Mass Measurements

Lecture 1 – October 7, 2015

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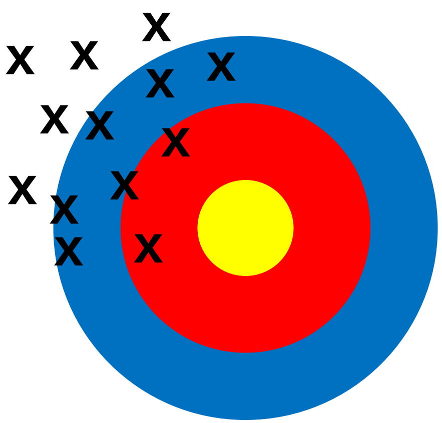
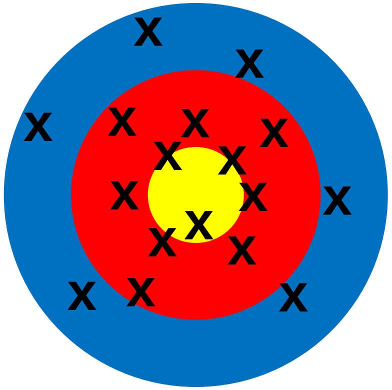
Precision v. Accuracy



Not Accurate
Not Precise



Accurate
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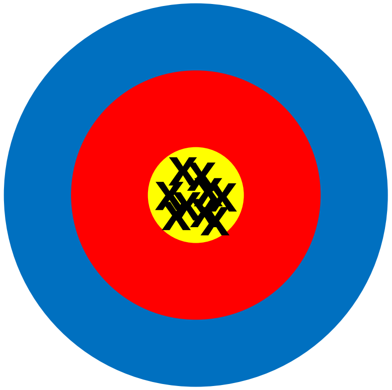
Precise



Accurate
Precise

Mass

- The property of a body to attract another body via gravitation
  - Gravitational mass

$$F = \frac{Gm_1 m_2}{r^2}$$

- Object's resistance to changing acceleration
  - Inertial mass

$$\vec{F} = m\vec{a}$$

- The sum of all the rest energy in a body
  - Mass-energy relation

$$m = \begin{pmatrix} Nm_n + Z(m_p + m_e) - \\ -\frac{1}{c^2}\sum_i BE_i \end{pmatrix}$$

Why measure masses?

^{17}**Cl**
Chlorine
35.453

- Molar mass
  - 35.453 ± 0.002 g/mol
    35.453(2) g/mol
    - $\frac{\delta m}{m} = \frac{0.002}{35.453} \approx 6 \times 10^{-5}$
  - $N_A = 6.022140857(74) \times 10^{23}$ particles/mol
    - $\frac{\delta m}{m} = \frac{0.000000074}{6.022140857} \approx 1.2 \times 10^{-8}$
  - If you measure out 35.453 g *precisely*
    - $6.02214(34) \times 10^{23}$ atoms
  - **Does it matter?**
    - **Depends on the application**

Why measure masses?

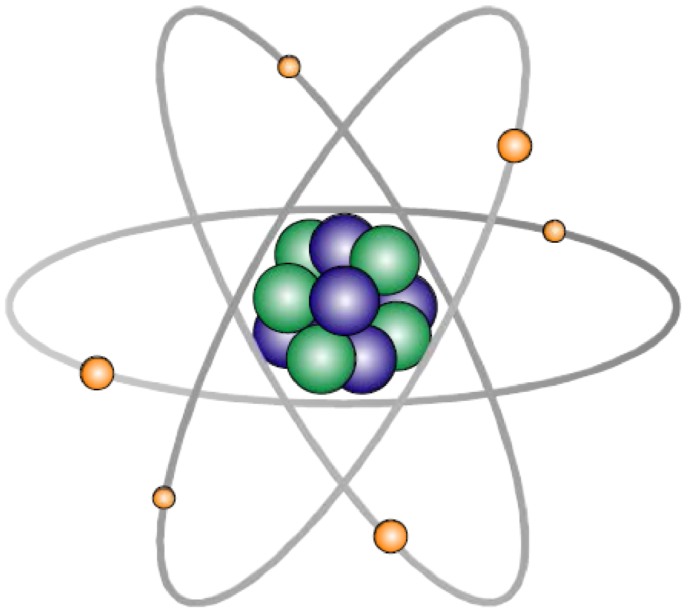
- Particle/molecular identification
  - What is the difference between:
    - ^{180}Hf
    - ^{179}HfH
    - $C_6H_{12}O_6$ – Glucose
    - $C_9H_8O_4$ – Caffeic Acid (Aspirin)
    - $C_{10}H_{12}O_3$ - Tyrosol, acetate
    - $C_7H_7F_3O_2$ – No idea but it exists in nature

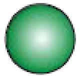
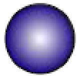
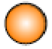
Why measure masses?

	Mass (AMU)	m - M(^{180}Hf) (AMU)	$\frac{\Delta m}{M}$
^{180}Hf	179.9466	--	--
^{179}HfH	179.9538	0.0071	3.94×10^{-5}
$C_6H_{12}O_6$	180.0648	0.1168	6.49×10^{-4}
$C_9H_8O_4$	180.0432	0.0957	5.32×10^{-4}
$C_{10}H_{12}O_3$	180.0800	0.1321	7.34×10^{-4}
$C_7H_7F_3O_2$	180.0406	0.0933	5.18×10^{-4}

Atomic and nuclear masses

Masses determine the atomic and nuclear binding energies reflecting all forces in the atom/nucleus.



$= N \cdot$  $+ Z \cdot$  $+ Z \cdot$ 

– binding energy

$$M_{\mathrm{Atom}} = N \bullet m_{\mathrm{neutron}} + Z \bullet m_{\mathrm{proton}} + Z \bullet m_{\mathrm{electron}} - (B_{\mathrm{atom}} + B_{\mathrm{nucleus}})/c^2$$

$\delta m/m < 10^{-10}$

$\delta m/m = 10^{-6} - 10^{-8}$

Slide courtesy of K. Blaum
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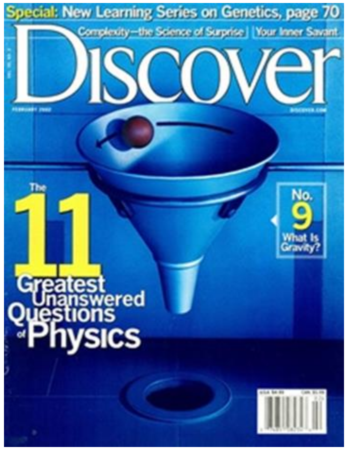
Most of physics requires masses

- General physics and chemistry
- Nuclear structure physics
- Astrophysics
- Weak interaction studies
- Fundamental neutrino physics
- Fundamental symmetries
- Tests of QED

Nuclear Astrophysics

- Big Bang
  - ^{1}H, ^{2}H, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li
- Cosmic Ray Spallation (interstellar medium)
  - Many elements, including most Li, Be, B
- Stellar Burning
  - Up to ^{56}Fe
- p Process (core-collapse supernovae)
  - Some heavy elements
- rp/vp Process (x-ray bursts)
- s Process (AGB stars)
  - Heavy elements up to A=208 (half of all heavy nuclei)
- r Process (??)
  - Heavy elements A>56 (the other half)

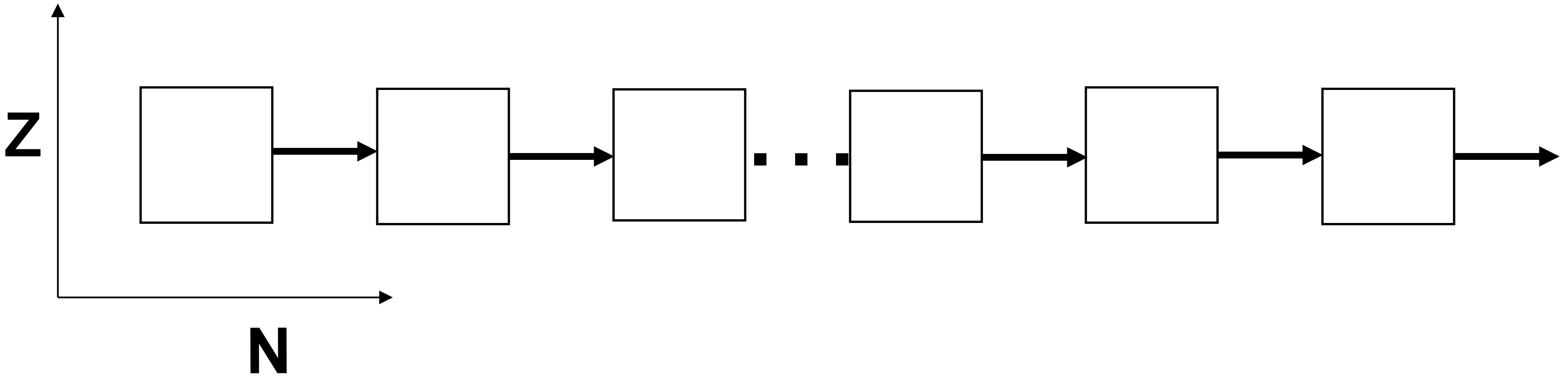


Neutron separation energy and mass

$$\boxed{S_n = M(A,Z) + M(n) - M(A+1,Z)}$$

The Astrophysical *R*-Process

- **Temperatures are very high ($T \sim 10^9 K$)**
- **Neutron densities are very high ($n_n > 10^{20}$ cm^{-3})**
- **(n,γ) rate is high (< 1 μs/capture)**



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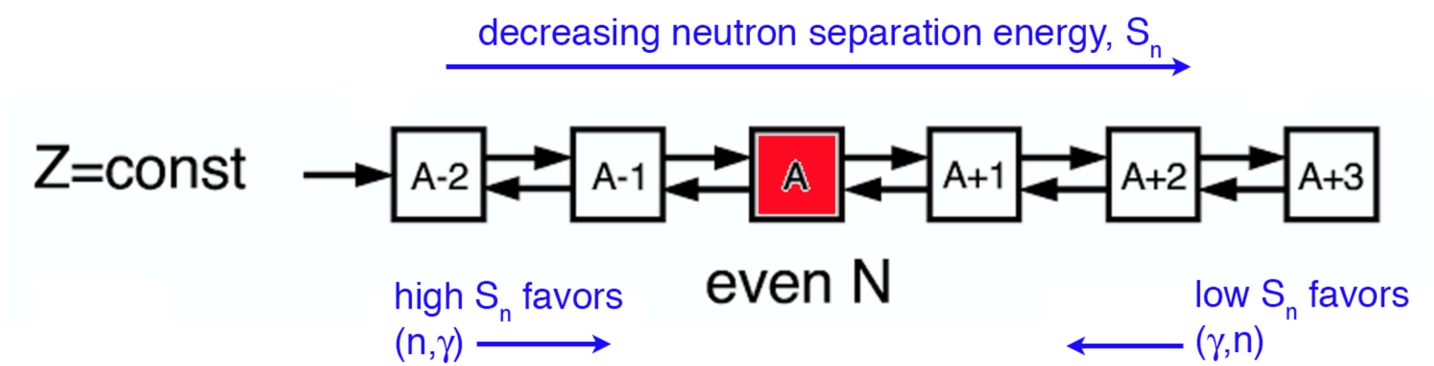

[A. Champagne, RIA Summer School '06]

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- **Equilibrium point defined by Saha Equation**

Partition Function – Ratio can vary between $10^{\pm 4}$ but normally between $10^{\pm 1}$

$$\frac{Y(Z,A+1)}{Y(Z,A)} = n_n \left(\frac{2\pi\hbar^2}{m_u kT}\right)^{3/2} \underbrace{\left(\frac{A+1}{A}\right)^{3/2}}_{\approx 1} \frac{G^*(Z,A+1)}{2G^*(Z,A)} \exp\left[\frac{S_n(Z,A+1)}{kT}\right]$$

The Astrophysical *R*-Process

- Temperatures are very high ($T \sim 10^9 K$)
- Neutron densities are very high ($n_n > 10^{20}\ cm^{-3}$)
- (n,γ) rate is high (< 1 μs/capture)
- As region of low neutron separation energy (S_n) reached $(n,\gamma) \leftrightarrow (\gamma,n)$ equilibrium achieved
- Equilibrium point defined by Saha Equation

$$S_n^0 \approx kT \ln\left[\frac{2}{n_n}\left(\frac{m_u kT}{2\pi\hbar^2}\right)^{3/2}\right]$$

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- **Temperatures are very high ($T \sim 10^9$K)**
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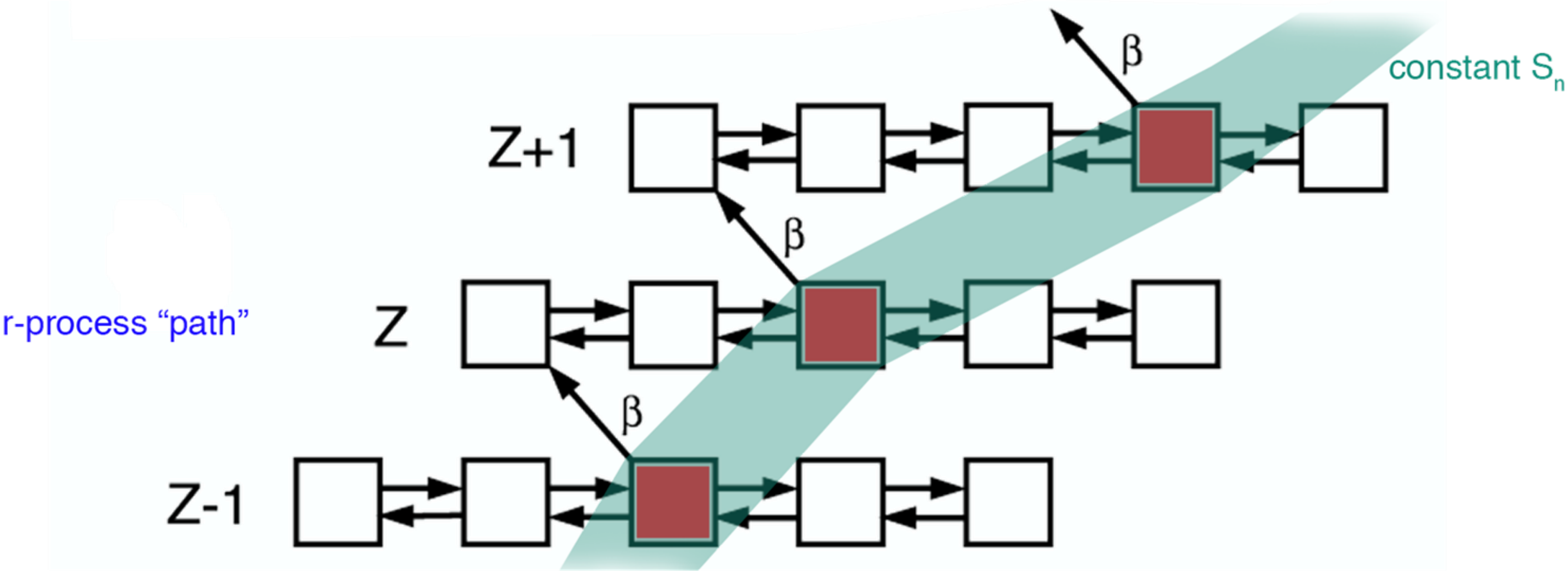
- **Equilibrium point defined by Saha Equation**

$$S_n^0 \approx kT \ln\left[\frac{2}{n_n}\left(\frac{m_u kT}{2\pi\hbar^2}\right)^{3/2}\right] \approx 3\ \mathrm{MeV}$$

So we wait for a β-decay

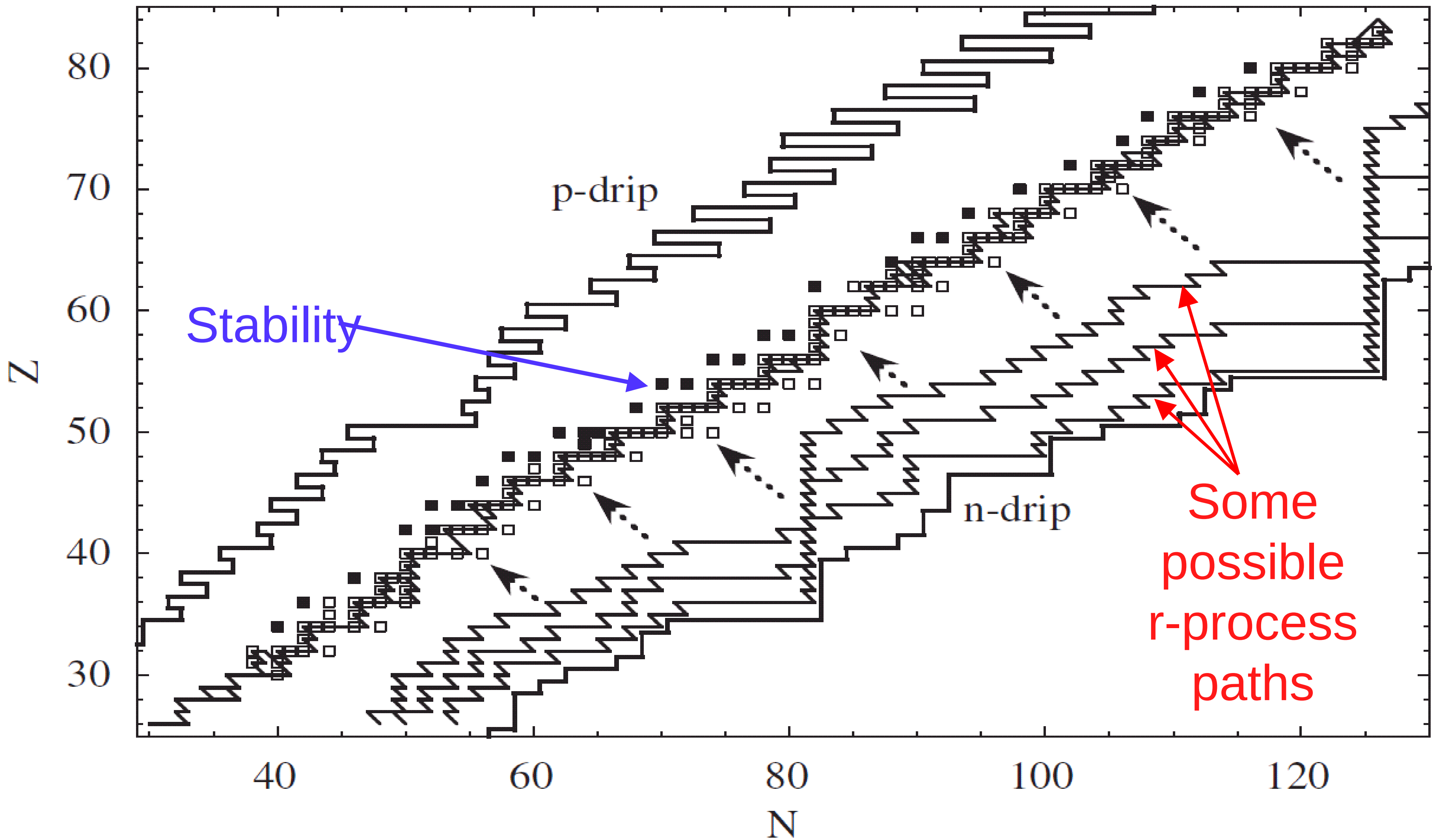


[A. Champagne, RIA Summer School 06]

The S_n where equilibrium occurs depends only on T and n_n

$$S_n^0 \approx kT \ln\left[\frac{2}{n_n}\left(\frac{m_u kT}{2\pi\hbar^2}\right)^{3/2}\right]$$

M. Arnould et al. / Physics Reports 450 (2007) 97–213

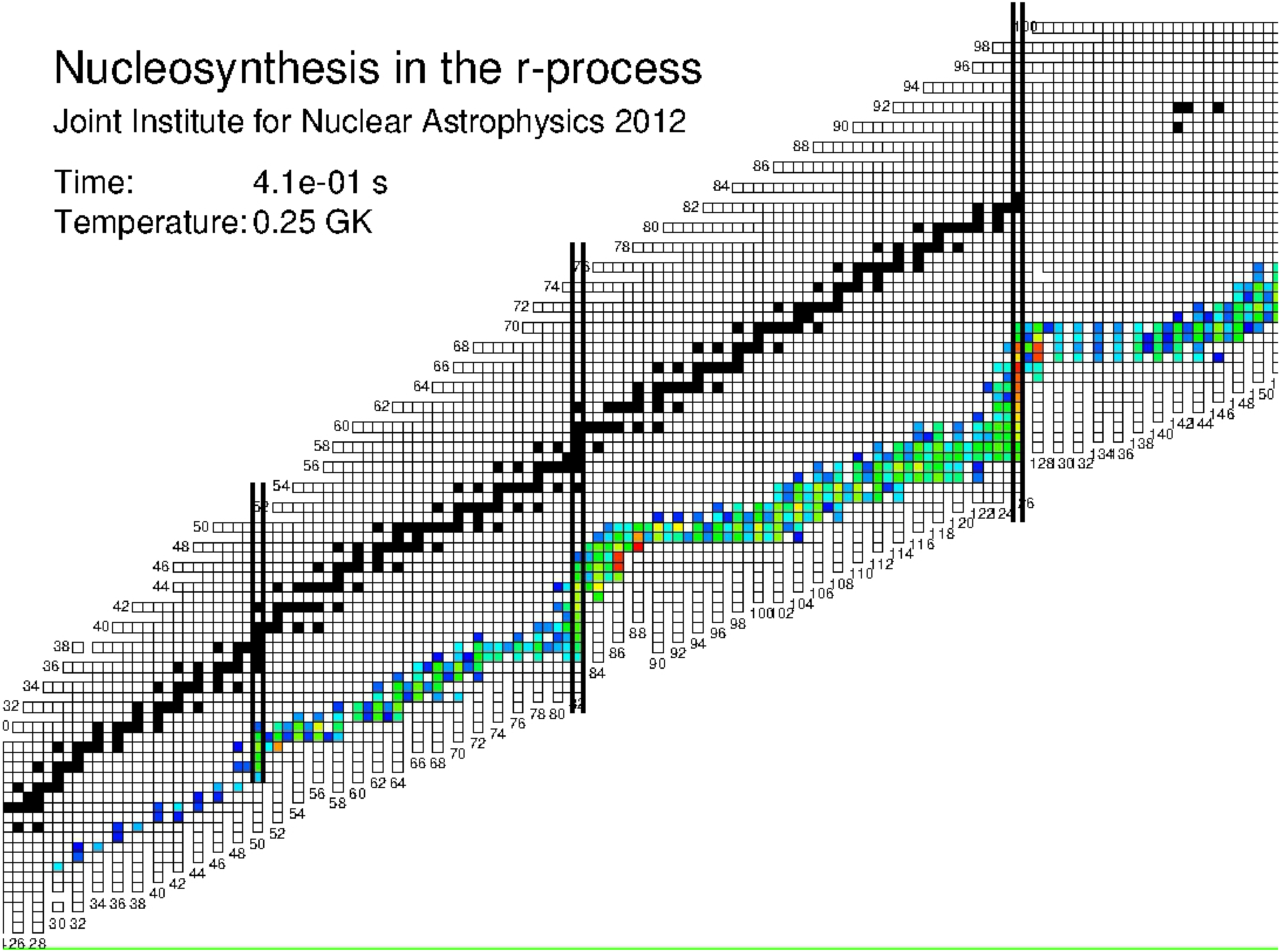


Nucleosynthesis in the r-process

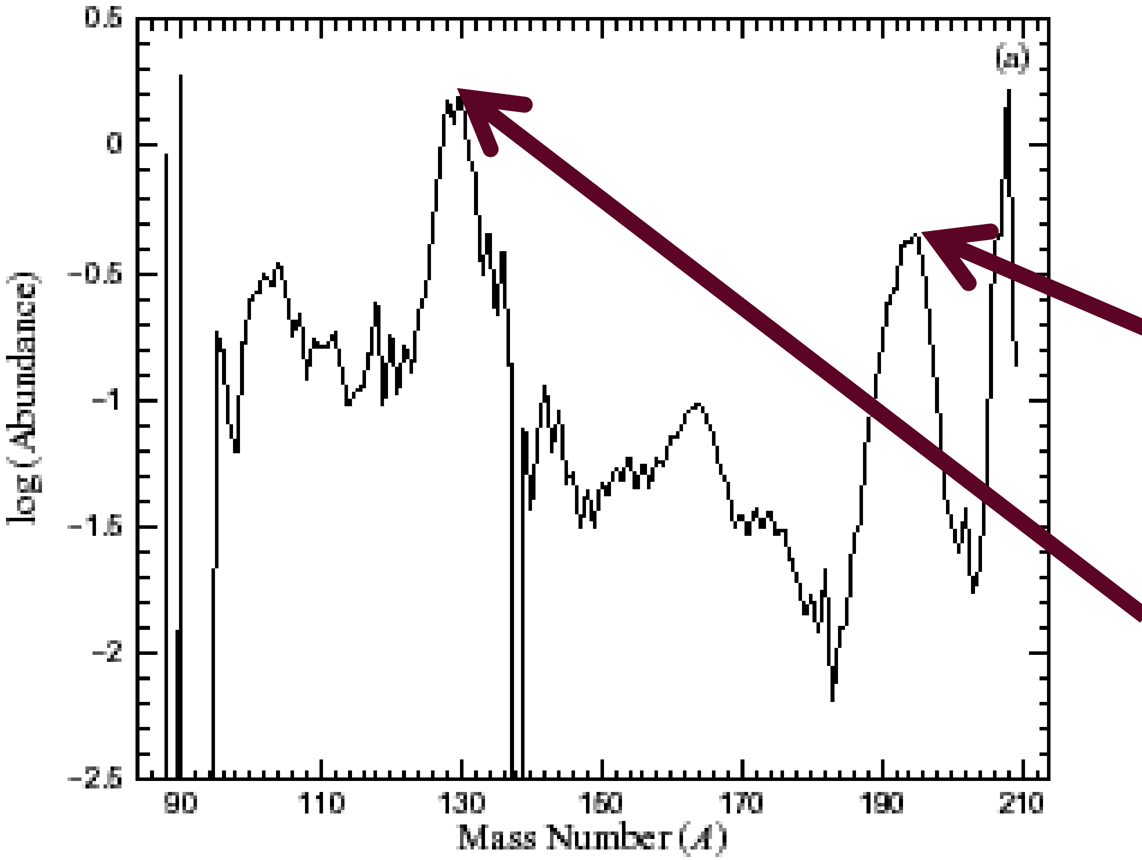
Joint Institute for Nuclear Astrophysics 2012

Time: 4.1e-01 s

Temperature: 0.25 GK



Evidence From Elemental Abundances



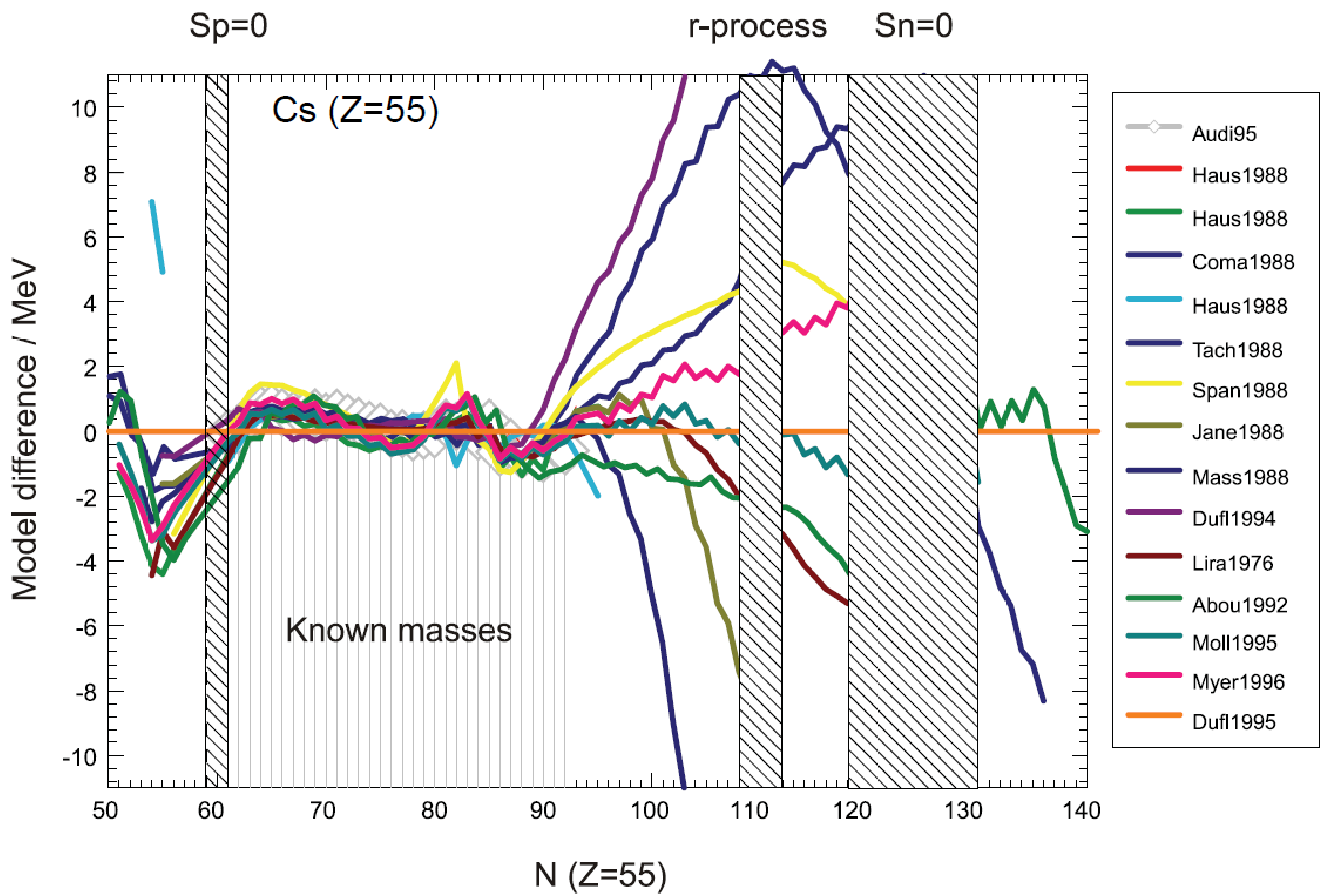
A	N
195	126
130	82

Y.-Z. Qian, *Prog. Part. Nucl. Phys.*, 50 (2003) 153

Neutron separation energy and mass

$$\boxed{S_n = M(A,Z) + M(n) - M(A+1,Z)}$$

Test of nuclear mass models…(Hint: They stink!)



How to measure masses? $\vec{B} \otimes$

Dipole mass spectrometry

- Charged particle in a uniform magnetic field
- $\vec{F} = q\vec{v} \times \vec{B}$ (Magnetic force)
- $\vec{F} \perp \vec{v}$ (F is a centripetal force)

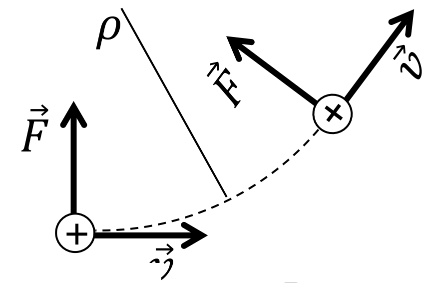
$$qvB = \frac{mv^2}{\rho}$$

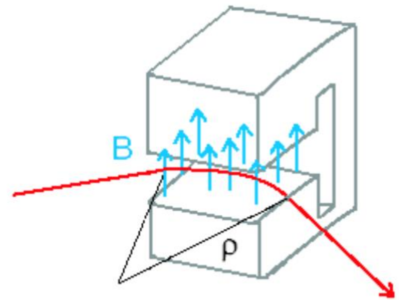
Radius of curvature

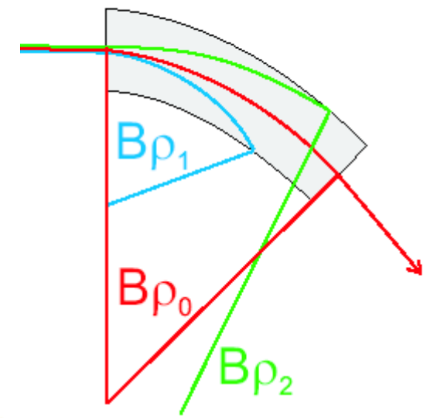
- Magnetic Rigidity ($B\rho$):

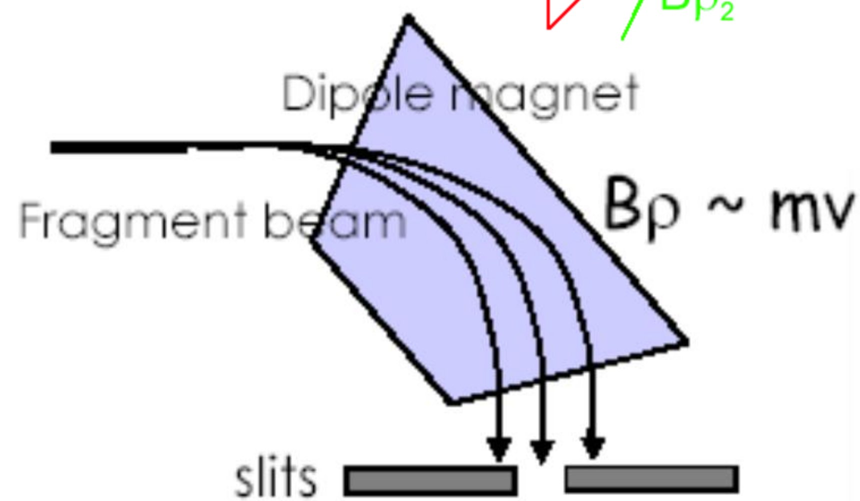
$$B\rho = \frac{p}{q}$$

- Best $\frac{\delta m}{m} \sim 10^{-5}$







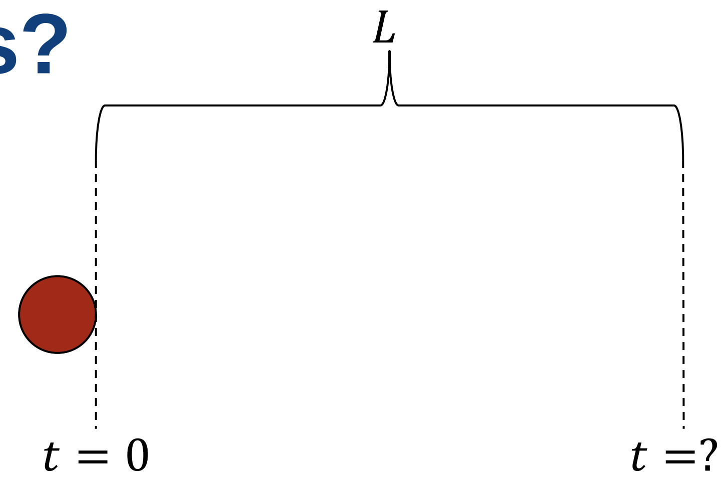


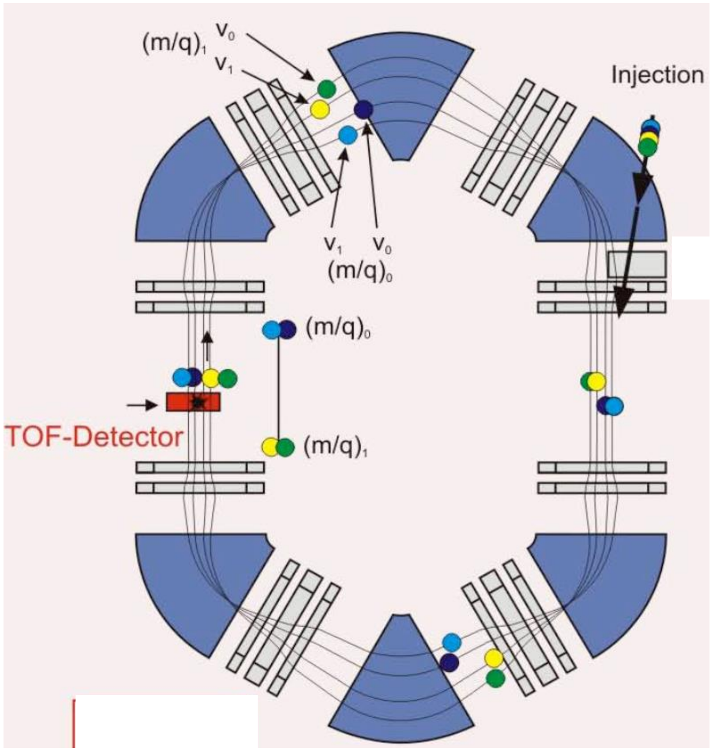
How to measure masses?

Time of flight mass spectrometry

- Measure the time that an ion at known energy will travel a known distance.
- Can be linear
  - Space limitation
- Storage rings better
  - Distance unlimited
- Best precision:

$$\frac{\delta m}{m} \sim 10^{-6}$$





We can do better than ${}^{\delta m}/_{m} \sim 10^{-6}$

- Actually we need
  - For nuclear astrophysics $\frac{\delta m}{m} \sim 10^{-7}$ *minimum*
  - For fundamental symmetries: $\frac{\delta m}{m} \sim 10^{-10}$ *minimum*
- What is the physical quantity that we can measure to the highest precision?

Time

- Time is the physical quantity we can measure most precisely
- For this clock:
- We can do better than $\frac{\delta t}{t} \sim 10^{-15}$
- When making a precision measurement, the goal is to measure time (frequency)

WIRED

ADAM MANN SCIENCE 04.04.14 6:30 AM

HOW THE U.S. BUILT THE WORLD'S MOST RIDICULOUSLY ACCURATE ATOMIC CLOCK

$$\frac{\delta t}{t} \sim 10^{-14}$$

For next class:

- How can you relate a time (frequency) measurement to mass?

Thank you!
Merci

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