Physics 718 Problem Set 6

Due: May 1, 2017

Problem 1 GRB Numbers

Assume that GRBs occur at a fixed luminosity L_0 and at a constant rate per comoving volume per Myr r. Assuming Euclidean geometry, generate 10^4 "fake" GRBs uniformly distributed in volume between distances of 0 and 1 Gpc. Show that a $\log N$-$\log S$ plot (i.e., a plot of number brighter than some flux S against the flux S) has a power-law shape. Empirically determine the slope of the power-law.

Now repeat the same exercise but in a realistic ΛCDM cosmology. Over what range in distance does the power-law relation still hold? What is the minimum distance you need to observe GRBs to in order to determine that they are cosmological in origin?

Problem 2 Energy in Pulsars

We discussed how pulsars convert rotational energy into electromagnetic energy. The supernova explosion that made the pulsar had a whole lot of energy, and only a small part of it went into rotation. A pulsar will also have energy stored in its magnetic field. Pulsars also typically move pretty quickly, with velocities of hundreds of km/s, so they have kinetic energy of motion too.

What magnetic field B, translational velocity v, and period P are necessary for the magnetic, translational, and rotational energies of a pulsar all to be equal to its gravitational binding energy GM^2/R. Assume a mass of $1.4\,M_\odot$ and a radius of $10\,\mathrm{km}$. For the magnetic field, you can assume that the field is only inside the neutron star and that it is constant.

How do the results you got compare to the total energy released during a supernova, and are they reasonable given what else we know about pulsars?

Problem 3 Pulsar Spin-down

Gravitational radiation from an ellipsoidal neutron star (with ellipticity ϵ) gives rise to a torque that could spin-down a pulsar.

a. Starting from the GW energy loss rate:

$$\dot{E}_{\rm GW} = -\frac{32}{5}\frac{G}{c^5}I^2\epsilon^2\Omega^6$$

integrate the spin-down for gravitational radiation to get:

$$\Omega(t) = \Omega_0 \left(1 + 4\frac{\Omega_0^4}{\Omega_{\rm now}^4}\frac{t}{T_{\rm GW}}\right)^{-1/4}$$

With $T_{\rm GW} = \Omega_{\rm now}/|\dot{\Omega}_{\rm now}|$ measured now. Recast that law in terms of period P, initial period P_0, and period derivative $\dot{P}$.

b. A pulsar could have a combination of magnetic and gravitational radiation. Numerically integrate the spin-down equation to find $P(t)$ for a pulsar with moment of interia $I = 10^{45}\,{\rm g\,cm^2}$ and both $\epsilon > 0$ and $B > 0$. First, verify that as either ϵ or B goes to 0 you recover the analytic forms for the $P(t)$ evolution. Next, plot the evolution in the P-$\dot{P}$ diagram for $\epsilon = 10^{-7}$ and $B = 10^8$ G, 10^9 G, 10^{10} G, and 10^{11} G. What is the effective braking index for each of those curves? You might find Wade et al. (2012, PRD, 86, 4011) helpful.

Problem 4 Cosmic Rays

In class we discussed the Hillas plot which is based on confinement of high energy particle in accelerators. Lets try an energy argument. If the acceleration is due to electromagnetic processes, the minimum potential drop to accelerate particles to 10^{20} eV is $U = 10^{20}$ V. Assuming that the accelerating region is nearly a vacuum, so that the resistance is given by the impedance of the vacuum, $R = 1/(\epsilon_0 c) \approx 400\,\Omega$, what is the luminosity of the source? Given the luminosity density of UHECRs of $\mathcal{L}_{\rm cr} = 3\times10^{46}\,{\rm ergs\,Mpc^{-3}\,yr^{-1}}$, what is the number density of these sources?