

# Fault Detection and Diagnosis in the DAMADICS Benchmark Actuator System – A Hidden Markov Model Approach

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**Abstract:** Early fault detection and diagnosis in chemical process monitoring represents a challenge to be overcome. Another one concerns the spatial overlapping problem among distinct fault classes, once some events may only be distinguished from the others by taking into account its order of occurrence. The hidden Markov model (HMM) technique is capable of providing information about the tendency of the process and of modelling ordered data. Hence, the goal is to investigate the contribution of this technique to both aspects related to process monitoring activities. The case study is based on the DAMADICS benchmark actuator system. Both abrupt and incipient faulty events were investigated. To the former, detection and diagnosis tasks were immediately satisfied; and to the latter, they were carried out in a progressive and correct course.

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## 1. INTRODUCTION

Process monitoring tasks in chemical industries are more and more usual in order to guarantee economic, safety and/or environmental goals. Undesirable situations may result in lower production, higher level of emissions, and equipment and personnel damages. A process monitoring activity has three major tasks: detection, diagnosis, and process recovery to a normal or at least a safety condition. Once it is unfeasible by visual inspection to identify such occurrences in its initial stage, computer-based systems play an important role with regard to early detection, fact that contributes to at least mitigate the potential risk of losses (Chiang, 2001). Due to complex plants, integrated operations, multivariable scenarios, and non-linear relationships, data-based methods are a matter of common sense concerning chemical process monitoring activities. A review about the fault detection and diagnosis subject is described in Venkatasubramanian *et al.* (2003a, b, c). The applications are usually based on residue metrics, and another way to approach the fault detection and diagnosis matter is making use of signal processing tools (Patton *et al.*, 2006).

Besides early detection, another challenge in process monitoring is the spatial overlapping problem among distinct fault classes. Therefore, once some events may only be distinguished from each other by taking into account its order of occurrence, it is worthy to consider a times series modelling. In this context, the so-called hidden Markov model (HMM) method appears as a promising decision-making tool for helping control room operators to accomplish chemical process monitoring tasks. This data-driven technique belongs to the signal processing field and constitutes an alternative approach for the development of Fault Detection and Diagnosis (FDD) systems. Thus, this work investigates the

performance of the hidden Markov model approach on accomplishing chemical process monitoring tasks. The case study is based on the DAMADICS actuator system, a control engineering benchmark (Syfert *et al.*, 2003; Bartys and de las Heras, 2003). Studies concerning applications of HMMs on chemical process monitoring are those by Almeida and Park (2005), Tokatli and Cinar (2004), Yangsheng and Ming (2004), Sun *et al.*, (2003), Chen and Chang (2000), Wong *et al.*, (2001, 1998), Bakhtazad *et al.* (2000).

The text is organized as follows. Next section presents the concept of hidden Markov models and section 3 describes the case study. Section 4 presents the methodology and section 5 discusses the results. Final considerations are summarized in section 6.

## 2. HIDDEN MARKOV MODELS

Every chemical process is under random influences due to an inherent variability present in, e.g. raw material lots, external temperature, feed stream compositions, and air humidity. Therefore, measurements of process variables may be considered realizations of an underlying stochastic process. This way, processes under normal operations are described by characteristic probability distributions, and changes in its conditions are responsible for modifying such underlying distributions. In case of using parametric ones, such as Gaussians, it means a deviation in at least one of its parameters, i.e. its mean and/or standard deviation (Venkatasubramanian *et al.*, 2003c). This is the motivation of putting together a signal processing tool and the chemical process monitoring activity once the hidden Markov model (HMM) method is capable of identifying changes of statistical nature in signals (composed by measurements of process variables). The most successful applications of HMMs are in the speech processing field, including both speech recognition

and speaker verification, since the seventies. Other areas of applications are handwriting recognition, image and video processing, financial market, telecommunications, and, recently, computational biology.

## 2.1 Mathematical Formulation

Hidden Markov models (HMMs) are a particular kind of Bayesian networks. Equation (1) is the factorization of its joint probability distribution for 1<sup>st</sup> order HMMs, where  $q_{1..T} = \{q_1, q_2, \dots, q_T\}$  is a sequence of states,  $o_{1..T} = \{o_1, o_2, \dots, o_T\}$  is a sequence of observations (outputs), and  $t$  is an integer-valued index.

$$P(q_{1..T}, o_{1..T}) = P(q_1)P(o_1 | q_1) \prod_{t=2}^T P(q_t | q_{t-1})P(o_t | q_t) \quad (1)$$

It can be noticed that HMMs are a doubly stochastic process, in which the former is responsible for the state-transitions ( $P(q_t | q_{t-1})$ ), whereas the latter is related to the observation-emissions ( $P(o_t | q_t)$ ). The HMM concept is an extension of Markov chains once the state-transitions rule follows the Markov property, i.e.  $q_t$  depends only on  $q_{t-1}$ . The difference between both classes of models concerns the second process, which does not exist in Markov models, for which the relationship between states and observations is deterministic. The *hidden* term in HMMs is exactly due to its introduction since the underlying sequence of states, i.e. the Markov chain, is not directly observable. Fig. 1 depicts the Bayesian network representation according to the factorization of the joint probability distribution in (1) (Ghahramani, 2001).

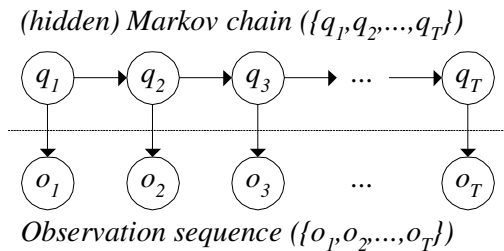


Fig. 1. Bayesian network representation for 1<sup>st</sup> order HMMs.

Table 1 shows the three parameters to specify discrete HMMs, where  $M_D$  is the number of distinct observation symbols in the emission probability distributions (each state has a particular distribution), and  $N$  is the size of the discrete state space. A compact notation for the parameters is given by  $\lambda$ , i.e.  $\lambda = (\pi, A, B)$ .

Table 1. Elements of discrete HMMs.

Parameters	Description
$A = \{a_{ij}\}$	State-transition probability distribution. $a_{ij} = P(q_{t+1} = j   q_t = i), 1 \leq i, j \leq N$ .
$B = \{b_j(k)\}$	Emission (symbol) probability distribution. $b_j(k) = P(o_t = v_k   q_t = j), 1 \leq k \leq M_D, 1 \leq j \leq N$ .
$\pi = \{\pi_i\}$	Initial state distribution. $\pi_i = P(q_1 = i), 1 \leq j \leq N$ .

For the continuous case, the  $B$  matrix is replaced by probability density functions. The usual representation for them is a finite mixture of Gaussian distributions, as in (2), where  $o_t$  is the observation vector at time  $t$ ,  $M_C$  is the number of mixture components per state,  $c_{jk}$  is the mixture component,  $\mu_{jk}$  is the mean vector, and  $\Sigma_{jk}$  is the covariance matrix, for the  $k$ th mixture component in the state  $j$ . Equation (3) is the stochastic constraints for  $c_{jk}$ . The parameters  $\pi$  and  $A$  are the same as in the discrete case.

$$b_j(o_t) = \sum_{k=1}^{M_C} c_{jk} N(o_t, \mu_{jk}, \Sigma_{jk}), 1 \leq j \leq N \quad (2)$$

$$\sum_{k=1}^{M_C} c_{jk} = 1, 1 \leq j \leq N \quad (3a)$$

$$c_{jk} \geq 0, 1 \leq j \leq N, 1 \leq k \leq M_C \quad (3b)$$

## 2.2 Fault Detection and Diagnosis with HMMs

An event ( $\lambda$ ) classification is more reliable when also considering observed data ( $O$ ), i.e. its conditional probability ( $P(\lambda | O)$ ) rather than only its *a priori* probability ( $P(\lambda)$ ). The former can be given by the Bayes rule, as in (4), where  $P(O | \lambda)$  is the likelihood of  $\lambda$  with respect to  $O$ . The probability distribution of the data ( $P(O)$ ) is independent of  $\lambda$ ; consequently (4) can be rewritten as in (5). Since in a fault detection task there is a single model, that characteristic of normal operation,  $P(\lambda) = 1$ , and hence the decision-making is based on the observed data, i.e. the likelihood function ( $P(O | \lambda)$ ), which is exactly the output of HMMs.

$$P(\lambda | O) = P(O | \lambda)P(\lambda) / P(O) \quad (4)$$

$$P(\lambda | O) \propto P(O | \lambda)P(\lambda) \quad (5)$$

In regard to a diagnosis task the winner HMM ( $\lambda^*$ ) is the one that maximizes the product between the likelihood function ( $P(O | \lambda)$ ) and the *a priori* probability distribution for the models ( $\lambda$ ). As it is in general assumed to be uniform, subjected to (7), once its determination is arduous, the decision-making process is also based on the likelihood function.

$$\lambda^* = \max_{\lambda} [P(O | \lambda)P(\lambda)] \quad (6)$$

$$\sum_j P(\lambda_j) = 1 \quad (7)$$

Thus, the goal of HMMs is to model sequential data. Fig. 2 shows the input-output relation for them, in which the input is a temporal sequence of  $T$  vectors ( $O = \{o_1, o_2, \dots, o_T\}$ ), and the output is a likelihood value ( $-\log[P(O | \lambda)]$ ), which measures the capacity of the model ( $\lambda$ ) in generating the observed data ( $O$ ). Hence, it can be defined as a sequential pattern recognition tool. The sequence (or pattern) can be a set of symbols (discrete case) or real vectors of same size (continuous case). These elements are called frames and each

one carries a piece of information about the system at a given time  $t$ . The logarithmic form is preferable in order to avoid underflow computational problems (Rabiner, 1989).



Fig. 2. Input-output relation for HMMs: the observation sequence ( $O$ ), and the likelihood function ( $-\log[P(O|\lambda)]$ ), respectively.

### 3. CASE STUDY

The case study is the DAMADICS benchmark (acronym for Development and Applications of Methods for Actuator Diagnosis in Industrial Control Systems) (Syfert *et al.*, 2003; Bartys and de las Heras, 2003). Its purpose is the development of fault detection and isolation methods for final control elements in the industry environment. It is possible to simulate nineteen abnormal events from three actuators, and a fault scenario is characterized by the fault type in conjunction with the failure mode, which can be abrupt (A) or incipient (I). The focus of this study is the actuator responsible for controlling the thin juice flow rate into the first stage of the sugar evaporation station, as in Fig. 3. The goal of this unit is to concentrate the syrup from 14 % to 70 %. An alarm system warns if the level of syrup in the evaporator goes beyond safety operating limits. In case it is too low, an overheating of the evaporator chamber may occur, which represents a risk of explosion; on the other hand, a carry-over may contaminate subsequent processing units in the mill.

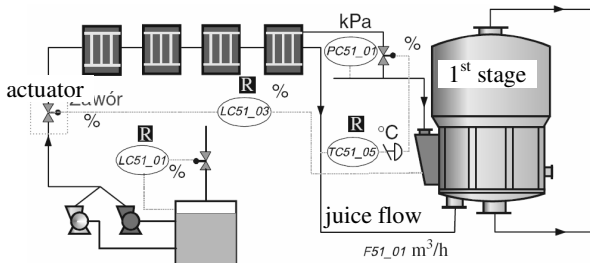


Fig. 3. Actuator responsible for controlling the thin juice flow rate into the 1<sup>st</sup> stage of the sugar evaporation station.

Fig. 3 shows the actuator model block. The input variables are the controller output signal for the valve stem position (CV), the upstream and downstream pressures ( $P1$  and  $P2$ , respectively), the fluid temperature ( $T1$ ), and the vector of faults ( $f$ ), and the outputs are the juice flow through the valve ( $F$ ), and the position of the rod displacement ( $X$ ). A detailed description about the DAMADICS benchmark is in Bartys *et al.* (2006). The output variables of the actuator model ( $F$  and  $X$ ) are employed to construct the observation sequences ( $O = \{o_1, o_2, \dots, o_p, \dots, o_T\}$ , where  $o_i = [F_{i=1} \ X_{i=1}]'$  and so on).

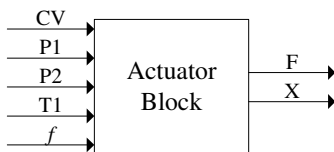


Fig. 3. General scheme for the actuator model.

Table 2 presents the three faults under analysis, which belongs to the class of external faults, and the four fault scenarios derived from it. Since it is unfeasible for mills' operators to directly monitor root causes of faulty events (the hidden process), the idea is to infer about the state of the evaporation station by monitoring key process variables (the observable process). A single fault is considered to occur at a time.

Table 2. Fault scenarios.

Fault	Fault description	Failure mode <sup>†</sup>
<i>f16</i>	Positioner supply pressure drop	A
<i>f17</i>	Unexpected pressure change across the valve	A / I
<i>f18</i>	Fully opened by-pass valve	A

<sup>†</sup> A: Abrupt, I: Incipient.

### 4. METHODOLOGY

#### 4.1 Data Generation Step

A simulator of the actuator model (available for MatLab-Simulink) is employed to generate both normal and faulty data. Three sets are obtained (training, validation and test), each one containing observation sequences ( $O$ ) for each operating condition, whose number depends on the failure mode. To consider process disturbance and measurement noise artificial noise are added to all signals. Due to the choice of using discrete HMMs a vector quantization procedure on the original signals is needful. The  $k$ -Means algorithm is used in this task and the number of distinct output symbols is a parameter to be determined.

#### 4.2 Model Identification Step

The aim of this step is to obtain a HMM for each operating condition: the normal and the three abrupt faults. The data set concerning the incipient fault is only used in the test step. Initially, a plenty of models are generated by varying the number of distinct symbols ( $M_D$ ), from 2 up to 64 in step of 2, and the number of states of the Markov chain ( $N$ ), from 2 up to 12. The topology of the models is a fixed parameter being used the left-to-right one. In this particular case,  $a_{ij} = 0$  for  $j < i$ , i.e. it is not possible to jump to a state with a lower index. Furthermore, for the initial state probabilities;  $\pi_i = 0$  if  $i \neq 1$  and  $\pi_i = 1$  if  $i = 1$ , once the state sequence begins in state 1 and ends in state  $N$ . This topology is useful whenever the statistical properties of the signals changes in a periodic way as is the current case. This procedure generates 1408 models (i.e. number of distinct symbols  $\cdot$  number of hidden Markov models  $\cdot$  number of states =  $32 \cdot 4 \cdot 11 = 1408$ ). The model selection is based on the likelihood function ( $-\log[P(O|\lambda)]$ ) calculated onto the validation set. The final result is a single model for each operating condition: the normal ( $N$ ) and the abrupt faulty events (*f16A*, *f17A* and *f18A*). In order to refine the models parameters a reestimation procedure using both sets (training and validation) is accomplished.

### 4.3 Test Step

A data set representing an operating condition in particular is fed simultaneously to all previous identified HMMs. This action is repeated for each fault scenario (cf. Table 2). The first eight observation sequences ( $O_1, O_2, \dots, O_8$ ) are related to the normal operation, and the fault start up is at  $O_9$ . A continuous feed of sequences allows the investigation of both the performance of the HMM representing the normal operation ( $N$ -HMM), issue regarding the detection task, and the behaviour of the faulty models ( $f16A$ -,  $f17A$ - and  $f18A$ -HMM), issue concerning the diagnosis task. The Hidden Markov Model Toolbox for MatLab (Murphy, 1998) was employed in both the model identification and the test steps.

## 5. RESULTS AND DISCUSSION

### 5.1 HMMs Identification

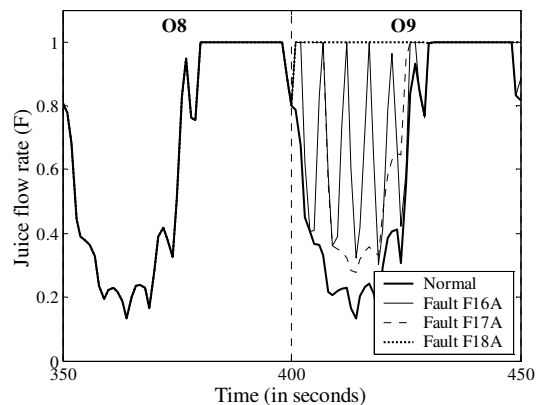
Fig. 4(a) shows the temporal patterns related to the normal operation and the abrupt fault events to the juice flow rate signal ( $F$ ). It can be observed the changes in the signals trajectories after the faults start up, i.e. from  $O_8$  to  $O_9$ . The data are scaled to the range  $[0, 1]$ . Fig. 4(b) shows the spatial overlapping among these distinct conditions mainly between the normal operation and the abrupt fault  $f16$  ( $f16A$ ). Therefore, it can be verified the value of considering a sequential data modelling, such as HMM, since the temporal order of the events (given by the observations,  $o_i$ ) is relevant information regarding the discrimination of operating conditions. Table 3 summarizes the results for the model identification step. The final number of distinct symbols ( $M_D$ ) was 46, that is, each state of the Markov chain is capable of emitting 46 distinct observations. It can be noted that there is a single codebook and what distinguishes one operating condition from the others is the temporal order of the events. Finally, the discrepancy among the mean likelihood values suggests that these models may be employed to perform a fault isolation task (a subject beyond the scope of this paper), an action that limits the search space of possible causes for abnormal events, which contributes to mitigate potential losses.

Table 3. Final result for the model identification step (corresponding to the codebook of size 46).

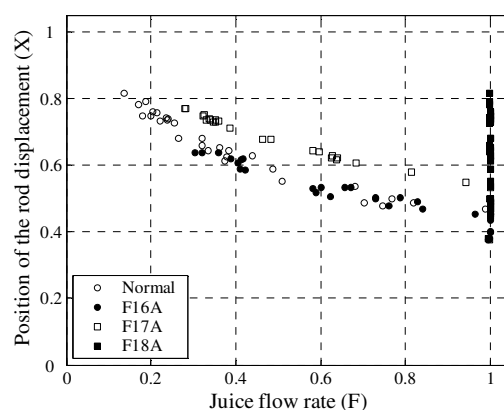
HMM	Operating condition	Code	Number of states (of the Markov chain)	$-\log[P(O \lambda)]$ (mean value)
1	Normal	$N$	9	112.2
2	Abrupt fault	$f16A$	2	138.3
3	Abrupt fault	$f17A$	7	133.4
4	Abrupt fault	$f18A$	10	104.2

### 5.2 Detection and Diagnosis Tasks

A set of observation sequences ( $O$ ) belonging to a particular fault scenario are simultaneously fed to all four models. The results for both monitoring tasks, detection and diagnosis, are shown at the same plot. Its presentation was split according to the failure mode (abrupt and incipient).



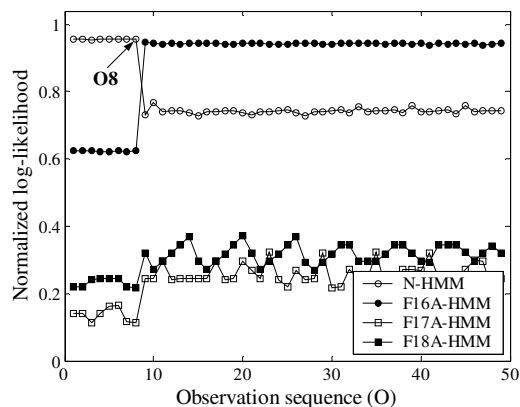
(a)



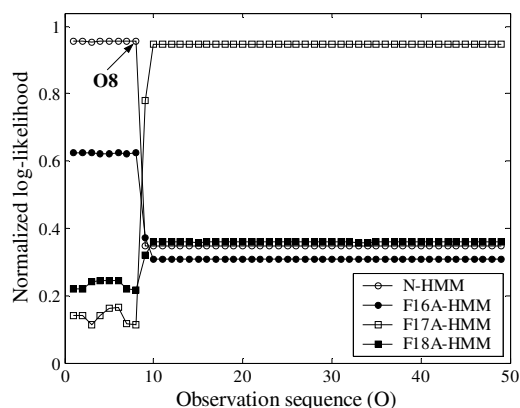
(b)

Fig. 4. (a) Temporal patterns for both the abrupt fault events ( $f16A$ ,  $f17A$  and  $f18A$ ) and the normal operating condition to the juice flow rate signal, and (b) the spatial overlapping problem among them.

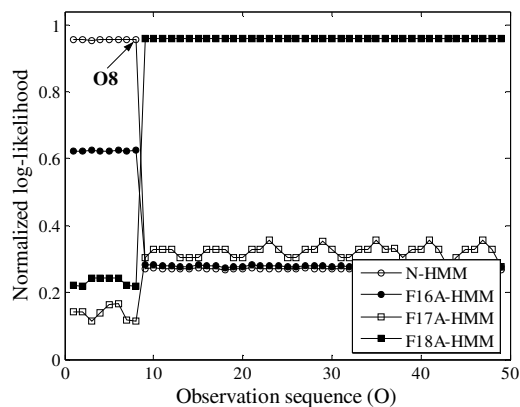
**Abrupt faults.** Fig. 5 shows the behaviour of all HMMs when subjected to the abrupt faults,  $f16$ ,  $f17$ , and  $f18$ , respectively. Each point represents a likelihood value for an observation sequence ( $O$ ) in particular. The following comments refer to  $f16$ . (A similar analysis is valid for  $f17$  and  $f18$ .) At first, as it was expected, the highest output values were generated by the model characteristic of normal operations ( $N$ -HMM), once the first eight sequences ( $O_1, O_2, \dots, O_8$ ) are composed by measurements collected with the actuator operating in normal conditions. After the fault occurrence, the model identified for the abrupt fault  $f16$  ( $f16A$ -HMM) becomes the winner, whereas the values generated by the other faulty models are kept low. In other words, the probability of  $N$ -HMM in generating the observation sequences after the fault occurrence diminishes, on the other hand  $f16A$ -HMM becomes the most probable generator of such sequences, which is indicative of the occurrence of this fault, related to the positioner supply pressure drop (cf. Table 2) in the actuator responsible for controlling the thin juice flow rate into the 1<sup>st</sup> stage of the evaporation station. Therefore, both monitoring tasks (detection and diagnosis) were immediately satisfied after faults occurrence, in response to abrupt changes in at least one of the monitored variables ( $F$  and  $X$ ) that compose the observations ( $o_i$ ) in the sequences ( $O$ ).



(a)



(b)



(c)

Fig. 5. Fault detection (given by the model characteristic of normal operations: *N*-HMM) and diagnosis (given by the faulty models: *f16*-, *f17*-, and *f18*-HMM) when the process is subjected to the abrupt faults: (a) *f16*, (b) *f17*, and (c) *f18*.

**Incipient fault.** The model identified for the abrupt fault *f17* was employed to detect the incipient fault *f17*. The first eight observation sequences also refer to the normal condition. Fig. 6 shows the results when all models are subjected to this event. After fault start up, the probability of *f17A*-HMM in generating the sequences becomes higher in comparison to *N*-HMM. This fact can be explained by the analysis of the monitored signals, once they go towards the pattern for the

abrupt fault *f17*. Lower likelihood values from the model characteristic of normal operations are indicative of possible upsets. Such tendencies are valuable information for mills' operators so that to take decisions in advance in order to recover the process. About the detection task, there is a delay of one observation sequence, since the first lower output value from *N*-HMM corresponds to the 10<sup>th</sup> sequence (*O*<sub>10</sub>). The fault strength (*f*) at this time is small, equal to 5.6%. *f17A*-HMM becomes the winner-model after the 18<sup>th</sup> observation sequence (*O*<sub>18</sub>) when *f* is equal to 27.8%. The output of this model becomes flat after more six sequences, i.e. at *O*<sub>24</sub>, where *f* equals 44.5%. Such stability for the likelihood function confirms that there is an unexpected pressure change across the valve (cf. Table 2). This incipient event is only completely developed (i.e. *f* = 100.0%) at *O*<sub>44</sub>. This example illustrates the possibility of reaching early fault detection and diagnosis, with a monitoring system based on the hidden Markov model method, once it provides information about the tendency of the process.

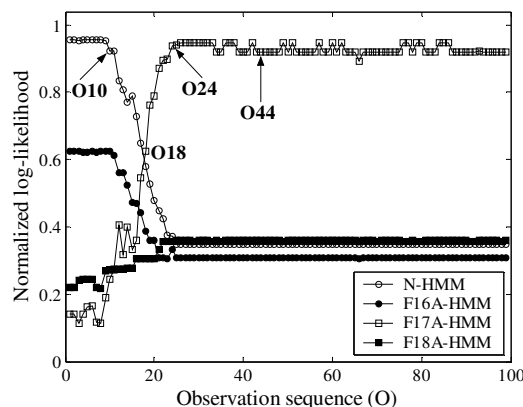


Fig. 6. Fault detection (given by the model characteristic of normal operations: *N*-HMM) and diagnosis (given by the faulty models: *f16*-, *f17*-, and *f18*-HMM) when the process is subjected to the incipient fault *f17*.

## 6. FINAL CONSIDERATIONS

A fault detection and diagnosis system based on hidden Markov models was developed. Using the DAMADICS benchmark actuator system it was possible to verify its capacity in accomplishing process monitoring tasks. The model characteristic of normal operations was able to detect abrupt events immediately after its occurrence in response to abrupt changes in at least one of the monitored variables (i.e. *F* and/or *X*). In the same way, faulty models correctly diagnosed the corresponding undesirable events. (This study can be expanded to all possible abnormal events in the benchmark.) This sequential pattern recognition tool plays an important role when the identification of different operating conditions requires a temporal evolution analysis, once some patterns may only be distinguished from each other by considering the sequence of events. In addition, by using a temporal sequence of observations as model input, a reinforcement of the discrepancy between normal and abnormal conditions may occur, which suggests early fault detection in comparison to methods where the decision-making is based on residues. Another positive aspect is the

resulting trend plot, given by the likelihood values, which may provide information about the current/future state of the process condition. In brief, the results are very encouraging concerning the application of hidden Markov models in chemical process monitoring activities.

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