Derive the expression $\rho(E)dE = 4\pi\left(\frac{2m_e}{h^2}\right)^{3/2}\sqrt{E}dE$. This is problem 5.6 in Tavernier.

You begin with the the expression for the energy levels of the electron in a 3-D well.

$$E = \frac{p^2}{2m} = \frac{\pi^2(hc)^2}{a^2 2mc^2}(n_1^2 + n_2^2 + n_3^2) \tag{1}$$

where n_i are all positive integers. The following text is taken directly from Tavernier's solution: The energy levels of an electron enclosed in an infinitely deep potential well in one dimension can be found by solving the Schrdinger equation. This problem is solved in any good textbook on quantum mechanics. Here we only give a simple and heuristic derivation of the result. A wave is associated with every electron and the corresponding wavelength is given by

$$\lambda = \frac{h}{p} \tag{2}$$

The energy eigenstates of the electron correspond to the values of the momentum such that the associated wavelength satisfies

$$\frac{n\lambda}{2} = a \tag{3}$$

where n is a positive integer and a the dimension of the well.We therefore have the following relations

$$p = \frac{\pi\hbar}{a}n \tag{4}$$

$$E = \frac{p^2}{2m} = \frac{\pi^2(\hbar c)^2}{a^2 2mc^2}n^2 \tag{5}$$

In the three-dimensional well this becomes Equation 1.

The energy eigenstates of the electron are given by the positive integer values of the quantum numbers n_1, n_2 and n_3. To derive the number of electron states in the infinitesimal energy interval $E, E + dE$, we slightly rewrite this as

$$R^2 = (n_1^2 + n_2^2 + n_3^2) \qquad \text{with} \qquad R^2 = E\frac{a^2 2mc^2}{\pi^2(\hbar c)^2} \tag{6}$$

Each electron state corresponds to a point of a regular grid in a three- dimensional Euclidian space. The probability to have an electron state in the infinitesimal energy interval $E, E + dE$ is equal to the volume of one octant of the skin of a sphere with radius R and with thickness dR, divided by the number of points per unit volume. Since the number of points per unit volume is equal to one, the probability to have one electron state in the interval is simply given by.

$$\rho(R)dR = \frac{1}{8}4\pi R^2 dR \tag{7}$$

To obtain the probability density of states per unit volume in the variable E rather than in the variable R, we only need to make a change of variables and divide by a^3.

$$\rho(E) = 4\pi \left(\frac{2m}{h^2}\right)^{3/2} \sqrt{E}dE \tag{8}$$

In this last result we have multiplied the density of states by a factor two to take into account the fact that an electron can have two spin states.