each problem quickly first.

Please bring a calculator (no smartphones).

Official notice such as a doctor's note must be provided to support any absences.

Exam Format:

- True/False
- Multiple choice (like activities)
- Short answer (like Kutner problems on problem set)
- Two longer problems
- One order-of-magnitude

Topics: you should have at least general familiarity with these areas.

- Seasons, orbit of the Earth, coordinates
- Parallax, small-angle approximation
- Fluxes, luminosities, inverse-square law
- Orbits, binary stars, Kepler's laws
- Time-scales for the Sun: free-fall, Kelvin-Helmholtz

- Stellar energy sources, basics of fusion
- Getting energy out of the Sun, random walks, optical depth
- Ideal gas law, hydrostatic equilibrium
- Blackbodies, light, radiation pressure
- Virial theorem
- Stellar scalings (how temperature, radius, luminosity depend on mass and why)
- Evolution of the Sun, main sequence, HR diagrams, star clusters
- White dwarfs, degenerate matter: mass & radius of WD
- Neutron stars, pulsars: mass & radius of NS
- Black holes. getting energy via accretion, Innermost Stable Circular Orbit
- Supernovae
- Star formation, Jeans mass
- The solar system (composition, age)
- Formation of the solar system, minimum mass solar nebula, frost line
- Methods for extrasolar planet detection, hot jupiters
- Diffraction limit, seeing, optical telescopes
- Signal-to-noise, Poisson noise
- Radio telescopes and interferometry
- Astronomy without photons
- Cosmological principles, redshift, Hubble law
- Components of the Milky Way; masses, luminosities, stellar populations, mass-to-light ratios
- Kinematics of the Milky Way: rigid body rotation, Keplerian rotation, constant orbital speed. Differential rotation.
- Measuring the Milky Ways rotation curve; rotation curves and dark matter
- Galactic center: Sagittarius A*, evidence for supermassive black hole

- Galaxy classification: the Hubble sequence, properties of spirals and ellipticals
- Spiral galaxies: rotation curves and circular velocities
- Spiral structure: the winding problem, spiral density waves
- Elliptical galaxies; type of ellipticals, velocity dispersion and Virial mass
- Galaxy clusters
- Groups, rich and poor clusters
- The Local Group
- Dark matter, mass-to-light ratios on different scales
- Active galaxies
- Luminosity and accretion rate of AGN, Eddington limit
- The extragalactic distance scale: distance indicators
- Expansion of the universe: Hubble flow, cosmological redshift
- Cosmology: Newtonian derivation of the Friedmann equation for a matter-only universe
- Critical density, open vs. closed Universe
- Scale factor and redshift
- Cosmic microwave background radiation: energy density and peak wavelength of blackbody radiation
- Temperature evolution with redshift
- The cosmological constant Λ
- equivalent mass density of dark energy
- Density parameters for matter, radiation, dark energy; total density parameter
- Measuring cosmological parameters: the cosmological constant and accelerating universe from Type Ia supernovae

Physics Synopsis:

Newton's Laws : most importantly, $F = ma$

Kinetic Energy : $K = \frac{1}{2}mv^2$

Gravitation : $F = GM_1M_2/r^2$ (on the surface of the Earth, $F = gm$)

Potential Energy : $U = -GM_1M_2/r$ (from gravity; on the surface of the Earth, $U = gmh$)

Centripetal Acceleration : $a = v^2/r = \omega^2 r$

Ideal Gas Law : $PV = nk_BT$

Circumference of a Circle : $2\pi r$

Area of Circle : πr^2

Surface Area of a Sphere : $4\pi r^2$

Volume of a Sphere : $\frac{4}{3}\pi r^3$

Radians : $180\,\text{deg} = \pi\,\text{radians}$, $\sin(\pi/2) = 1$, $\sin\pi = 0$, etc.

Arcseconds : $1\,\text{deg} = 60' = 3600''$

Small Angles : $\sin x \approx x$ for x very small and measured in radians. Also, $\tan x \approx x$, and $\cos x \approx 1$ (**draw these**)

Scientific notation : $A \times 10^a \cdot B \times 10^b = (AB) \times 10^{a+b}$

Formulas:

Distance $d[\mathrm{pc}] = 1/\mathrm{parallax[arcsec]}$

Doppler shift $(\lambda_{\mathrm{obs}} - \lambda_{\mathrm{emit}})/\lambda_{\mathrm{emit}} = \Delta\lambda/\lambda = -\Delta\nu/\nu = v_{\mathrm{rad}}/c$

Gravity and Tides accel$= GM/r^2$, $\mathrm{accel}_{\mathrm{tide}} \approx (GM/r^2)(2R/r)$; escape speed $v_{\mathrm{esc}} = \sqrt{2GM/R}$

Kepler $GM = \Omega^2 a^3 = \left(\frac{2\pi}{P}\right)^2 a^3$; $\frac{a_1}{a} = \frac{v_1}{v} = \frac{m_2}{M}$; for circular orbit $v = \sqrt{GM/a}$; for Solar units (years, AU, $M_\odot$): $P^2 = a^3/M$

Virial Theorem $E_{\mathrm{kin}} = -\frac{1}{2}E_{\mathrm{pot}}$; $E_{\mathrm{tot}} = E_{\mathrm{kin}} + E_{\mathrm{pot}} = \frac{1}{2}E_{\mathrm{pot}}$, [where $E_{\mathrm{pot,binary}} = -\frac{Gm_1m_2}{a}$ and $E_{\mathrm{pot,star}} \approx -\frac{GM^2}{R}$]

Number density $n = \rho/\mathrm{mean\,mass}$

Ideal Gas $P = nk_BT = \frac{\rho}{\mu m_H}k_BT$; typical KE per particle is $\frac{3}{2}k_BT$; energy density $u = \frac{3}{2}nk_BT = \frac{3}{2}P$

Degenerate Gas $\Delta x \Delta p \sim \hbar$; $E_{\mathrm{F}} = \frac{1}{2}\frac{p_{\mathrm{F}}^2}{m_e} \propto n_e^{2/3}$; $P \propto n_e E_{\mathrm{F}} \propto n_e^{5/3} \propto (\rho/\mu)^{5/3} \rightarrow R \propto M^{-1/3}$ [non-relativistic]

Photon Propagation $l_{\mathrm{mfp}} = \frac{1}{n\sigma} = \frac{1}{\kappa\rho}$; $t_{\mathrm{randomwalk}} = \frac{R}{l_{\mathrm{mfp}}}\frac{R}{c}$

Blackbody $L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$, $F = \sigma T_{\mathrm{eff}}^4$; $\lambda_{\mathrm{peak}} = 0.29\,\mathrm{cm}/T$

Light $c = \lambda\nu$, $E = h\nu = hc/\lambda$, $p = E/c$, energy density $u = aT^4$, pressure $P = (a/3)T^4$

Hydrostatic Equilibrium $\frac{\Delta P}{\Delta r} = \rho\frac{GM}{r^2} = -g\rho \rightarrow P \propto M^2/R^4$

Stars $T_c \propto M/R$, $\rho_c \propto M/R^3$, $P_c \propto M^2/R^4$

Timescales $\tau_{\mathrm{free-fall}} \sim \sqrt{1/G\rho}$; $\tau_{\mathrm{Kelvin-Helmholtz}} \sim \frac{GM^2/R}{L}$

Hydrogen Fusion $E = \Delta mc^2 \approx 0.7\%c^2$

Hydrogen Atom $E_n = -13.6\,\mathrm{eV}/n^2$

Jeans mass $M_J = 3k_BTR/2G\mu m_{\mathrm{H}}$; $\rho_J = (3/4\pi M^2)(3k_BT/2G\mu m_{\mathrm{H}})^3$

Equilibrium Temperature $T_p = T_s(1-A)^{1/4}(R_s/2d)^{1/2}$

Diffraction Limit $\theta = \lambda/D$

Poisson noise uncertainty on n counts is $\sqrt{n}$

redshift $z \equiv (\lambda_{\rm obs} - \lambda_{\rm emit})/\lambda_{\rm emit}$, $z = v/c$ for small z

Hubble law $v = H_0 d$, $H_0 = 70.4^{+1.3}_{-1.4}$km/s/Mpc

Hubble time $t_H = 1/H_0 = 13.8\,{\rm Gyr}$

Schwarzschild radius $R_{\rm Sch} = 2GM_{\rm BH}/c^2$

Circular velocity $v_c^2 = GM/r$

Virial mass $M = 5R\sigma^2/G$

AGN accretion rate $\dot{M} = L/\eta c^2 = 0.018 M_\odot\,{\rm yr}^{-1}(L/10^{37}\,{\rm W})(\eta/0.1)^{-1}$, where η is efficiency

Critical density $\rho_c = 3H_0^2/8\pi G$

Density from cosmological constant $\rho_\Lambda = \Lambda c^2/8\pi G =$constant

Constants:

bolometric absolute mag of the Sun $M_{\mathrm{bol},\odot} = 4.74$

Solar Mass $M_\odot = 2 \times 10^{30}\,\mathrm{kg}$

Solar Luminosity $L_\odot = 4 \times 10^{26}\,\mathrm{W}$

Solar Radius $R_\odot = 7 \times 10^{8}\,\mathrm{m}$

Earth Mass $M_\oplus = 6 \times 10^{24}\,\mathrm{kg}$

Earth Radius $R_\oplus = 6.4 \times 10^{6}\,\mathrm{m}$

AU $1.5 \times 10^{11}\,\mathrm{m}$

parsec $3.1 \times 10^{16}\,\mathrm{m} = 206265\,\mathrm{AU}$

year $3.16 \times 10^{7}\,\mathrm{s}$

c $3 \times 10^{8}\,\mathrm{m\,s^{-1}}$

G $6.7 \times 10^{-11}\mathrm{N\,m^2\,kg^{-2}}$

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$

Permittivity of free space $\epsilon_0 = 1/\mu_0 c^2$

Electric Charge $e = 1.6 \times 10^{-19}\,\mathrm{C}$

Electron volt $\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$

Planck's constant $h = 6.6 \times 10^{-34}\,\mathrm{J\,s}$, $\hbar = h/2\pi$

Boltzmann's constant $k_B = 1.4 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$

Radiation constant $a = 4\sigma/c$

Proton mass $m_p \approx m_n \approx m_H = 1.7 \times 10^{-27}\,\mathrm{kg}$

Electron mass $m_e \approx m_p/1800 = 9.1 \times 10^{-31}\,\mathrm{kg}$