Physics 718 Problem Set 5

Due: April 17, 2017

Problem 1 Compactness Problem

In this problem, we will find out why gamma-ray bursts have to be beam. For this problem consider a gamma-ray burst with timescales of $\delta t \sim 100$ and $L_{\rm iso} \sim 10^{53}$ erg/s, and observed gamma rays $dN/dE = f(E/E_0)^{-\alpha}$ where f is some constant and $\alpha = 2$. Let say that the observed energy of the photons go from $E_{\min} = 10keV$ to $E_{\max} = 1$ GeV. (a) Calculate the density of photons in the region $R = c\delta t$ assuming it is static. This calculation fixes f. Note that the total luminosity is

$$L = \int \frac{E}{dN} dE dE = L_{\rm iso}$$

(b) Find the fraction of photons f_e that might produced an electron-positron pair. To make this simple assume the incoming photons are of the same energy. Calculate the optical depth of this region to production of photons. (c) You should find that the optical depth is huge, which will give a BB spectrum, which is not seen. Now assume that radiation is produced in a moving region (toward the observer) with Lorentz factor Γ. The source region is now larger by $\Gamma^2 c\delta t$ (can you guess why the 2 factors of Γ?). In this case, redo part (a). You should find that photon density is small by a factor of Γ^{-4} (d) Redo part (b). You should find that f_e is small by a fraction of $\Gamma^{-2\alpha}$. For what value of Γ does the optical depth fall below unity.

Problem 2 2D Supernova Lightcurves

Your colleague, I.M. Insane, suggests that we have it all wrong with Type Ia supernova and that in fact that they are explosions of cosmic strings. He/She suggests that the explosion releases so much energy that it produces radioactive nickel from conversion of energy to matter. As we have done in class, work out the behavior of light curve of such an explosion by computing the time to peak brightness, early time light curve, and late time light curve. The important point is to get the scaling in relation to time, t. Note that we considered a 3-D spherical explosion in class, whereas we will consider a 2-D cylindrical explosion here. Also note that we will have to parameterize the mass as the mass per unit length, σ, because of the cylindrical nature.

Problem 3 Shock Breakout of SN

As discussed in class, the early time light curve is a useful way of constraining the physical size of the original progenitor. Take a look at Figure 4 of Piro, Chang, and Weinberg (2010). We will reproduce this simple plot to some degree for this problem. But we will simplify it and assume that it is a constant density sphere. (a) Show that the optical depth to the layer of photons that escape is always $\tau = c/v_{\rm exp}$. (b) Show that the luminosity decays as t^{-2}. (c) Make a plot of absolute magnitude in the wavelength range 300-700 nm. To do so, you will need to do the following: Normalized L to be with the initial total energy to be 10^{51} ergs and $R = 1000$ km. Compute the effective temperature of the radiation. Numerically integrate I_ν for $T_e(t)$ and $R(t)$ over a range of t that captures it rise and fall.

Problem 4 TDE

(a) When the tidally disrupted star falls back on the hole. Its angular momentum is always the same, i.e., determined by the tidal radius, r_t. Show that for these eccentric orbits that the resulting disk has a semimajor axis that $a = 2r_t$, assuming that angular momentum is conserved during the circularization process. (b) Ignoring the fact that the initial debris falls back at a super-Eddington rate, compute the disk spectrum of a disk that starts at $r_{t,0}$ down to the ISCO for a 1 solar mass star and a 10^6 solar mass black hole at a function of time. You should plot this for $t = 50$ d, 100 d, and 300 d. What waveband is the peak of the radiation in each epoch? (c) Now consider the optical signature. Assuming that the optical is in the Rayleigh Jeans side of the spectrum, show that the optical luminosity falls as $t^{-5/12}$