ECSE 493 — Controls&Robotics Lab
Lab 1 Report

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(a) The equation of the DC motor given in the description is:

$$J_m\ddot{\theta} + \left(b + \frac{K_t K_m}{R_a}\right)\dot{\theta} = \frac{K_t}{R_a}v_a$$

Substituting the numbers provided in the equation above, we get:

$$0.01\ddot{\theta} + \left(0.001 + \frac{0.02 * 0.02}{10}\right)\dot{\theta} = \frac{0.02}{10}v_a \quad (1)$$

$$0.01\ddot{\theta} + 0.00104\dot{\theta} = 0.002v_a \quad (2)$$

We then apply the Laplace Transform and get the following:

$$0.01s^2\Theta + 0.00104s\Theta = 0.002V \quad (3)$$

$$\frac{\Theta}{V} = \frac{1}{s}\frac{0.002}{0.01s+0.00104} \quad (4)$$

$$\frac{\dot{\Theta}}{V} = \frac{0.002}{0.01s+0.00104} \quad (5)$$

(b) Next we input this equation in Matlab to plot the steady-state response:

```
num = 0.002;
den = [0.01  0.00104];
T = tf(num,den)
step(T)
```

Which yields the plot in Figure 1

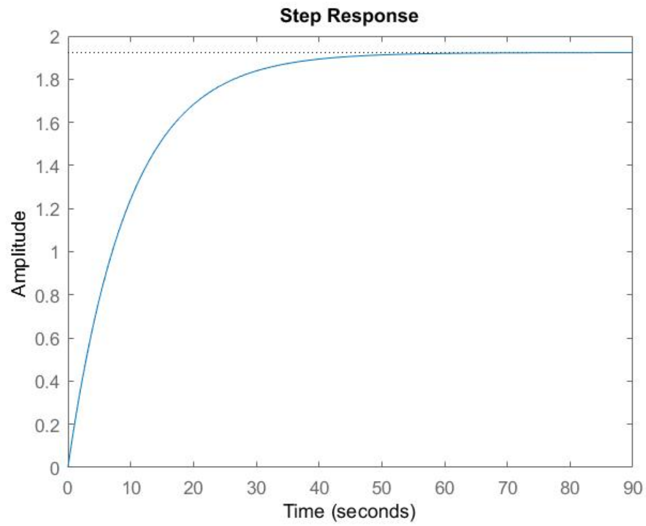


Figure 1: Step response of motor shaft speed

With this plot, we can estimate the steady-state speed of the motor to be about 1.95 rad/sec.

(c) If we assume our estimate in part (b), then 99% of the final speed would be 1.88 rad/sec, which I estimate arrives at around 45 seconds

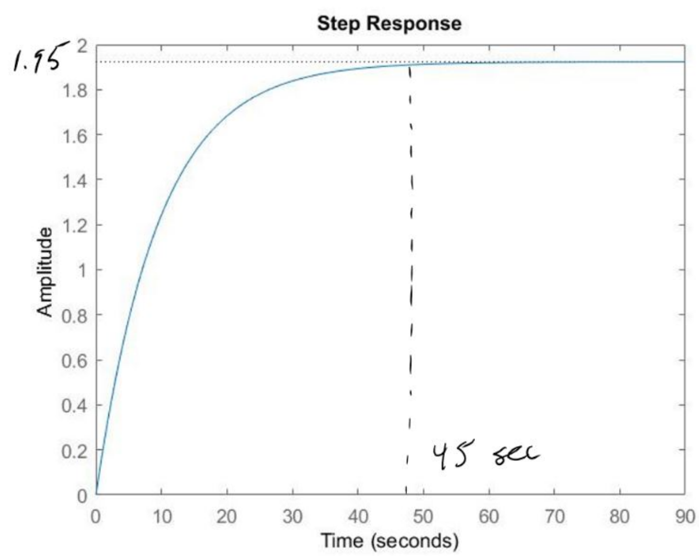


Figure 2: Steady-state and rise-time estimation

(d) The final value theorem goes as follow

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) \tag{6}$$

$$\lim_{s\to 0} s\frac{1}{s}\frac{0.002}{0.01s+0.00104} = \frac{0.002}{0.00104} = 1.923 \tag{7}$$

This is 0.27 rad/sec error, or a 14% overshoot, so my estimate was not that great.

(e) As we derived earlier, the transfer function between shaft angle and voltage is

$$\frac{\Theta}{V} = \frac{1}{s}\frac{0.002}{0.01s+0.00104}$$

(f) If we add feedback to our system, we get the following block diagram, where $G(s)$ is our plant, in this case, the motor. This system can be modeled by the following transfer function:

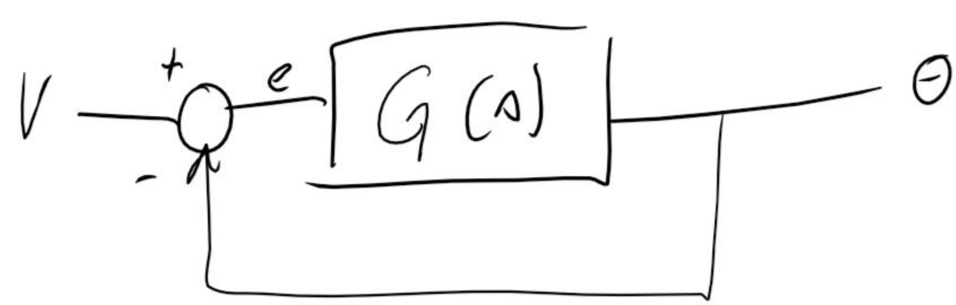


Figure 3: Motor with feedback system

$$H(s) = \frac{G(s)}{1+G(s)}$$

Next we want to add a gain of K to the forward path of the system, this yields the following diagram and is described by the following transfer function:

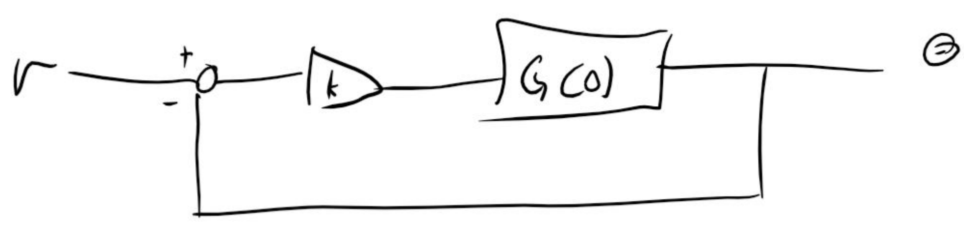


Figure 4: Motor with feedback system and gain on forward path

$$H(s) = \frac{kG(s)}{1+kG(s)}$$

(g) k is gain, unit-less, or V/V

(h) From the system diagrams in figure 4, we can clearly see the input to the plant $G(s)$ is $k * e(s)$. We can a value for this expression like so. First we find an expression for $e(s)$ and one for $\Theta(s)$:

$$e(s) = V(s) - \Theta(s) \tag{8}$$
$$\Theta(s) = e(s) * k * G(s) \tag{9}$$

We isolate $\Theta(s)$ in both expression, equate the equations and isolate $k * e(s)$

$$\Theta(s) = V(s) - e(s) \tag{10}$$
$$V(s) - e(s) = e(s) * k * G(s) \tag{11}$$
$$e(s)\left(k * G(s) + 1\right) = V(s) \tag{12}$$
$$e(s) = \frac{V(s)}{k * G(s) + 1} \tag{13}$$
$$k * e(s) = \frac{k * V(s)}{k * G(s) + 1} \tag{14}$$

(i) Next, we will replace out plant equation in the transfer function of the feedback system

$$H(s) = \frac{kG(s)}{1 + kG(s)}$$

where $G(s) = \frac{1}{s}\frac{0.002}{0.01s+0.00104}$

$$H(s) = \frac{k\left(\frac{0.002}{0.01s^2+0.00104s}\right)}{1 + k\left(\frac{0.002}{0.01s^2+0.00104s}\right)} \tag{15}$$
$$H(s) = \frac{\frac{k0.002}{0.01s^2+0.00104s}}{\frac{0.002k+0.01s^2+0.00104s}{0.01s^2+0.00104s}} \tag{16}$$
$$H(s) = \frac{k0.002}{0.01s^2 + 0.00104s}\frac{0.01s^2 + 0.00104s}{0.002k + 0.01s^2 + 0.00104s} \tag{17}$$
$$H(s) = \frac{k0.002}{0.002k + 0.01s^2 + 0.00104s} \tag{18}$$

(j) We can use Matlab and plot this transfer function for different values of k : We find that a maximum

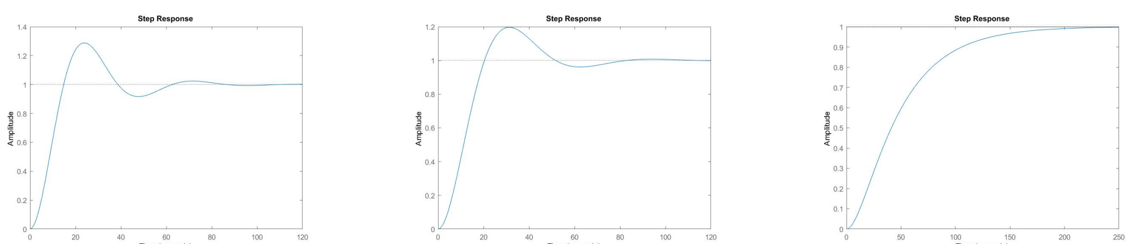


Figure 5: Servo motor response with different gains

value of 0.064 is acceptable to not overshoot past 20%.

(k) In order to reduce our rise-time, we will increase our gain k. If we increase it to a value or 1, our rise-time goes down under 4 seconds, and lead to the followinf step response:

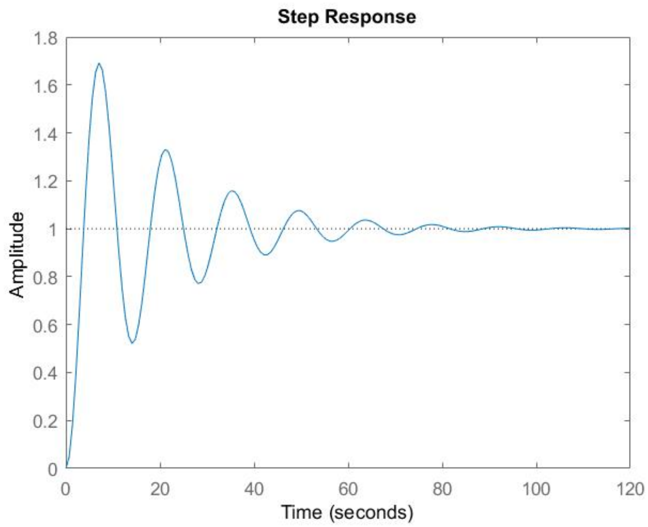
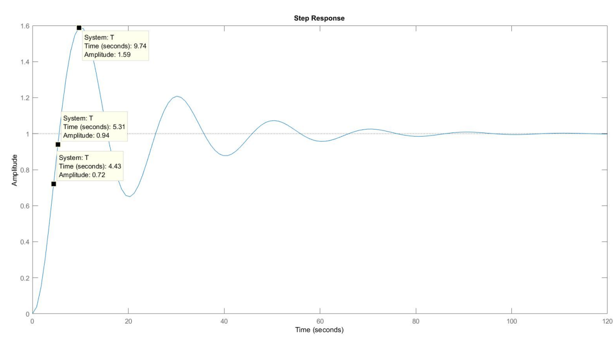
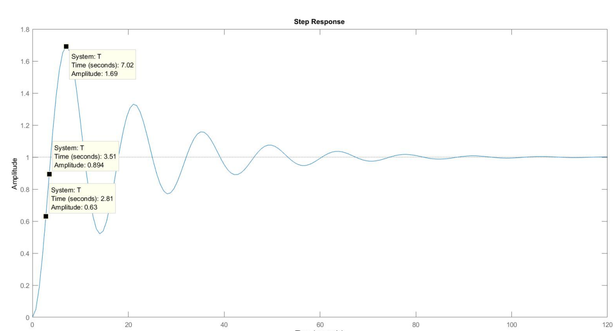


Figure 6: Motor with feedback system and a gain 10

(l) With a value of 0.5 for k, we get the following response: For this diagram, we can approximate a

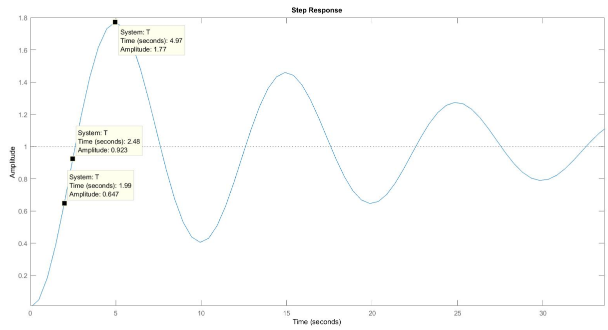


rise-time of 4.7 seconds and an overshoot of 59%.
With a value of 1 for k, we get the following response: For this diagram, we can approximate a rise-time



of 3.2 seconds and an overshoot of 69%.

With a value of 2 for k, we get the following response: For this diagram, we can approximate a rise-time



of 2.4 seconds and an overshoot of 77%.

(m) With this experiment, we see that the parameter k influences the rise-time of the system, more specifically, how aggressive the corrections are. We can see that a large k value means a very fast acting system, but the system has a high inertia, so it is not able to stop when it approaches it's target. A large gain leads to fast rise-time, large oscillations, and long settling time.

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(a) In this code p and q are the coefficients of the numerator and denominator of a transfer function, specifically

$$\frac{s+1}{s^3+5s^2+6s}$$

(b) The root locus diagram show how the poles of a system will change as some parameters in the transfer function change from 0 to ∞.