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Detector Physics

Semiconductor Detectors

GAPS Postdoc Lecture Series – November 2, 2015

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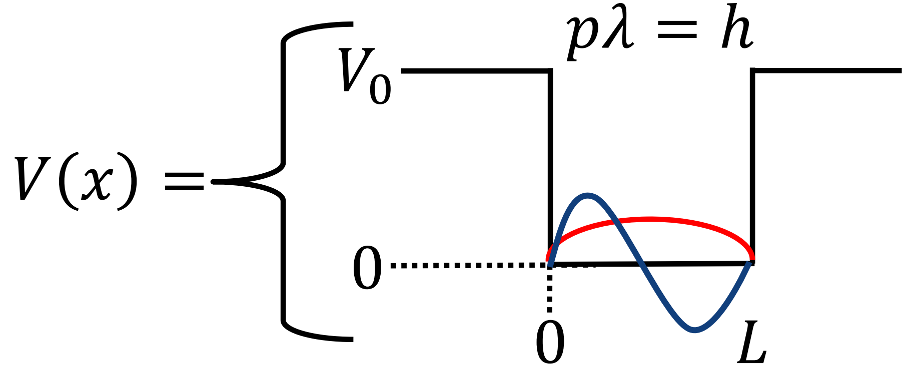
Outline

- Semiconductor refresher
- Doping
- Junctions
- Detectors
  - Si and Si(Li)s
  - HPGes

Particle in a finite potential well

$$p\lambda = h$$

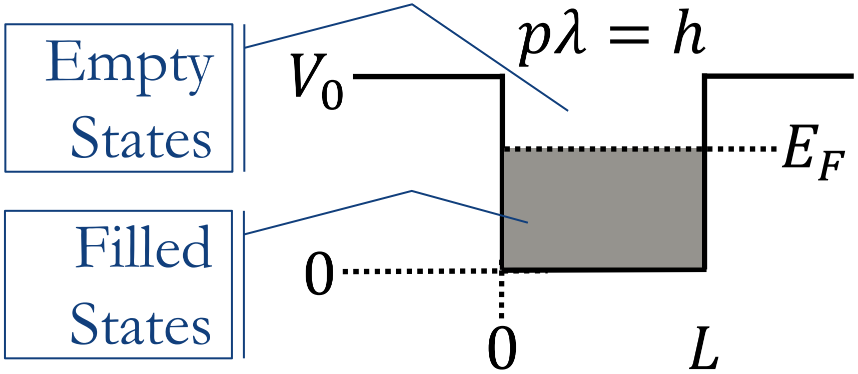


- Consider particle with $E < V_0$
  - $E_{\text{kinetic}} = E - V(x)$
  - $E_{\text{kinetic}} > 0$ for 0<x<*L*
- Solving the Schrödinger Eq:
- Boundary conditions:
  - $\Psi(0) = \Psi(L) = 0$

$$\Psi(x) = A\sin\frac{2\pi x}{\lambda}$$

$$\lambda = \frac{2L}{n} \rightarrow \Psi_n(x) = A\sin\frac{n\pi x}{L}$$

Many electrons in the well



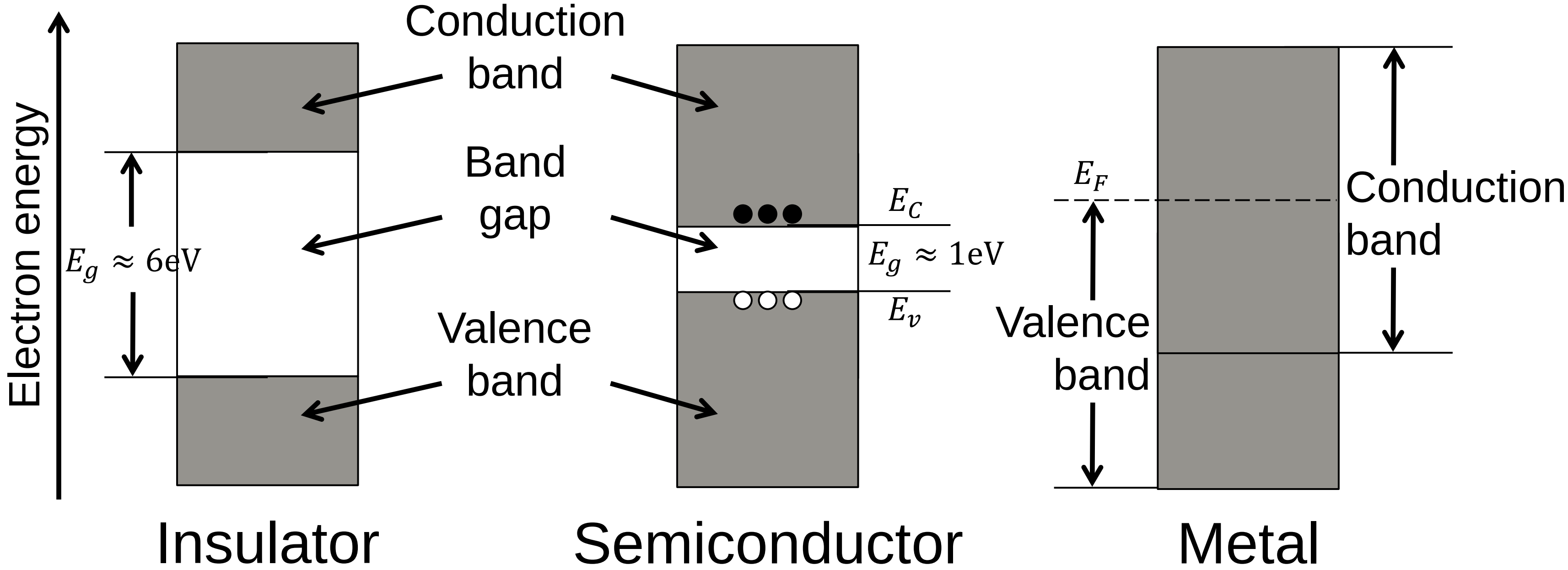
- For a given electron:
  - $E_{\text{kinetic}} = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{n^2h^2}{8mL^2}$
- The Fermi energy for N_0 electrons & 2 e^-/state:

$$E_F = \frac{N_0^2 h^2}{32mL^2}$$

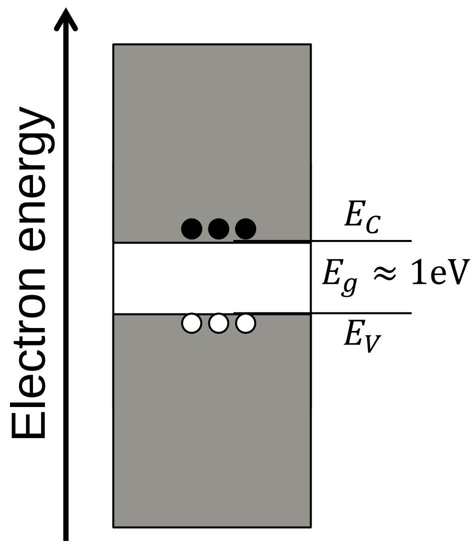
- At absolute zero:
  - All levels below E_F **filled**
  - All levels above E_F **empty**
- Energy to remove one electron: $V_0 - E_F$

Crystals

- E-levels of atoms form closely spaced levels called "bands"
- Energy level spacing creates "band gaps"



Crystals



Semiconductor

E_g(Si) = 1.1 eV

E_g(Ge) = 0.7 eV

- The probability of an energy level being filled:

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

- Electron density in the conduction band:

$$\frac{dn(E)}{dE} = \rho(E) f(E)$$

Density of e⁻ states

$$\boxed{\rho(E)dE = 4\pi \left(\frac{2m_e}{h^2}\right)^{3/2} \sqrt{E}\,dE}$$

For HW: Derive This

e⁻ density in the conduction band (n_e)

$$\rho(E)dE = 4\pi\left(\frac{2m_e}{h^2}\right)^{3/2}\sqrt{E}dE$$

$$\rho(E)dE = 4\pi\left(\frac{2m_e^*}{h^2}\right)^{3/2}\sqrt{E-E_c}dE$$

Effective mass

e⁻ density

$$n_e = \int \rho(E)f(E)dE$$

$$= 4\pi\left(\frac{2m_e^*}{h^2}\right)^{3/2}\int_{E_c}^{\infty}\sqrt{\frac{E-E_C}{e^{\frac{E-E_C}{kT}}+1}}dE$$

$$= 4\pi\left(\frac{2m_e^*}{h^2}\right)^{3/2} e^{-\frac{E_C-E_F}{kT}}(kT)^{3/2}\int_0^{\infty}\sqrt{x}e^{-x}dx$$

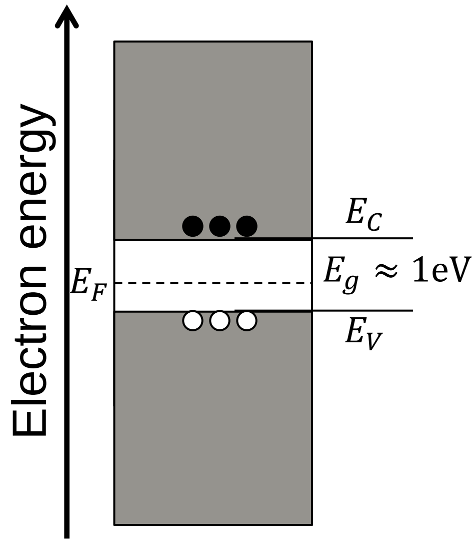
e⁻ density in the conduction band (n_e)

$$n_e = 4\pi\left(\frac{2m_e^*}{h^2}\right)^{3/2} e^{-\frac{E_C-E_F}{kT}}(kT)^{3/2}\int_0^\infty \sqrt{x}e^{-x}dx$$

$$n_e = 4\pi\left(\frac{2m_e^*}{h^2}\right)^{3/2} (kT)^{3/2}\frac{\sqrt{\pi}}{2}e^{-\frac{E_C-E_F}{kT}}$$

$$n_h = 4\pi\left(\frac{2m_h^*}{h^2}\right)^{3/2} (kT)^{3/2}\frac{\sqrt{\pi}}{2}e^{-\frac{E_F-E_V}{kT}}$$

Density of holes



Semiconductor

e⁻ density in the conduction band (n_e)

$$n_e = 4\pi \left(\frac{2m_e^*}{h^2}\right)^{3/2} e^{-\frac{E_C-E_F}{kT}} (kT)^{3/2} \int_0^\infty \sqrt{x} e^{-x} dx$$

$$n_e = 4\pi \left(\frac{2m_e^*}{h^2}\right)^{3/2} (kT)^{3/2} \frac{\sqrt{\pi}}{2} e^{-\frac{E_C-E_F}{kT}}$$

$$n_h = 4\pi \left(\frac{2m_h^*}{h^2}\right)^{3/2} (kT)^{3/2} \frac{\sqrt{\pi}}{2} e^{-\frac{E_F-E_V}{kT}}$$

Density of holes

- In a pure semiconductor:

$$n_e = n_h = n_i \propto T^{3/2} e^{\left(-\frac{E_g}{2kT}\right)}$$

- Impure:

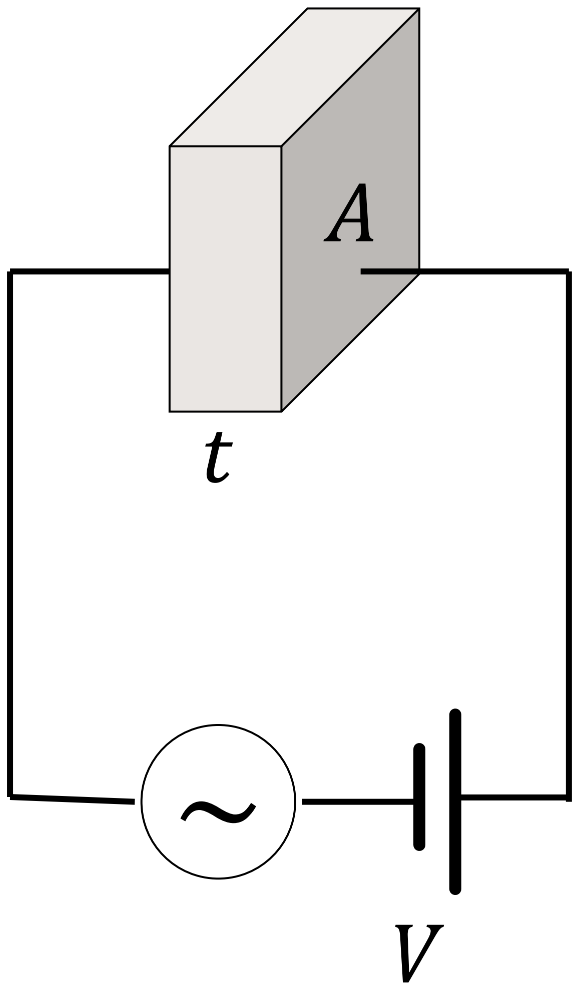
$$\sqrt{n_e n_h} = n_i \propto T^{3/2} e^{-\frac{E_g}{2kT}}$$

Intrinsic charge carrier concentration

e^- density in the conduction band (n_e)

- Typical values of n_i
  - 2.5×10^{13} cm^{-3} for Ge at T = 300 K
  - 1.5×10^{10} cm^{-3} for Si at T = 300 K
- Typically 10^{22} atoms/cm^3
- Ion to atom ratio:
  - ~10^{-9} for Ge at 300K
  - ~10^{-12} for Si at 300K

Current in a semiconductor



$$V = IR$$

$$V = I\frac{\rho t}{A}$$

$$I = \frac{AV}{\rho t}$$

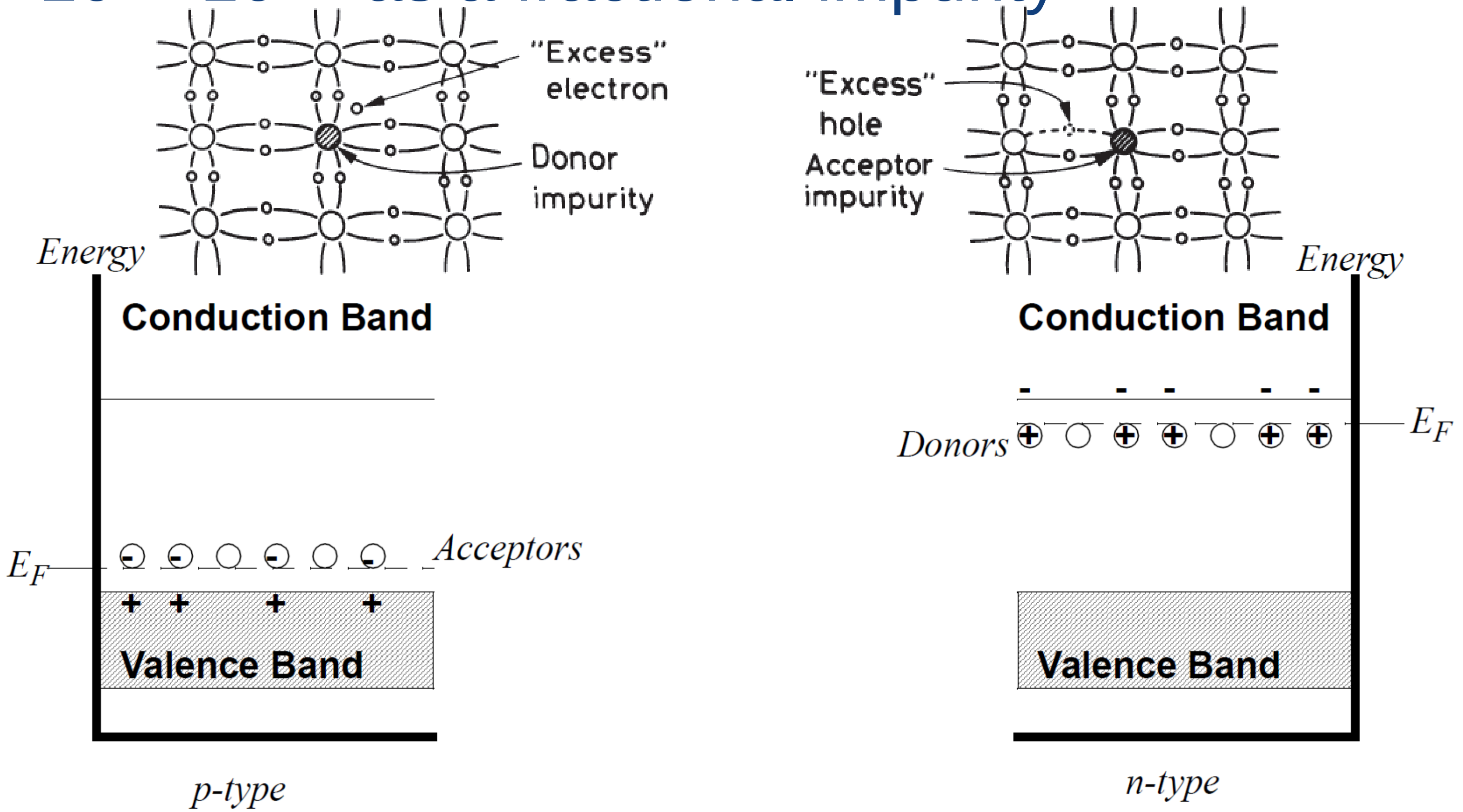
$$I = I_e + I_h$$

Electron mobility

- Under an externally applied electric field
  - Electron drift velocity: $v_e = \mu_e E = \mu_e \frac{V}{d}$
  - Material resistivity: $\rho = \frac{1}{e(n_e\mu_e + n_h\mu_h)}$
- Silicon
  - Pure: Intrinsic ρ = 230,000 Ω cm
  - Dope at 2 x 10^{-10}: ρ = 463 Ω cm

Semiconductor doping

- Addition of impurities can create additional states within an energy gap.
- 10^{-7} -10^{-10} as a fractional impurity



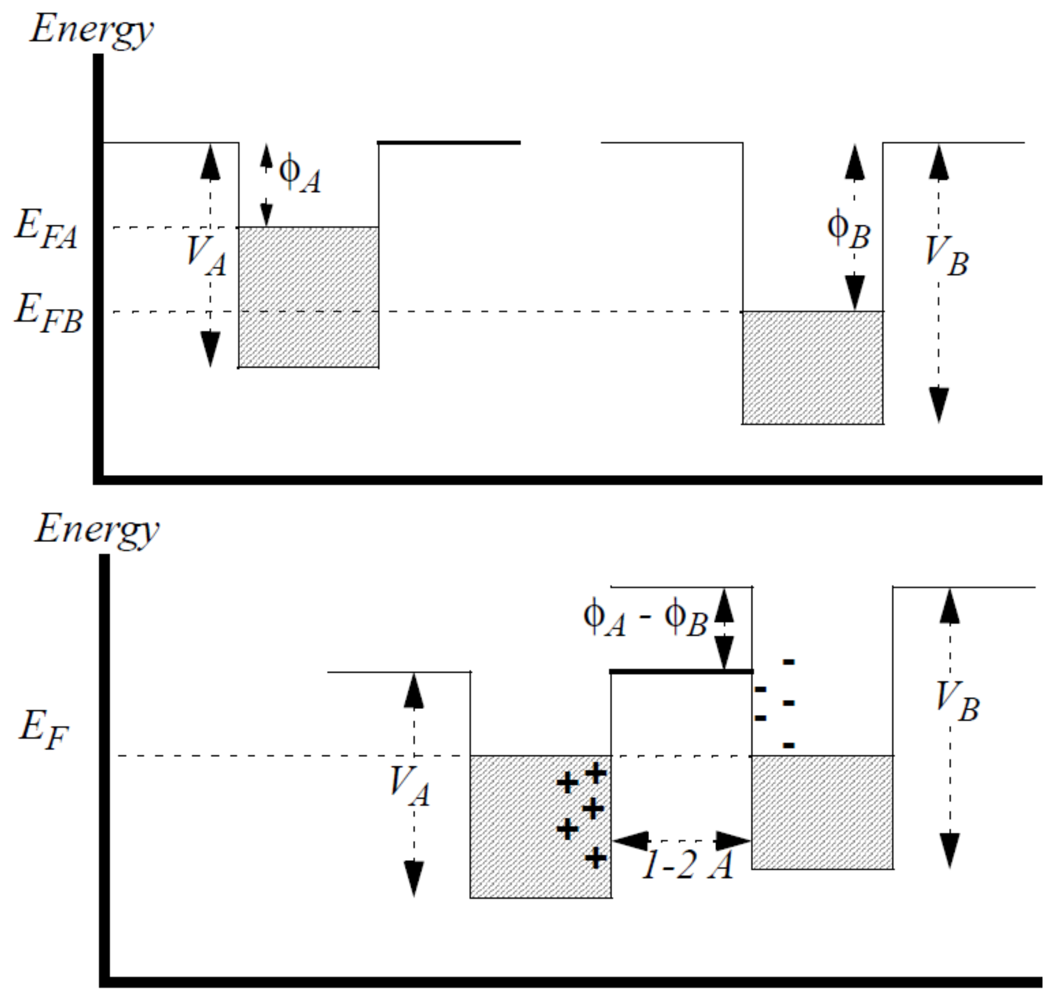
Detector material properties

Table 5.1 Some physical properties of silicon and germanium. Unless otherwise noted, data for silicon are at room temperature and data for germanium at 77 K

Physical property	Si	Ge
Atomic number Z	14	32
Atomic weight A	28.1	72.6
Density [g/cm^2]	2.33	5.32
Radiation length [mm]	93.6	23
Dielectric constant (relative)	12	16
Energy gap [eV)]	1.115	0.72
Intrinsic carrier density at 300 K [cm^{-3}]	1.5×10^{10}	2.4×10^{13}
Intrinsic resistivity at 300 K [Ω cm]	230,000	47
Electron mobility [cm^2 V^{-1} s^{-1}]	1350	36,000
Hole mobility at [cm^2 V^{-1} s^{-1}]	480	42,000
Energy/e–h pair [eV]	3.62	2.96
Fano factor	≈0.1	≈0.1
Energy loss min. ionising particles [MeV/cm]	3.87	7.29

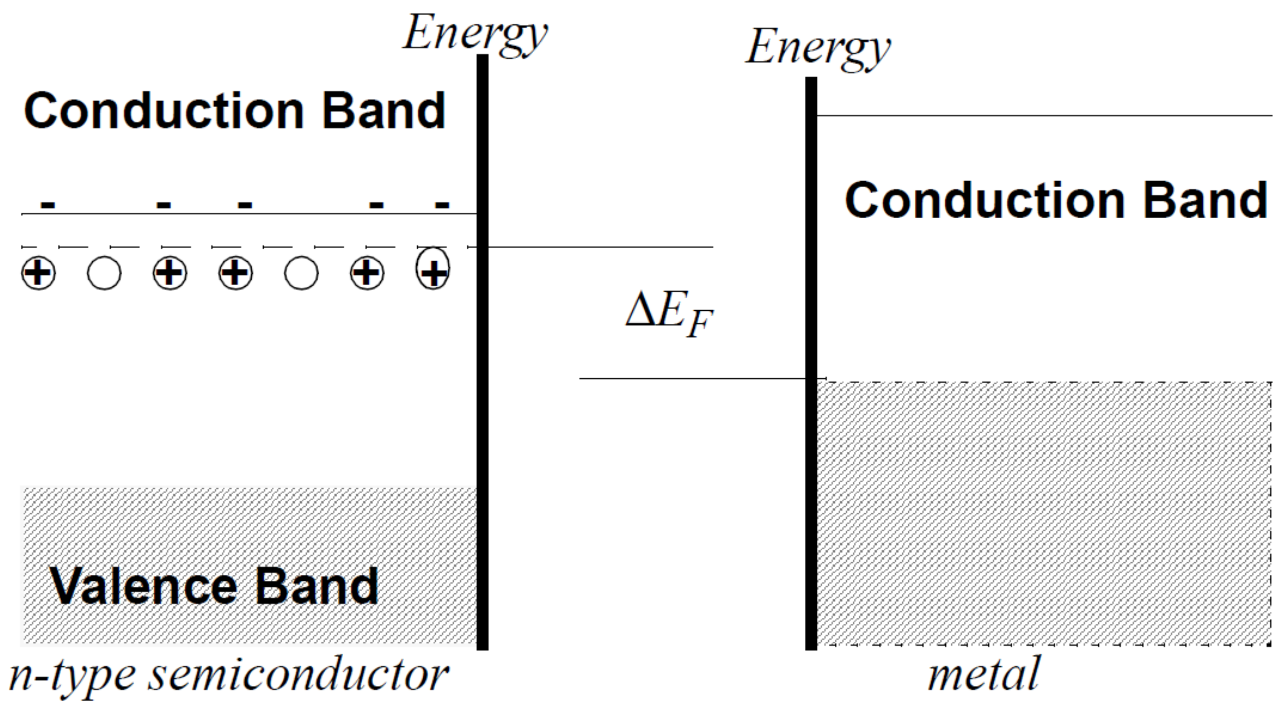
Metal-Metal Junction

- Each metal has its own E_f and Φ
- Both metals are neutral
- If placed in contact electrons will move from A to B until the energy levels are equal
  - Metals become charged.



Slide courtesy of M. Fortner, N. Illinois University

Metal-Semiconductor junction

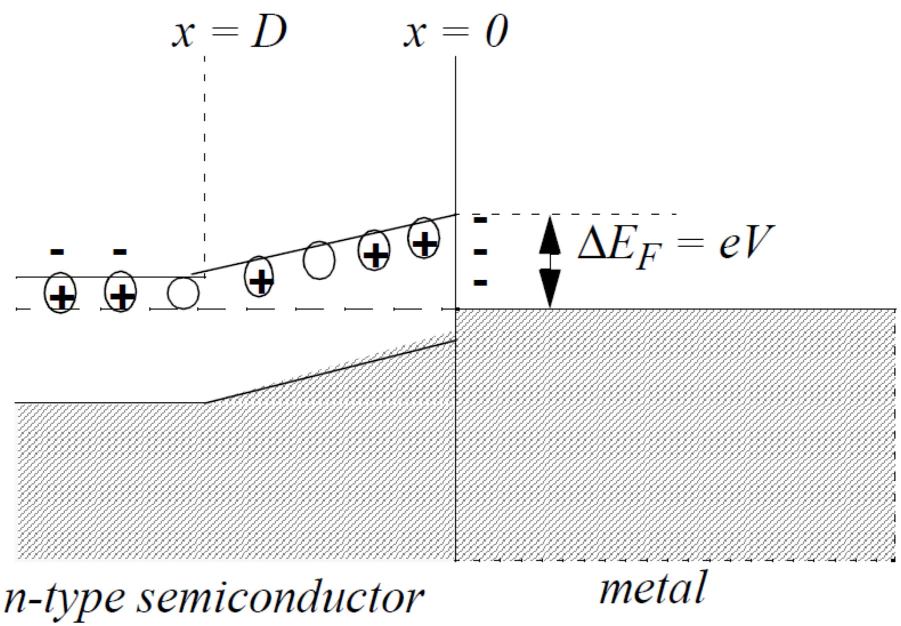


- The two materials begin with different E_f and Φ
  - Metal's E_F must be inside semi-conductor's bandgap
- Field in the semiconductor: $$E = \frac{Nex}{\varepsilon_r \varepsilon_0}$$

Depletion

Slide courtesy of M. Fortner, N. Illinois University

- Once materials in contact electrons flow semiconductor → metal



$$V = \int E\, dx = \int \frac{Nex}{\varepsilon_r \varepsilon_0} dx = \frac{NeD^2}{2\varepsilon_r \varepsilon_0}$$

- Barrier thickness:

$$D = \sqrt{\frac{2\varepsilon_r \varepsilon_0 V}{Ne}}$$

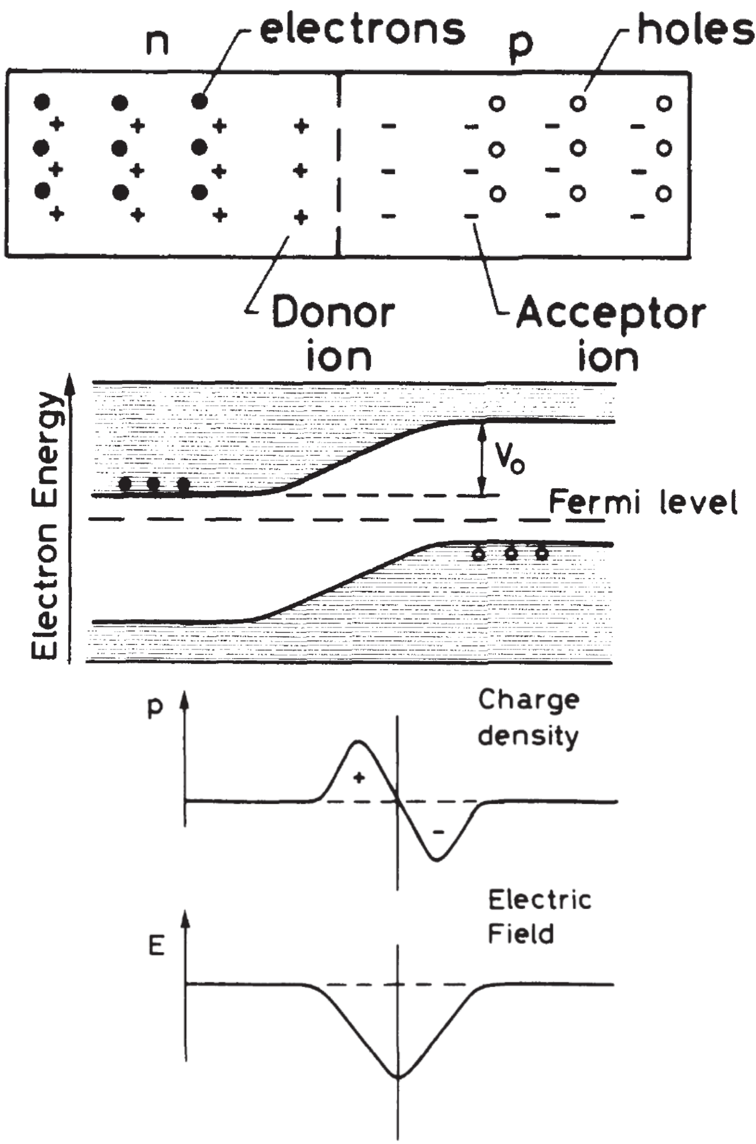
- Example: Ge with 10^{-7} donor/atom and 1 V applied across.
  - $\varepsilon_r = 16$, $N = 4.4 \times 10^{21}\ \mathrm{m}^{-3}$, $D = 630$ nm
  - ***E* = 30 kV/cm**

Punchline!!!!!

- Interactions in the depleted region create electron-hole pairs that move in opposite directions.
- However, metal-semiconductor depletion regions are small:
  - ~630 nm
- Current is constantly flowing (if no outside bias is applied)

n-p semiconductor junction - Schematic

- Thermal agitation moves electrons across to holes
- Soon E-field large enough to prevent movement
  - E_f now the same on both sides
- Charge deposited in high field region:
  - Moved quickly to one side



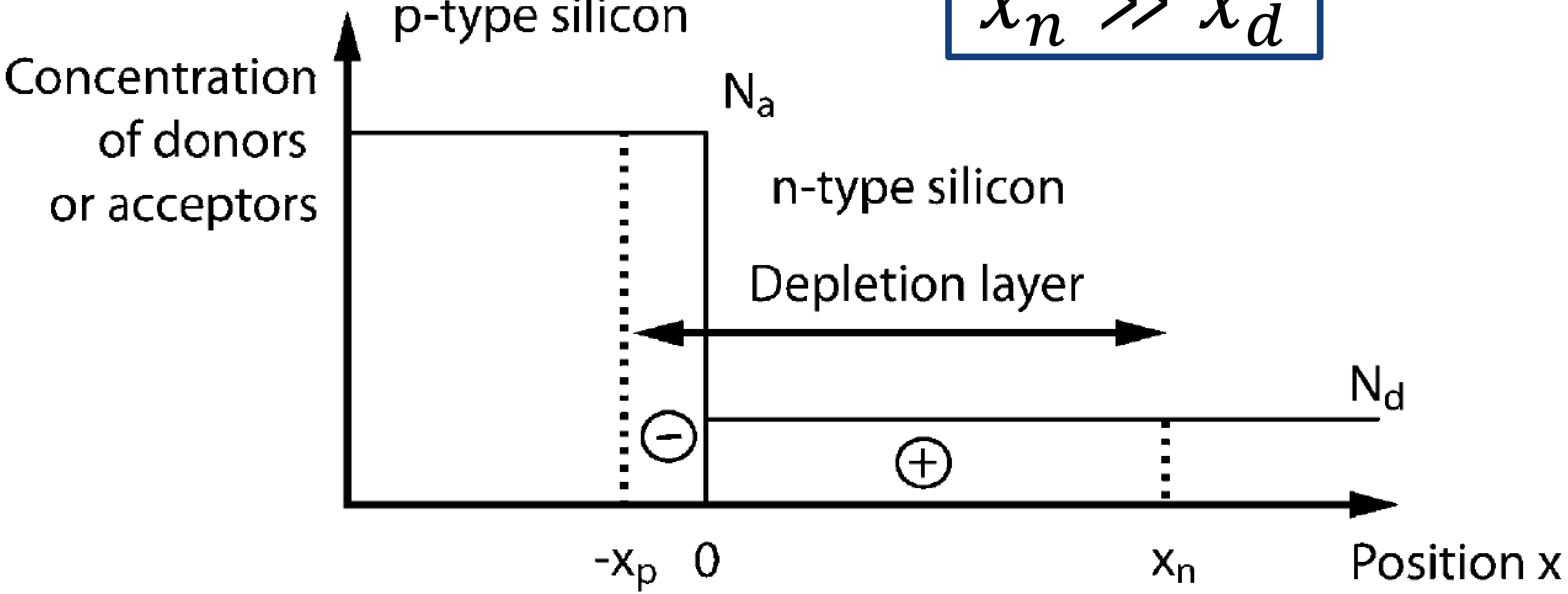
n-p junction – a little rigor

- Charge density
  - eN_d
  - $-eN_a$

$$N_a \gg N_d$$

$$N_d x_n = N_a x_p$$

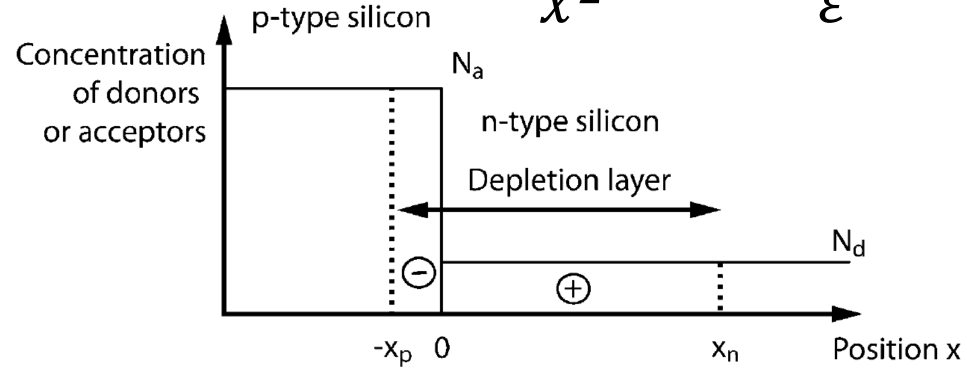
$$\boxed{x_n \gg x_d}$$



n-p junction – a little rigor

$$\frac{d^2V}{x^2} = -\frac{\rho(x)}{\varepsilon}$$

- We start out with 4 regions:
  1. $V_1(x): -\infty < x < -x_p$
     - $\rho = 0$
  2. $V_2(x): -x_p < x < 0$
     - $\rho = -eN_a$
  3. $V_3(x): 0 < x < x_n$
     - $\rho = eN_d$
  4. $V_4(x): x_n < x < \infty$
     - $\rho = 0$



- In regions 1 & 4

$$\frac{dV_1}{dx} = \frac{dV_4}{dx} = 0 \Rightarrow \begin{cases} V_1(x) = C_1 \\ V_4(x) = C_4 \end{cases}$$

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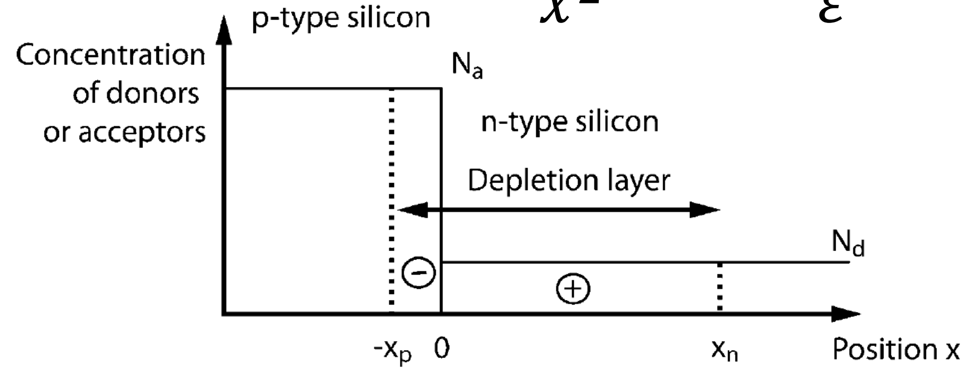


$$\int \frac{dV_3}{dx} dx = V_3(x) = -\frac{eN_d}{\varepsilon}\frac{x^2}{2} + C_{3a}x + C_{3b}$$

$$\left.\frac{dV_3}{dx}\right|_{x=x_n} = 0 \Rightarrow C_{3a} = \frac{eN_d}{\varepsilon} x_n$$

In region 3

$$\frac{d^2V_3}{dx^2} = -\frac{eN_d}{\varepsilon}$$

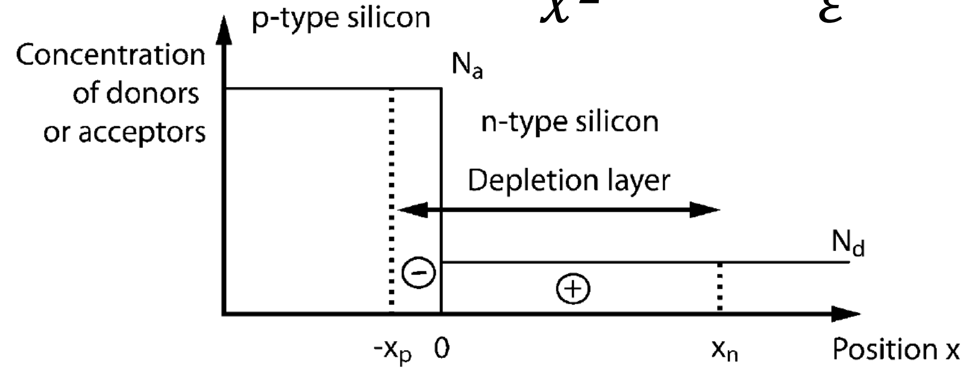
$$\int \frac{d^2V_3}{dx^2} dx = \frac{dV_3}{dx} = -\frac{eN_d}{\varepsilon} x + C_{3a}$$

n-p junction – a little rigor

$$\frac{d^2V}{x^2} = -\frac{\rho(x)}{\varepsilon}$$

We start out with 4 regions:

1. $V_1(x): -\infty < x < -x_p$
   - $\rho = 0$
2. $V_2(x): -x_p < x < 0$
   - $\rho = -eN_a$
3. $V_3(x): 0 < x < x_n$
   - $\rho = eN_d$
4. $V_4(x): x_n < x < \infty$
   - $\rho = 0$



In region 2, the same arguments get you:

$$V_2(x) = \frac{eN_a}{\varepsilon}\frac{x^2}{2} + \frac{eN_a x_p}{\varepsilon}x + C_{2b}$$

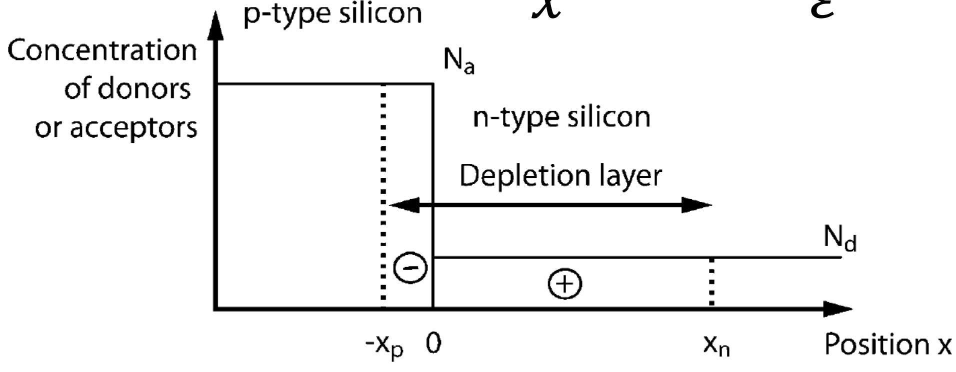
In region 3

$$\left.\frac{dV_3}{dx}\right|_{x=x_n} = 0 \Longrightarrow C_{3a} = \frac{eN_d}{\varepsilon}x_n$$

$$V_3(x) = -\frac{eN_d}{\varepsilon}\frac{x^2}{2} + \frac{eN_d x_n}{\varepsilon}x + C_{3b}$$

n-p junction – a little rigor

$$\frac{d^2V}{x^2} = -\frac{\rho(x)}{\varepsilon}$$



$$V_1(x) = C_1$$

$$V_2(x) = \frac{eN_a}{\varepsilon}\frac{x^2}{2} + \frac{eN_a x_p}{\varepsilon}x + C_{2b}$$

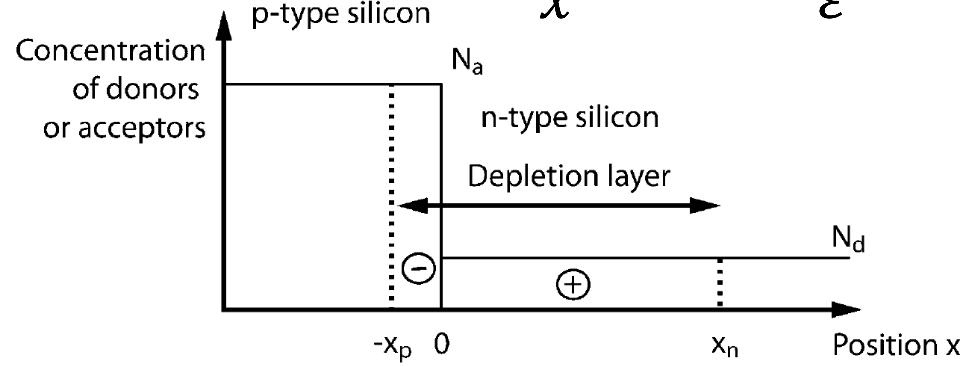
$$V_3(x) = -\frac{eN_d}{\varepsilon}\frac{x^2}{2} + \frac{eN_d x_n}{\varepsilon}x + C_{3b}$$

$$V_4(x) = C_4$$

- @ $x = 0$:
  - $V_2(0) = V_3(0)$
  - $C_{2b} = C_{3b} = C_b$

n-p junction – a little rigor

$$\frac{d^2V}{x^2} = -\frac{\rho(x)}{\varepsilon}$$



$$V_1(x) = C_1$$

$$V_2(x) = \frac{eN_a}{\varepsilon}\frac{x^2}{2} + \frac{eN_a x_p}{\varepsilon}x + C_b$$

$$V_3(x) = -\frac{eN_d}{\varepsilon}\frac{x^2}{2} + \frac{eN_d x_n}{\varepsilon}x + C_b$$

$$V_4(x) = C_4$$

$$\boxed{x_n \gg x_d}$$

- Potential across the junction:

$$V_0 = V_4 - V_1 = V_3(x_n) - V_2(-x_p)$$

$$V_0 = \frac{eN_d x_n^2}{2\varepsilon} + \frac{eN_a x_p^2}{2\varepsilon} \approx \frac{eN_d x_n^2}{2\varepsilon}$$

What have we learned?

- We can approximate the depletion depth, *D*
- If we want the largest *D*:
  - Increase the voltage
  - Decrease the dopant concentration
- How low can you go?
  - High purity n-type Si $\rho = 20$ kΩ cm
  - $N_d \approx 2.3 \times 10^{11}$ cm^{-3}

$$\boxed{x_n \gg x_d}$$

$$V_0 \approx \frac{eN_d x_n^2}{2\varepsilon}$$

$$D \approx x_n \qquad D = \sqrt{\frac{2\varepsilon V_0}{eN_d}}$$

- Without bias, ~0.7V across over depletion layer
  - D ≈ 64 μm
- With bias, breakdown @ 160 kV/cm
  - D ≈ 10 mm

Punchline 2 (semiconductor junction edition)

- Interactions in the depleted region create electron-hole pairs that move in opposite directions.
- ~~However, metal semiconductor depletion regions are small:~~
  - ~~630 nm~~ Up to 10 mm in Si. **10 cm in Ge**

Homework

- ~~Current is constantly flowing (if no outside bias is applied)~~
- Current only flows when charge deposited in depletion region
- Energy to remove an electron:
  - Ar gas: 30 eV/electron-ion pair
  - Si: 3.6 eV/electron-hole pair @ 300 K

General pros/cons of semiconductors

- Pros:
  - Denser material
  - Lower energy for charge generation
  - Fast signals (~ns)
- Cons
  - Difficult to manufacture
  - Hard to get large area detectors
  - More specialized detection schemes
  - No charge multiplication so electronics must be sensitive.

	Si	Ge
300 K	3.62 eV	------
77 K	3.81 eV	2.96 eV

Applications

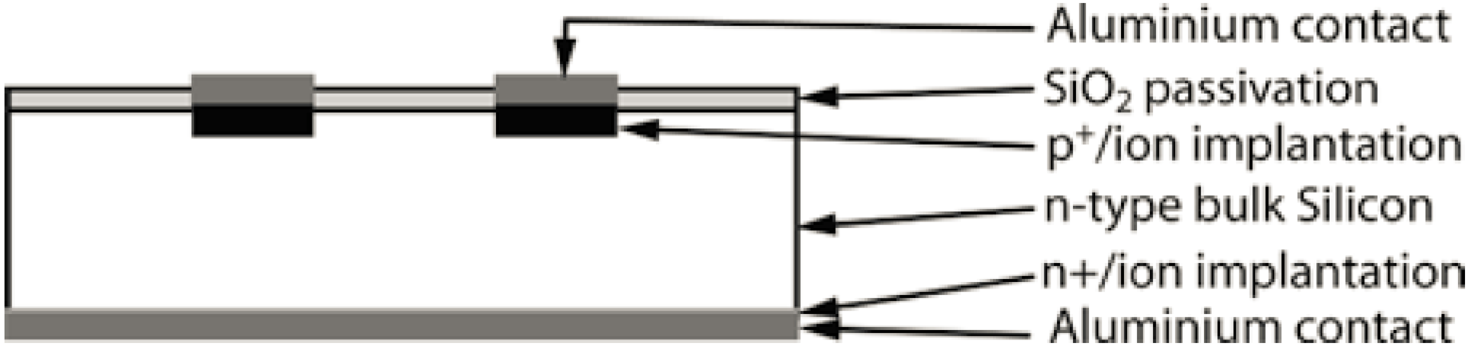
Si – Particle/Low E γ

- 10 mm max depth too small for high energy γ but perfect for α's
- Except for low energy applications, can be kept at room temp
- Resolution ~10 keV
  - Not great for 50 keV γ's
  - Fine for MeV α's
- http://dx.doi.org/10.1109/23.958703

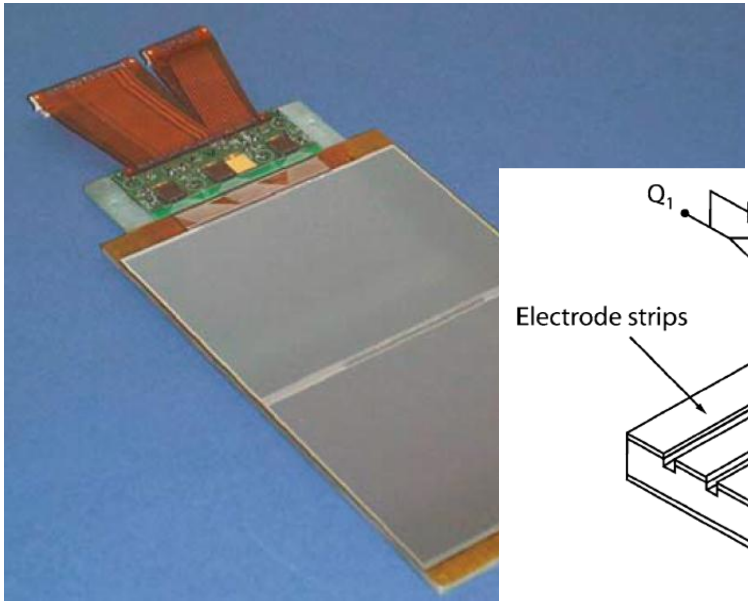
Ge – High-E γ

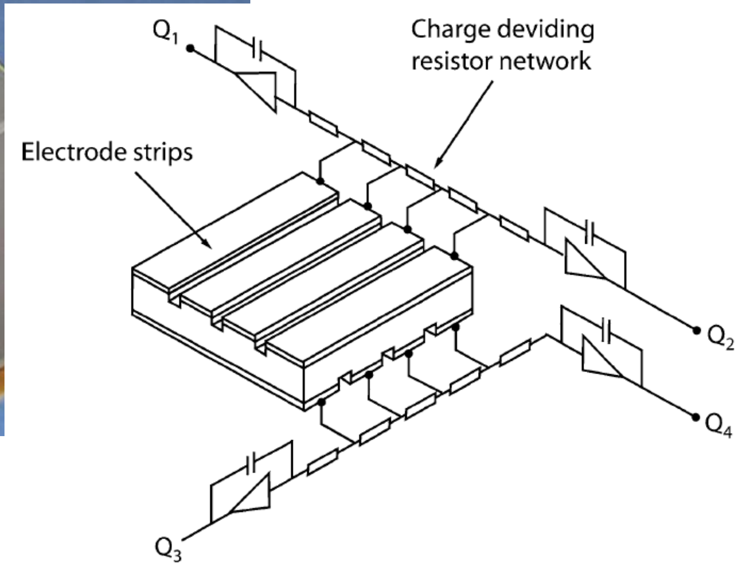
- 10 cm depth better for γ's
- Small (~0.7 eV) band gap energy
  - Kept cold to avoid thermal noise
  - More pairs created
- Resolution ~3 eV

The Si strip detector

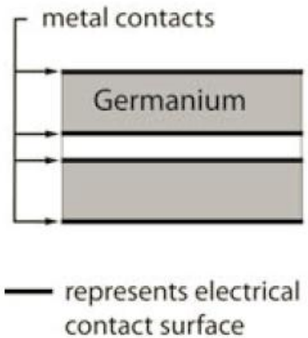


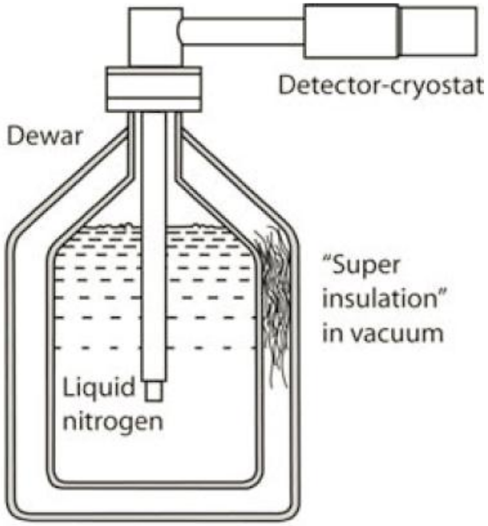
Charged particle tracking detector used in the CMS experiment. The detector consists of two wafers of silicon put side to side. Four amplifying chips with 512 amplifying channels each are visible in the top-left side of the picture. Each silicon wafer has strips as shown in Fig. 5.10, with a pitch of 180 µm. The r.m.s. spatial resolution is ≈25 µm.

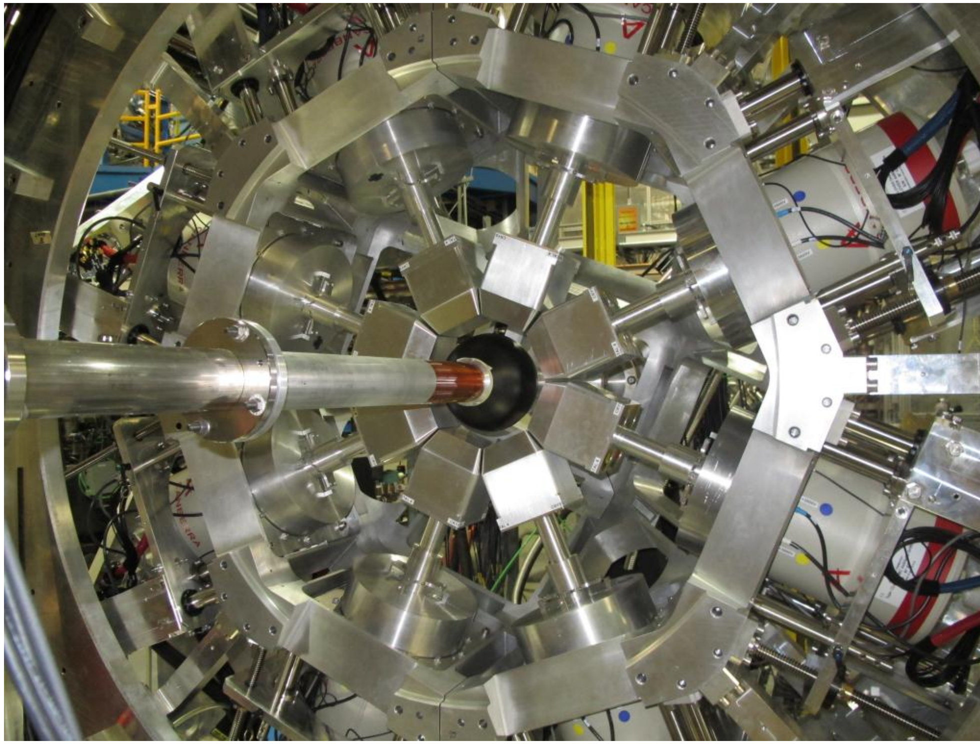




Germanium Detectors







Thank you!
Merci

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