ECSE 493 - Robotics and Control Laboratory
Lab Assignment 2

Posted: February 3, 2016
Due: February 16, 2016

February 2, 2016

Instructions:

The goal of this lab is to identify and model the cart system to be controlled. You will also investigate the frequency and step response of a proportional controller. **Please start this lab early.**

Please make sure all your plots are legible. **In your report, submit all the requested work making sure to explain and justify results.**

Part 1: Identification of System Parameters

1. Derive the open loop transfer function of the cart (pendulum removed) from mechanical and electrical first principles with **voltage as the input and velocity as the output**. Also, express the dynamic equation with **voltage as the input and position as the output**. Calculate the coefficients of these transfer functions. Find the relevant constants in the Lab Manual.

2. Using the step response of the cart's velocity with respect to the input voltage, experimentally calculate the transfer function (do not forget to differentiate the position encoder signal to estimate velocity). To this end, consider a first order model with unknown coefficients and use a square input pulse with an amplitude of $\pm 1\,\mathrm{V}$ and a frequency of $0.5\,\mathrm{Hz}$ to represent step commands. Plot the input and step responses (i.e. derive out an average response for your command)**. Estimate the coefficients of the transfer function** by measuring the time constant and gain of the step response. Label the times axis in terms of the time constant (i.e. $\tau, 2\tau, 3\tau, etc.$) and label the velocity axis in terms of velocity/voltage command. Is the voltage command in each step $1\,\mathrm{V}$ or $2\,\mathrm{V}$ (explain why)?

3. Compare the values obtained experimentally with those in the analytic model. Plot the step response of both models on the same plot and discuss the similarities or differences between the plots. **Use your engineering knowledge to explain differences between the two models.** Speculate why the theoretical and experimental values are different.

Part 2: Proportional Control

1. Derive the **closed loop transfer function of a proportional position controller** for the cart (analytically). Also derive an expression for ζ and ω_n from the closed loop transfer function and use them to compute the damping ratio and natural frequency for $K_p = 20, 100, 200$. Discuss how the natural frequency and damping change as K_p takes on different values. How does a proportional controller affect the response dynamics?

2. Implement (experimentally) a proportional position controller in your Simulink model and plot the step response for a 15cm displacement of the cart for $K_p = 20, 100, 200$. As K_p changes, how do the rise time, overshoot, steady state error and damping change? Compare the step response with the position response predicted by the damping ratio and natural frequency calculated in part 2.1. Again, use your engineering knowledge to explain your observations.

Part 3: Closed-Loop Frequency Response

1. Estimate the frequency response of the closed loop system (position control) using seven sinusoids with frequencies $\omega = [0.1, 0.5, 1, 2, 5, 10, 20, 30]$ $\frac{rad}{s}$ for $K_p = 20$ and $K_p = 200$. For each frequency, estimate the gain from the output by dividing the peak-to-peak amplitude of the output by the peak-to-peak amplitude of the input. Estimate the time lag between the output and input ($t^{peak}_{output} - t^{peak}_{input}$). The phase can be estimated from this time lag. Plot the frequency response (gain and phase) for both values of K_p on the same plot.

2. Discuss the effect of Kp on the frequency response of the system. How does the proportional gain affect the frequency response? Relate the estimated frequency response with the step response in Part 2.2. Use your engineering knowledge and explain the step response from the point of view of the frequency response.