

Robotics and Control Laboratory  
 ECSE 493 W2017  
 Lab Assignment 1  
 Analysis and Simulation of a Servo System  
**Due January 27<sup>th</sup> by 4pm.**  
**Please upload your assignment to the class webpage**

This assignment is a review. I will try to upload some review notes by Monday.

1. The equation of motion for a DC motor can be described by:

$$J_m \ddot{\theta} + \left( b + \frac{K_t K_e}{R_a} \right) \dot{\theta} = \frac{K_t}{R_a} v_a$$

*Handwritten annotations:*  
 Inertia · Acceleration (pointing to  $J_m \ddot{\theta}$ )  
 Motor Torque (pointing to  $\frac{K_t}{R_a}$ )  
 Back emf (pointing to  $\frac{K_t K_e}{R_a}$ )  
 Motor Torque (pointing to  $\frac{K_t}{R_a}$ )  
 Viscous Friction (pointing to  $b$ )  
 Armature Resistance (pointing to  $R_a$ )  
 Voltage applied (pointing to  $v_a$ )

where

$\theta$  is the shaft angle (in radians) of the motor and  $v_a$  is the applied voltage. Let


$J_m = 0.01 \text{ kgm}^2$  be the inertia of the rotor and the shaft

$b = 0.001 \text{ Nmsec}$  be the viscous friction coefficient

$K_e = 0.02 \text{ Vsec}$  be the back emf constant

$K_t = 0.02 \text{ Nm/A}$  be the motor torque constant

$R_a = 10 \Omega$  be the armature resistance

 Note that  $K_e = K_t$  when one uses SI units.

- (a) Find the transfer function between the applied voltage and the speed of the motor shaft. (Plug in the numbers above).
- (b) Repeat part (a) but use the Matlab command *tf* and the values above to define a transfer function between the applied voltage and the speed of the motor shaft. Use the step command to estimate the steady-state speed of the motor after a voltage is applied.
- (c) Use the step response plot to estimate how long it takes the motor to reach within 1% of steady state angular velocity.
- (d) Look up the Final Value Theorem and calculate the steady state speed of the motor after a voltage is applied. How close was your graphical estimate of the final value?
- (e) Find the transfer function between the applied voltage and the shaft angle.
- (f) Apply feedback to the transfer function in (d). Draw a block diagram of the system. Add a gain of  $K$  to the forward path of your block diagram. What is the transfer function of the feedback system?
- (g) What are the units of  $K$ ?
- (h) What is the input into the motor? (i.e., what is the applied voltage?)
- (i) Find the transfer function between  $v_a$  and  $\theta$ . (The system is now a position servo).
- (j) What is the maximum value of  $K$  that can be used if an overshoot of  $M_p < 20\%$  is desired?

- (k) What value of  $K$  provides a rise time of 4 seconds (ignore overshoot constraint above).
- (l) Plot the step response of the position servo for  $K = 0.5, 1$  and  $2$  and use the plots to find the rise time and % overshoot for each value of  $K$ .
- (m) What feature of the step response is  $K$  controlling?

2. Enter the following in Matlab;

```
p=[1 1];  
q=[1 5 6 0];  
sys=tf(p,q);  
rlocus(sys);
```

-these commands will produce a Root Locus plot.

- a) What are  $p$  and  $q$ ?
- b) Define and explain the function of the root locus. What is being depicted in the root locus plot of the system defined above?