Hyperbolic PDE's

In [1]:
```
%matplotlib notebook
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.animation as ani
import numpy as np
import sympy as sp
```

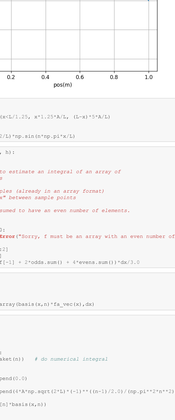
In [2]:
```
L = 1  # length of string
N = 51 # number of chunks
c = 1  # speed
A = 0.1 # amplitude of "pluck"

past = 0
present = 1
future = 2

x = np.linspace(0,L,N)
dx = x[1] - x[0] # what is dx?

dt = 0.5*dx/c  # this ensures that dx/dt > c

yp = np.where(x<L/1.25, x*1.25*A/L, (L-x)*5*A/L)

y = np.zeros((3,N),float)

y[present] = yp
y[future] = np.zeros(((1,N))

y
```

Out[2]:
```
array([[0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,],
       [0.    , 0.02255102, 0.04510204, 0.06765306, 0.09020408,
        0.0127551 , 0.01530612, 0.01785714, 0.02040816, 0.02295918,
        0.0255102 , 0.02806122, 0.03061224, 0.03316327, 0.03571429,
        0.0382653 , 0.04081633, 0.04336735, 0.04591837, 0.04846939,
        0.05102041, 0.05357143, 0.05612245, 0.05867347, 0.06122449,
        0.06377551, 0.06632653, 0.06887755, 0.07142857, 0.07397959,
        0.0765306 , 0.07908163, 0.08163265, 0.08418367, 0.08673469,
        0.08928571, 0.09183673, 0.09438776, 0.09693878, 0.09948980,
        0.09183673, 0.08163265, 0.07142857, 0.06122449, 0.05102041,
        0.04081633, 0.03061224, 0.02040816, 0.01020408, 0.        ],
       [0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,
        0.    , 0.    , 0.    , 0.    , 0.    , 0.    ,]])
```

In [3]:
```
plt.plot(x,yp)
```

Out[3]:
```
[<matplotlib.lines.Line2D at 0x1448eb793a30>]
```

In [4]:
```
fig1, ax1 = plt.subplots()
ax1.plot(x,yp,'r-')
ax1.grid()
ax1.set_aspect('equal')
plt.ylim(-.15,0.15)
```

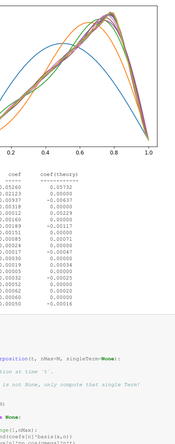


Out[4]:
```
(-0.15, 0.15)
```

In [5]:
```
def dotStep(y,isFirst=None, future=2, present=1, past=0):
    for i in range(1,N-1):
        if isFirst:
            y[future, i] = (2*y[present, i] + dt**2*c**2*(y[present,i-1] - 2*y[present,i] + y[present,i+1]))/dx**2/2.0
        else:
            y[future, i] = (2*y[present, i] - y[past, i] + dt**2*c**2*(y[present,i-1] - 2*y[present,i] + y[present,i+1]))/dx**2

    return y
```

In [6]:
```
y = np.zeros((3,N),float)

y[0] = yp

saveSTs = []

for i in range(200):
    y = dotStep(y, i==0, (i+1)%3, (i)%3, (i-1)%3)
    saveSTs.append(np.copy(y[(i+1)%3]))
```

In [7]:
```
fig, ax = plt.subplots()
ax.grid()
#ax.set_aspect('equal')
plt.xlabel('pos(m)')
plt.ylabel('y displacement (m)')
plt.title('Behavior of a Plucked String')

l, = ax.plot([x[0],x[-1]],[-1.1*A,1.1*A]) # note that this initial plot is just to establish the frame size.

animate = lambda i: l.set_data(x, saveSTs[i])

_ = ani.FuncAnimation(fig, animate, frames=len(saveSTs), interval=10, repeat=True)

plt.show()
```



In [8]:
```
def fs_vec(x):

    return np.where(x<L/1.25, x*1.25*A/L, (L-x)*5*A/L)

def basis(x, n):
    return np.sqrt(2/L)*sp.sin(n*sp.pi*x/L)
```

In [9]:
```
def simpson_array(f, h):
    """

Use Simpson's Rule to estimate an integral of an array of
    function samples

    f: function samples (already in an array format)
    h: spacing in "x" between sample points

    The array is assumed to have an even number of elements.

    """
    if len(f)%2 != 0:
        raise ValueError("Sorry, f must be an array with an even number of elements.")

    evens =  f[2:-2:2]
    odds = f[1:-1:2]
    return (f[0] + f[-1] + 2*odds.sum() + 4*evens.sum())*dx/3.0
```

In [10]:
```
def braket(ns):
    """
    Evaluate <n|f>
    """
    return simpson_array(basis(x,n)*fs_vec(x),dx)
```

In [11]:
```
N=20
coefs = []
coefs_th = []
ys = [[]]
sup = sp.zeros(N)
for n in range(1,N):
    coefs.append(braket(ns))   # do numerical integral

    if n%2==0:
        coefs_th.append(0.0)
    else:
        coefs_th.append(4*A*sp.sqrt(2*L)*(-1)**((n-1)/2.0)/(np.pi**2*n**2))  # compare theory

    ys.append(coefs[n]*basis(x,n))
    sup += ys[n]
    plt.plot(x,sup)

print("%10s\t%10s\t%10s" % ('n', 'coef','coef(theory)'))
print("%10s\t%10s\t%10s" % ('---','-----','------------'))

for n in range(1,N):
    print("%10d\t%10.5f\t%10.5f" % (n, coefs[n],coefs_th[n]))
```

```
         n          coef    coef(theory)
       ---          ----    ------------
         1       0.05232       0.05732
         2      -0.02123       0.00000
         3       0.00937      -0.00637
         4      -0.00318       0.00000
         5       0.00012       0.00229
         6       0.00160       0.00000
         7      -0.00189      -0.00117
         8       0.00151       0.00000
         9      -0.00085       0.00071
        10       0.00024       0.00000
        11       0.00030      -0.00049
        12      -0.00030       0.00000
        13       0.00019       0.00034
        14      -0.00005       0.00000
        15      -0.00032      -0.00025
        16       0.00052       0.00000
        17      -0.00062       0.00020
        18       0.00060       0.00000
        19      -0.00050      -0.00016
```

In [12]:
```
k1 = np.pi/L
k = 1.0

omegal = c*k1
T = 2*np.pi/omegal

def calcFourierSuperposition(t, nMax=N, singleTerm=None):
    #
    # get superposition at time 't'.
    #
    # if singleTerm is not None, only compute that single Term!
    #

    sup = np.zeros(N)

    if singleTerm is None:
        ys = [[]]
        for n in range(1,nMax):
            ys.append(coefs[n]*basis(x,n))
            sup += ys[n]*np.cos(omegal*t)
    else:
        sup += coefs[singleTerm]*basis(x,singleTerm)*np.cos(omegal*singleTerm*t)

    return sup

#
# Compute a solution with 20 terms
#

saveSTs = []
for t in np.linspace(0, T, 200):
    saveSTs.append(calcFourierSuperposition(t))
```

In [13]:
```
fig, ax = plt.subplots()
ax.grid()
#ax.set_aspect('equal')
plt.xlabel('pos(m)')
plt.ylabel('y displacement (m)')
plt.title('Behavior of a Plucked String, 20 terms')

l, = ax.plot([x[0],x[-1]],[-1.1*A,1.1*A]) # note that this initial plot is just to establish the frame size.

animate = lambda i: l.set_data(x, saveSTs[i])

_ = ani.FuncAnimation(fig, animate, frames=len(saveSTs), interval=10, repeat=True)

plt.show()
```


In [ ]:
```

```