Physics 718 Problem Set 4

Due: April 3, 2017

Problem 1 Common Envelope Evolution

In class, we discussed the energy formalism as applied to a particular WD system. Here let us explore the implications for the angular momentum formalism. Recall:

$$\frac{J_i - J_f}{J_i} = \gamma \frac{m_{1,\mathrm{env}}}{m_{1,i} + m_2}, \tag{1}$$

where J_i and J_f are the initial and final orbital angular momentum, $m_{1,i}$ and m_2 are the mass of the progenitor to 1 and the current WD mass of 2, and $m_{1,\mathrm{env}} = m_{1,i} - m_1$ is the mass of the envelope, and m_1 is the current WD mass of 1. Here we again consider WD 0135-052 which is a double WD system. The period of this system is $P = 1.5$ days and $m_1 = 0.47$ solar masses while the second WD is $m_2 = 0.52$.

a. Derive the formula for the ratio between ratio between the initial and final semi-major axis (also done in class). In particular prove:

$$\frac{a_f}{a_i} = \frac{m_{1,i}^2}{m_1^2} \frac{m_1 + m_2}{m_{1,i} + m_2} \left(1 - \gamma \frac{m_1 - m_1}{m_{1,i} + m_2}\right)^2 \tag{2}$$

b. Assume 1 in the CE evolution sits at the Roche radius. Following what we did in class find a_i; you may drop weak powers of the mass ratio here.

c. Find γ as a function of $m_{1,i}$. Plot γ as a function of $m_{1,i}$ between 1-3 solar masses. What values does γ fall in between?

d. Plot γ as a function of $m_{1,i}$ for the following systems: WD 0136+768, WD 1101+364, and WD 0957-666. Put it all on one plot if you can. What values does γ fall in between? You will find Table 1 of Nelemans et al (2000) A&A, 360, 1011 useful.

Problem 2 Supernovae in Binary Systems

Many stars are in binaries. If one of the two is massive enough, at some point its core will implode and its outer layers will be ejected in a supernova explosion. Here, we consider two stars, with masses M_1 and M_2, of which the first explodes, ejecting $M_{\rm env}$.

a. For simplicity, assume the envelope and its associated momentum disappear instantaneously. For what $M_{\rm env}$ (in terms of M_1 and M_2) will the binary unbind (i.e., have the individual stars fly apart)?

b. Brown et al. (2005, *Astroph. J.*, **622**, L33) found a "hypervelocity" star, which is likely a B9 main sequence star (mass of $3M_\odot$) traveling at $850\,\mathrm{km\,s^{-1}}$. They argue it is ejected from the Galactic center. But could it also have been ejected in a binary that became unbound in a supernova explosion? To verify, calculate the maximum velocity the star would have been left with if it was previously in a binary with a companion that exploded. For the companion, try both the lowest possible mass for a star than can go supernova ($\sim 8M_\odot$) and a really massive star ($\sim 50M_\odot$). *Note: for simplicity, you can assume the supernova left no remnant.*

Problem 3 Eccentric Binaries

Consider an initially circular binary system with masses M_1, M_2, and separated a. Star 1 goes supernova and loses ΔM of its mass on the way to becoming a neutron star. Assuming that the mass loss is symmetric, what is the eccentricity of the resulting binary e? What is the minimum that ΔM (and therefore e) could be?

Problem 4 Magnetic Supernova

In class, we discuss the neutrino mechanism for supernova. But the issue is coupling. Here we will consider a different mechanism – magnetic fields.

a. Consider a PNS with mass of 1.4 solar masses and radius of 30 km with an initial rotation period of T=10 ms. What is the rotational KE of this system?

b. Now imagine if there is a dipole magnetic field around the PNS with a magnitude of

$$B = B_0 \left(\frac{r_{\rm PNS}}{r}\right)^3 \tag{3}$$

If the magnetic field rotate rigidly, at what (cylindrical) radius does the speed of the magnetic field equal the speed of light – this is called light cylinder. What is the field strength at this radius?

c. Show that the total Poynting flux leaving through the light cylinder is to order of magnitude.

$$\dot{E} = B_{\rm lc}^2 c r_{\rm lc}^2, \tag{4}$$

where $r_{\rm lc}$ is the light cylinder radius and $B_{\rm lc}$ is the dipolar field strength at that radius. Find an expression in terms of T, B_0, $r_{\rm PNS}$.

d. For what values of B_0 and T for $r_{\mathrm{PNS}} = 30$ km, does the integrated energy loss over 10 s exceed 10^{51} ergs.