All together now: covariances for 3x2pt analysis

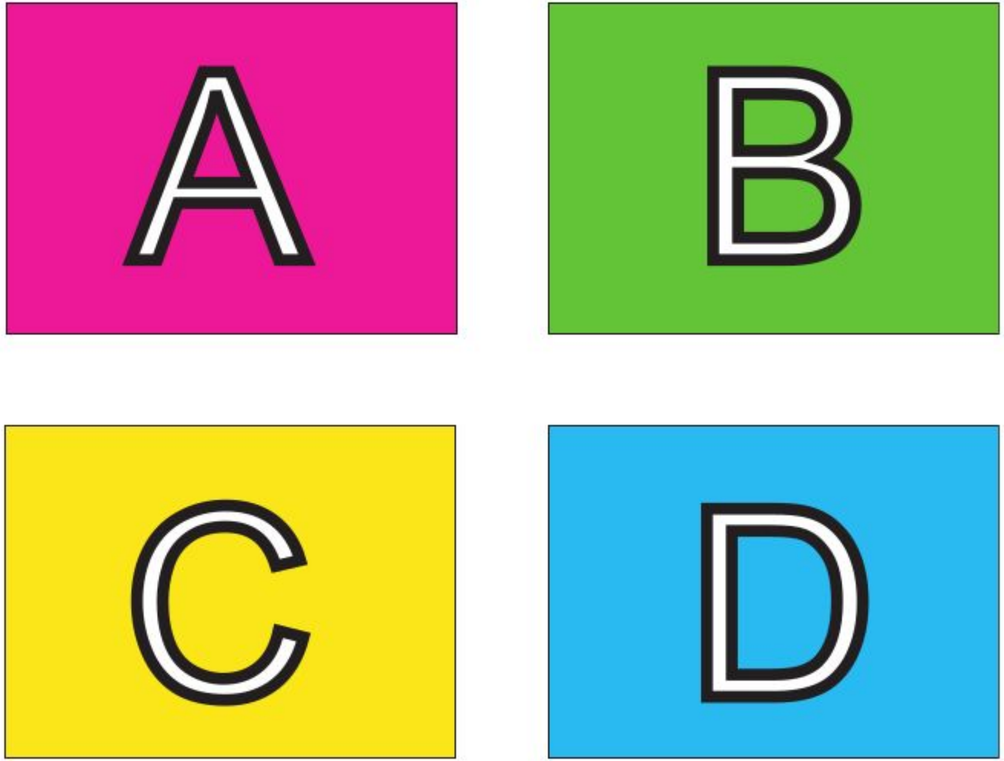
Danielle Leonard
DE School, August 1, 2022 (Chicago)

Housekeeping

- Have a piece of paper & pen in front of you.

- Think-pair-share questions

Think-pair-share: In-person



Think-pair-share: Online

- Type your answer in the chat box on zoom but don’t press enter straight away.

- When I ask for your answer, press enter.

- When it’s time to pair and share - use the zoom chat to discuss why you gave the answer you did. Ask questions.

- Then when I ask for an updated answer, put that in the chat.

- Make space for quieter folks - if you’ve started the discussion a couple times, let someone else go now.

Let’s try one:

Is this your first DE School?

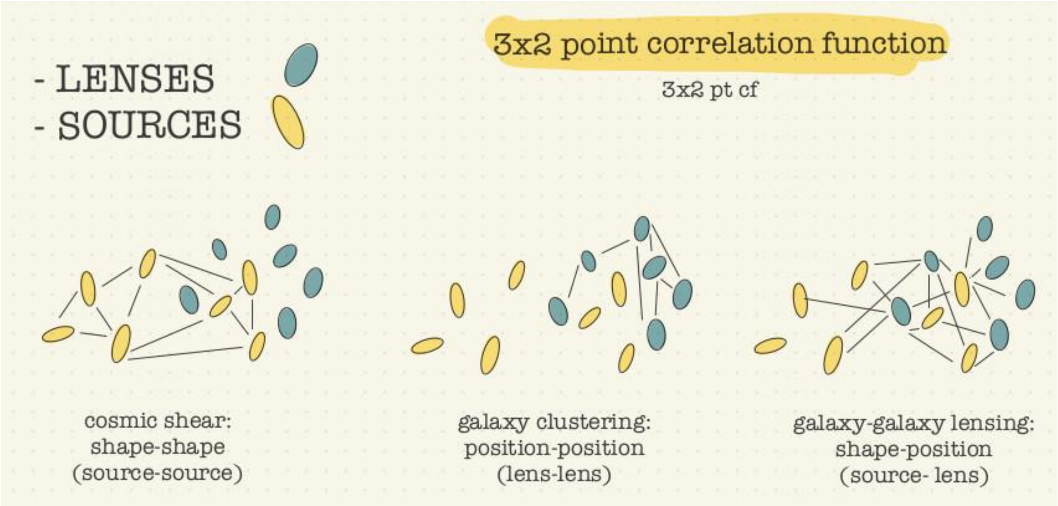
A: Yes

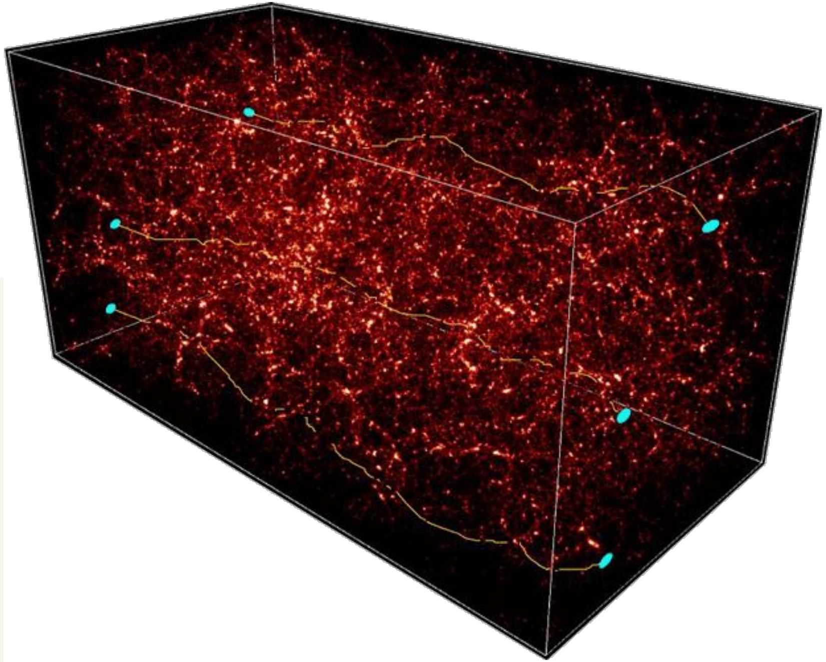
B: No

Lesson plan

- Part 0: Why do covariances matter for 3x2pt?
- Part 1: What is a covariance matrix?
- Part 2: 3x2pt analysis ingredients
- Part 3: What's inside a 3x2pt data covariance?

Part 0: Why do I care?





5+ lens and 5+ source z-bins (auto- and cross-correlation)

Each at tens of angular bins

→ 1000s of correlated data points

Graphic credit: left, Niko Sarcevic; right, hetdex.org

Part 1:
What is a covariance matrix?

Review: Mean and variance in the 1D case

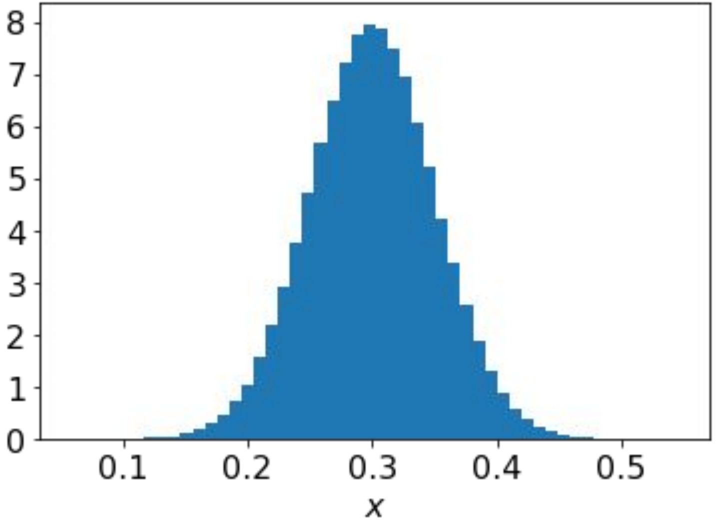
Mean
(1st moment):

$$\bar{x} = \frac{1}{n}\sum_{i}^{n} x_i$$

Variance
(2nd moment):

$$\sigma^2 =$$

What is the form of the equivalent expression for the variance? Write it down (don't worry about messing up an overall factor).



Histogram of points drawn from a 1D Gaussian distribution.

Review: Mean and variance in the 1D case

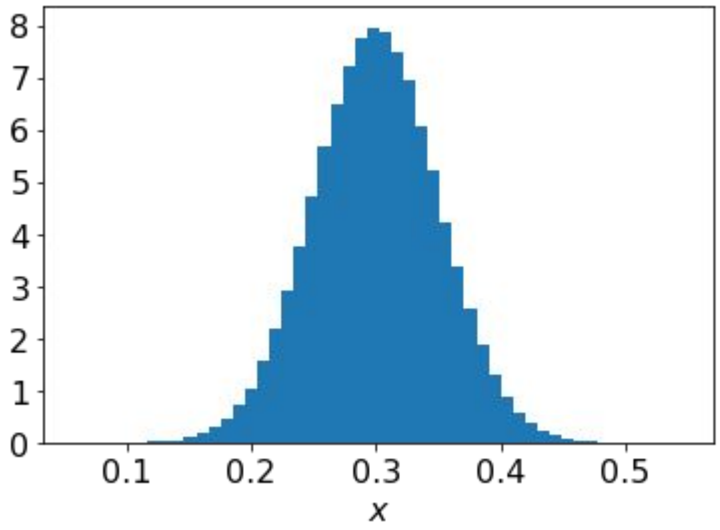
Mean
(1st moment):

$$\bar{x} = \frac{1}{n} \sum_{i}^{n} x_i$$

Variance
(2nd moment):

$$\sigma^2 = \frac{1}{n} \sum_{i}^{n} (x_i - \bar{x})^2$$

Variance: how much on average samples from a distribution deviate from the mean



Histogram of points drawn from a 1D Gaussian distribution.

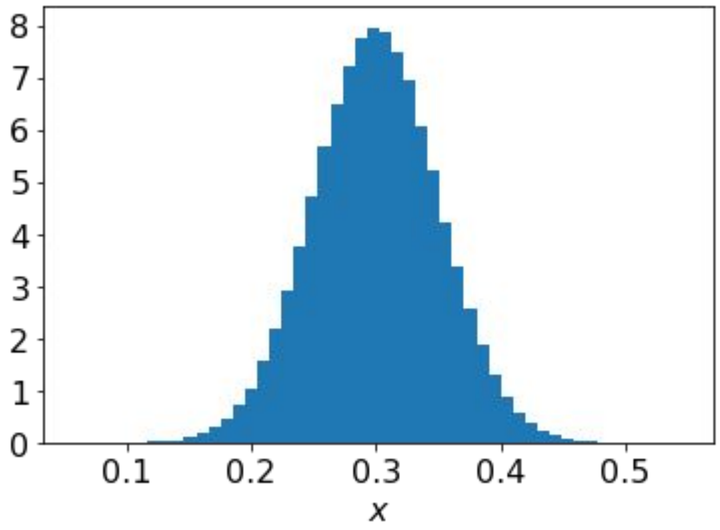
Review: Mean and variance in the 1D case

A 1D Gaussian distribution is **fully described** by its mean and variance.

What is the expression for a 1D Gaussian with mean $\bar{x}$ and variance σ_x ?

Write it down.

(Don't worry about the normalisation.)



Histogram of points drawn from a 1D Gaussian distribution.

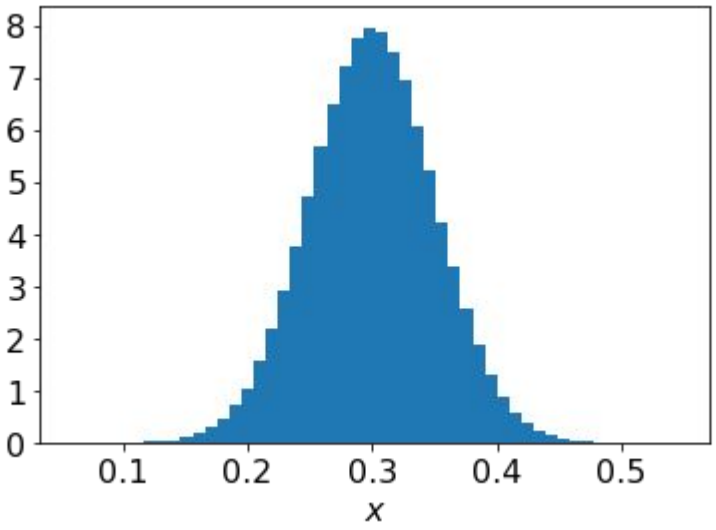
Review: Mean and variance in the 1D case

A 1D Gaussian distribution is **fully described** by its mean and variance.

$$P(x) \propto \exp\left(-\frac{(x-\bar{x})^2}{2\sigma_x^2}\right)$$

or

$$P(x) \propto \exp\left(-\tfrac{1}{2}(x-\bar{x})(\sigma_x^2)^{-1}(x-\bar{x})\right)$$



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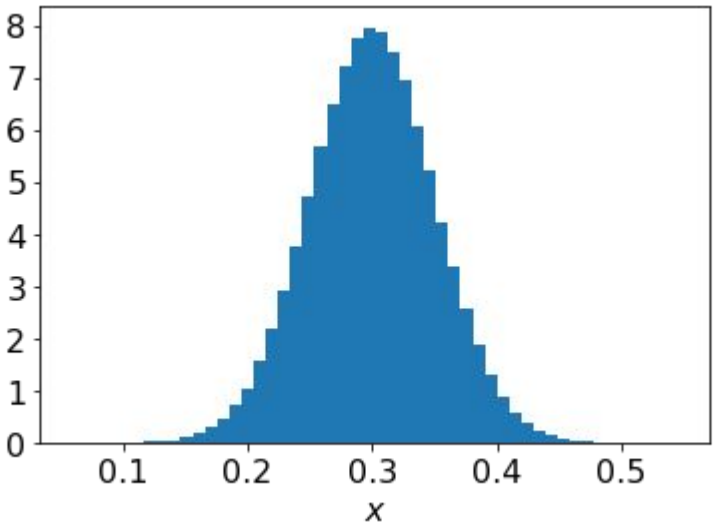


Histogram of points drawn from a 1D Gaussian distribution.

3rd moment (skewness), 4th moment (kurtosis), …. : 0 for a Gaussian but **not** 0 for general distributions.

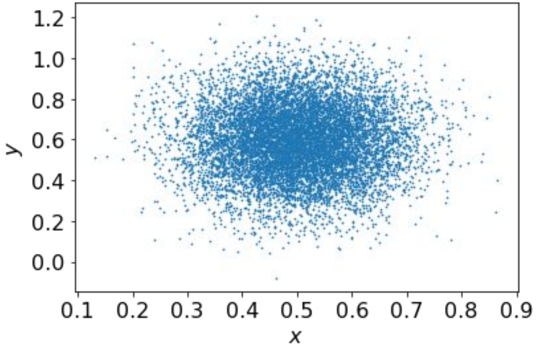
2D case: mean, variance and covariance

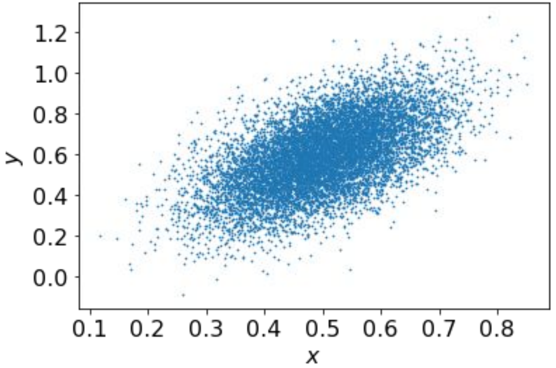
Means: $\bar{x} = \frac{1}{n}\sum_{i}^{n} x_i \qquad \bar{y} = \frac{1}{n}\sum_{i}^{n} y_i$

Variances: $\sigma_x^2 = \frac{1}{n-1}\sum_{i}^{n}(x_i - \bar{x})^2 \quad \sigma_y^2 = \frac{1}{n-1}\sum_{i}^{n}(y_i - \bar{y})^2$

Covariance: $\mathrm{Cov}(x, y) = ?$

What is the expression for the covariance between the x and y values? Write it down.





Same means, same variances, different covariance.

2D case: mean, variance and covariance

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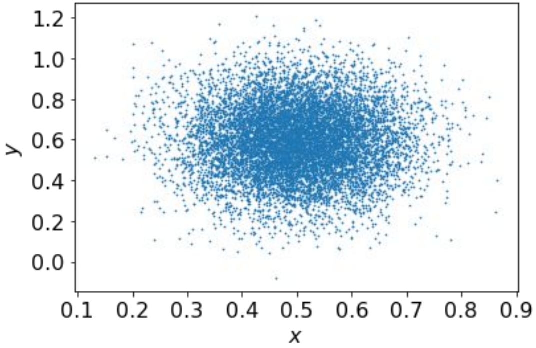
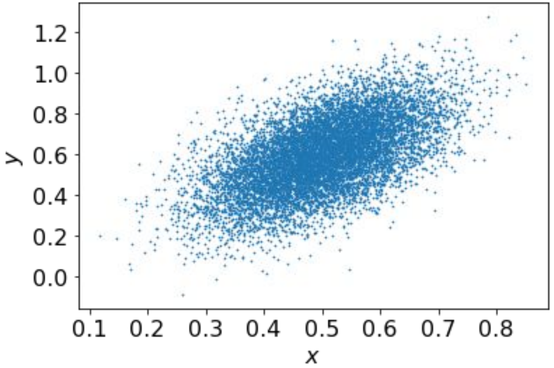
Covariance: $\mathrm{Cov}(x, y) = \frac{1}{n-1}\sum_{i}^{n}(x_i - \bar{x})(y_i - \bar{y})$





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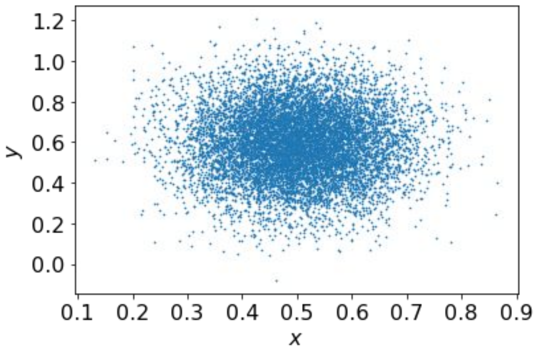
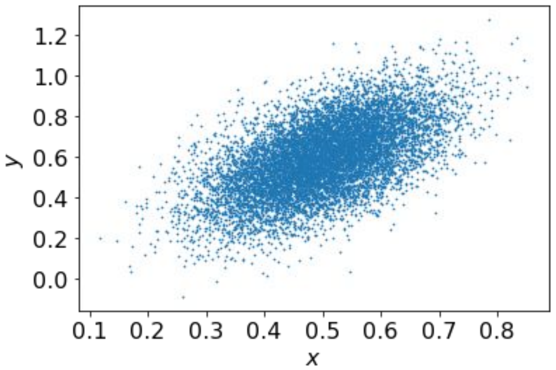
The covariance matrix:

$$C = \begin{bmatrix} \sigma_x^2 & \mathrm{Cov}(x,y) \\ \mathrm{Cov}(x,y) & \sigma_y^2 \end{bmatrix}$$





Same means, same variances, different covariance.

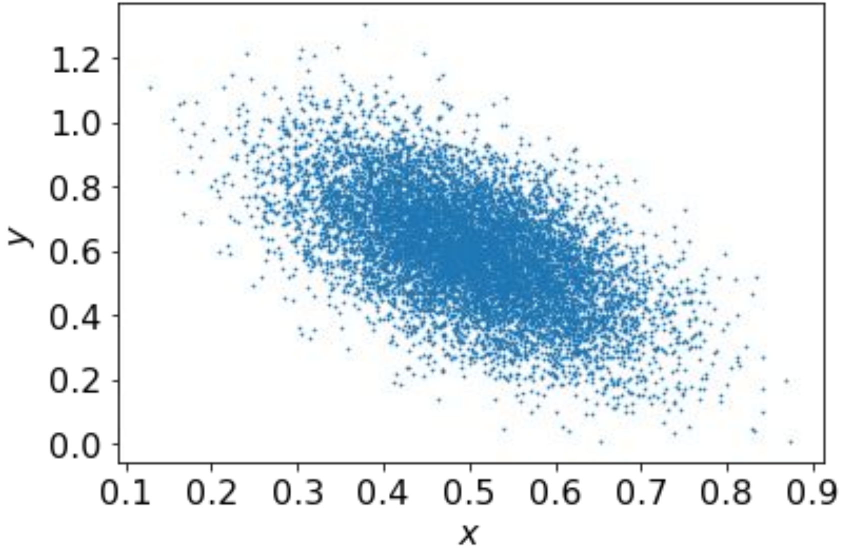
2D case: mean, variance and covariance

Q: Are the off-diagonal elements of the covariance matrix of this sample …

A: Zero?

B: Positive?

C: Negative?



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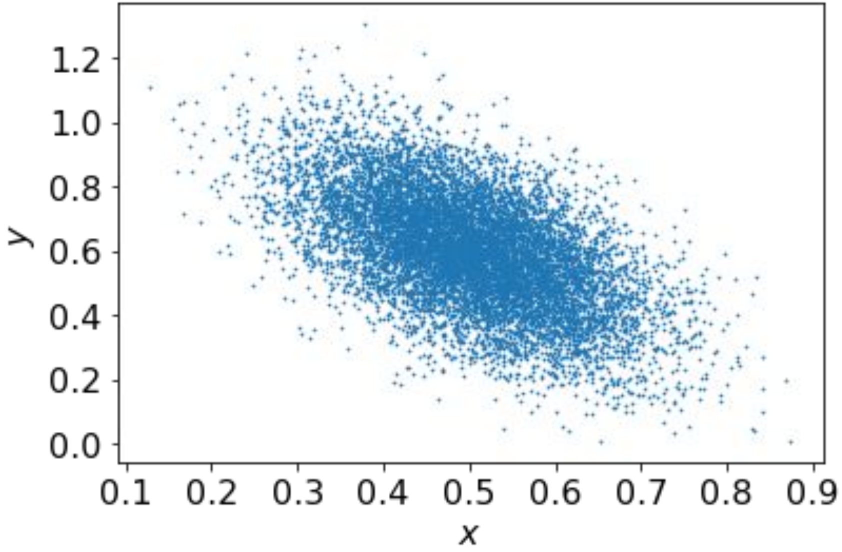

Part 2:
3x2pt analysis ingredients

Review: Bayes theorem

Bayes theorem:

$$P(\vec{\theta}|\vec{d}) \propto P(\vec{d}|\vec{\theta})P(\vec{\theta})$$

θ = parameters, d = data

We usually assume $P(\vec{d}|\vec{\theta})$ (the **likelihood**) is a multivariate Gaussian.

For a set of variables $\vec{d}$ with means $\vec{d}_{th}$ and a covariance matrix **C**, what is the form of a multivariate Gaussian likelihood? Try to write it down.

Review: Bayes theorem

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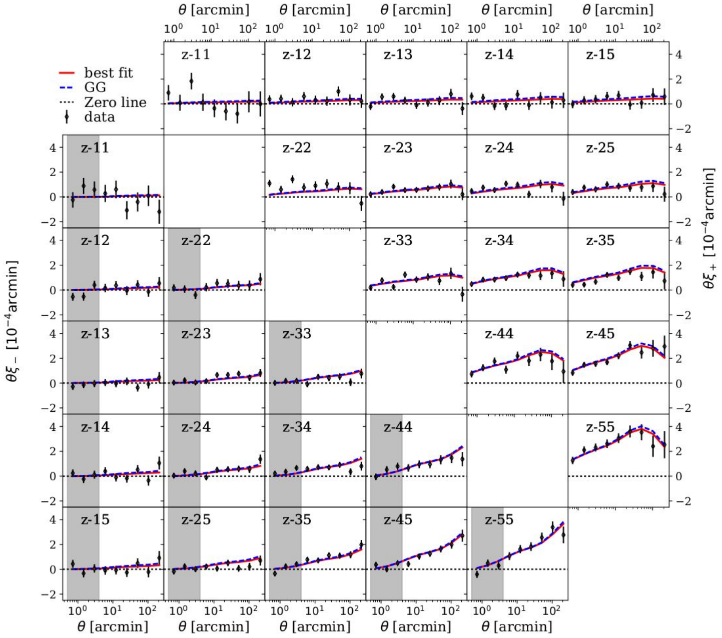
For a set of variables $\vec{d}$ with means $\vec{d}_{th}$ and a covariance matrix **C**, what is the form of a multivariate Gaussian likelihood? Try to write it down.

$$P(\vec{d}|\vec{\theta}) \propto \exp\left(-(\vec{d}-\vec{d}_{th}(\vec{\theta}))C^{-1}(\vec{d}-\vec{d}_{th}(\vec{\theta}))/2\right)$$

Constructing the 3x2pt likelihood: ingredients

$$P(\vec{d}|\vec{\theta}) \propto \exp\Big(-(\vec{d}-\vec{d}_{th}(\vec{\theta}))C^{-1}(\vec{d}-\vec{d}_{th}(\vec{\theta}))/2\Big)$$

1. Data vector (measurement), $\vec{d}$: $\xi_{\pm}^{i,j}(\theta)$ (shear x shear), $\gamma_t^{i,j}(\theta)$ (shear x position), $w^{i,j}(\theta)$ (position x position) where i, j run over tomographic bins.

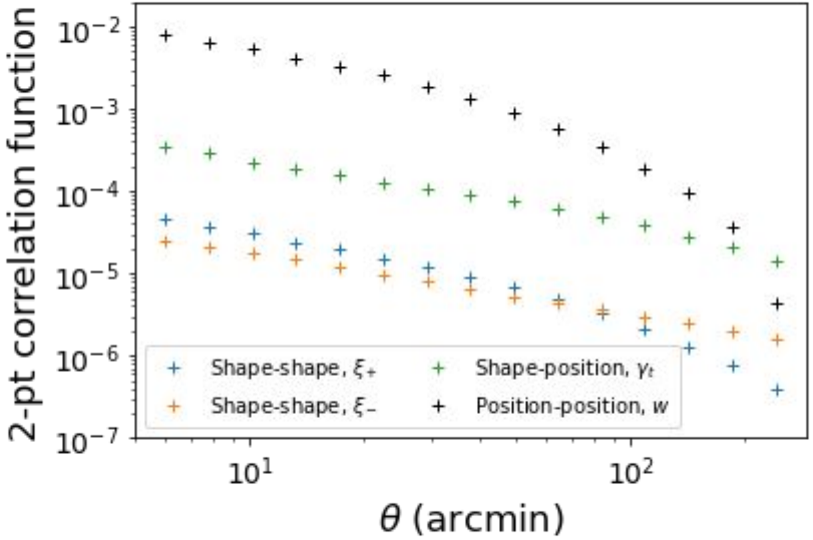


Cosmic shear 2-pt data vector, KiDS 1000 (Asgari+2021)

Constructing the 3x2pt likelihood: ingredients

$$P(\vec{d}\,|\vec{\theta}) \propto \exp\Big(-(\vec{d} - \vec{d}_{th}(\vec{\theta}))C^{-1}(\vec{d} - \vec{d}_{th}(\vec{\theta}))/2\Big)$$

2. $\vec{d}_{th}(\vec{\theta})$ Theoretically calculated data vector at a given set of parameters $\vec{\theta}$ and using a given model



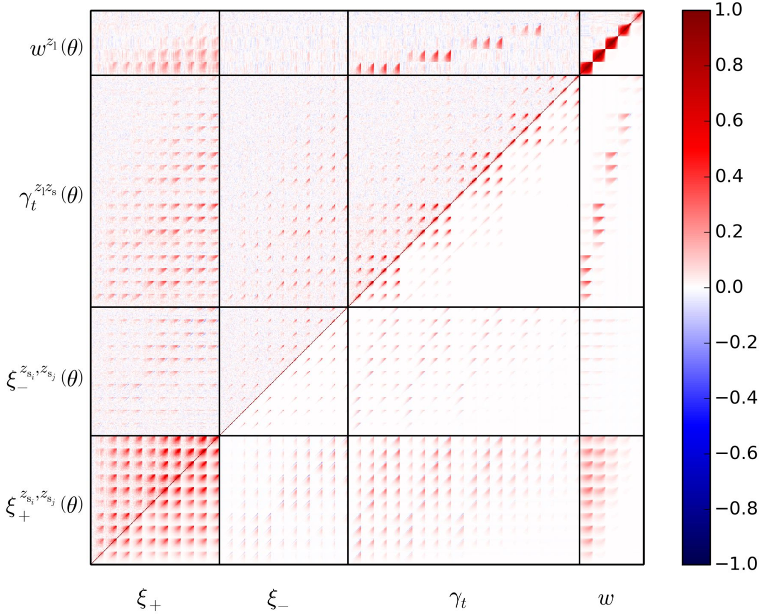
Examples of 2-point correlation functions calculated with CCL

Constructing the 3x2pt likelihood: ingredients

$$P(\vec{d}\,|\vec{\theta}) \propto \exp\left(-(\vec{d} - \vec{d}_{th}(\vec{\theta}))C^{-1}(\vec{d} - \vec{d}_{th}(\vec{\theta}))/2\right)$$

3. Data covariance matrix, **C**: characterises the variance and covariance on and between the points in the data vector.

On the right: Correlation matrix ρ for DES Y1 3x2pt analysis, $\rho_{ij} = C_{ij} / (\sigma_i \sigma_j)$ ('normalised covariance')



Constructing the 3x2pt likelihood: ingredients

Q: If $\vec{d}$ has 100 elements and $\vec{\theta}$ includes 20 parameters, what are the dimensions of **C**?

A: 10 x 10

B: 20 x 20

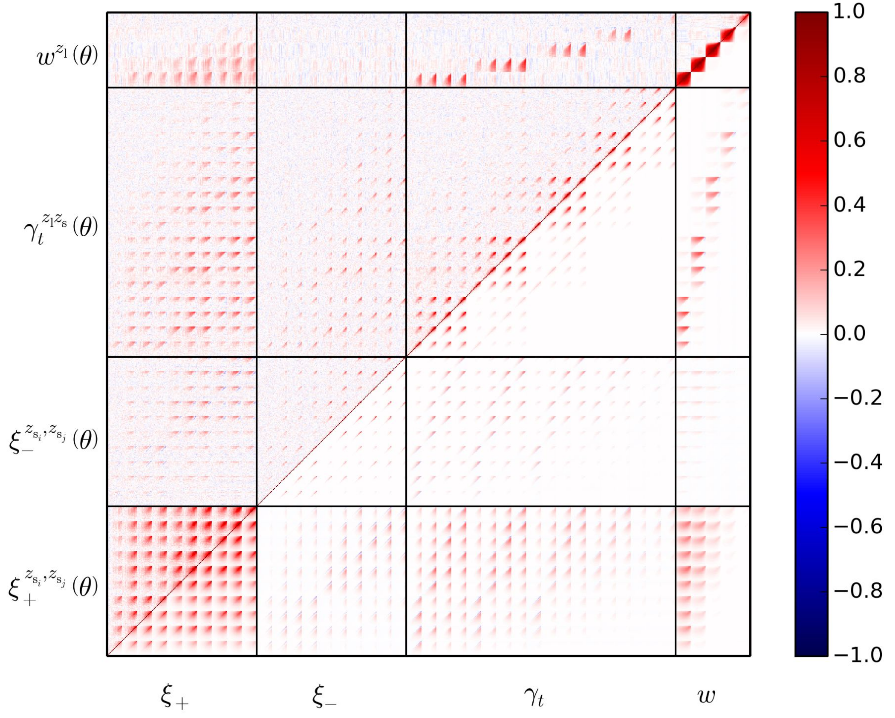
C: 100 x 20

D: 100 x 100

Group discussion: Structure in the correlation matrix

1. First look at the bigger blocks separated by solid black lines. Which one represents the single-probe correlation matrix for cosmic shear? For galaxy-galaxy lensing? For galaxy-clustering?

2. Within these bigger blocks, there is more block structure at a smaller level. Why? What varies within each block of this smaller structure?



Part 3:

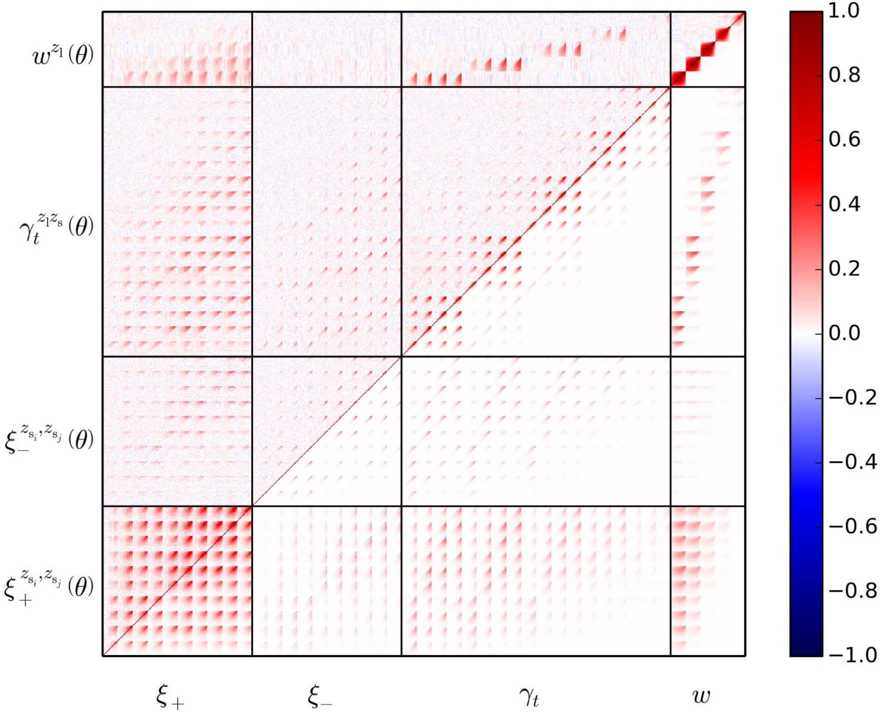
Unpacking the 3x2pt data covariance

What makes up the 3x2pt covariance matrix?

Qualitatively, we can think about what would induce either

- Uncertainty on the elements of our 3x2pt data vector and
- Correlations between these elements

Let’s introduce a few of the key culprits.



Poisson noise

Uncertainty associated with the occurrence (or non-occurrence) of **discrete random events.**

Think: “Is there a pair of galaxies in my sample within this angular separation bin, or is there not?”

Contribution ∝ $\frac{1}{n_i n_j}$, where n_i,n_j are the number of galaxy pairs in the relevant angular separation bin for each of the two 2-pt functions in question.

Cosmic variance

We could measure so many galaxies that Poisson noise is negligible - but the underlying fluctuations in the matter field are also the product of random processes.

We've only got one realisation of the Universe to observe.

The randomness associated with the 'selection' of that realisation introduces variance and covariance in 3x2pt measurements.

Poisson noise and Cosmic variance

Q: Which of these two contributions to the covariance do you think will produce contributions to the **diagonal** elements of the 3x2pt covariance matrix?

A: Poisson noise

B: Cosmic variance

Poisson noise and Cosmic variance

Q: Which of these two contributions to the covariance do you think will produce contributions to the **off-diagonal** elements of the 3x2pt covariance matrix?

A: Poisson noise

B: Cosmic variance

How can we calculate the data covariance?

- Analytically
  - Calculate all the terms up to a certain order in δ using theory.

- Simulations
  - Generate many different simulation realisations which capture the noise and covariance effects you care about. ‘Measure’ your data vector from these simulations and calculate the covariance.

- Jacknife
  - Take your galaxies and split up into patches on the sky.
  - Iteratively drop one of these patches at a time and calculate the data vector each time, then calculate the covariance between these data vectors.

Summary + Resources

- The data covariance is a key ingredient in our main 3x2pt cosmology analysis.
- Calculating it to the level we need for LSST is hard.
- Thinking about statistics is fun.

Questions to think about / ask experts this week which we didn't get to:

- We only talked about statistical uncertainty. How would we deal with systematic uncertainties?
- We only talked about 'Gaussian' terms in the covariance but there are also 'non-Gaussian' terms we need to calculate. What is 'non-Gaussian' for these terms?

Extra: Gaussian vs Non-Gaussian terms in the covariance

Poisson noise and cosmic variance are what we call Gaussian terms in the covariance: at most quadratic in δ (fluctuations in the matter field).

There are also non-Gaussian contributions: higher than quadratic order in δ.

These are a pain to calculate but for LSST we need at least some of them (see: MCPcov topical team's work).

Extra: Gaussian vs Non-Gaussian terms in the covariance

Group discussion:

In a 3x2pt analysis, what quantity are we assuming is Gaussian-distributed when we:

- Describe $P(d|\theta)$ with via a data covariance matrix, measured data vector and theoretical data vector?
- Calculate the data covariance matrix including only the Gaussian terms?

What are some examples of when each of these assumptions break down?