

Group member name(s): 魏珈民、羅啟帆、李嘉芸、黃耀霆、林雋哲
Group member UID(s): b07303025, b07902047, b07303087, b08902136, b08901064

ECON5107: Industrial Organization Assignment #8

Answer to Cement (1)

$$P = 225 - \frac{Q}{2}, MC_A = 50, MC_B = 40, Tech_{MC} = -6$$

1. We first calculate the profit of both firm without innovation, the calculations are as follows:

$$MR_B = 225 - \frac{Q_A}{2} - Q_B = MC_B = 40$$

$$MR_A = 225 - \frac{Q_B}{2} - Q_A = MC_A = 50$$

The Equilibrium Q, P:

$$Q_A = 110, Q_B = 130$$

$$P = 225 - \frac{Q_A + Q_B}{2} = 105$$

The profit without innovation:

$$Profit_A = (105 - 50) * 110 = 6050$$

$$Profit_B = (105 - 40) * 130 = 8450$$

2. We then calculate the profit of firm B, firm A with innovation, and compare the amount with the previously calculated profit. The calculations are as follows:

(1) Firm A:

$$MR_B = 225 - \frac{Q_A}{2} - Q_B = MC_B = 40$$

$$MR_A = 225 - \frac{Q_B}{2} - Q_A = MC_A = 50 - 6 = 44$$

The Equilibrium Q, P:

$$Q_A = 118, Q_B = 126$$

$$P = 225 - \frac{Q_A + Q_B}{2} = 103$$

The profit of firm A with innovation:

$$Profit_A = (103 - 44) * 118 = 6962$$

Thus the willingness to pay for the technology for firm A will be:

$$6962 - 6050 = 912$$

(2) Firm B:

$$MR_B = 225 - \frac{Q_A}{2} - Q_B = MC_B = 40 - 6 = 34$$

$$MR_A = 225 - \frac{Q_B}{2} - Q_A = MC_A = 50$$

The Equilibrium Q, P:

$$Q_A = 106, Q_B = 138$$

$$P = 225 - \frac{Q_A + Q_B}{2} = 103$$

The profit of firm B with innovation:

$$Profit_B = (103 - 34) * 138 = 9522$$

Thus the willingness to pay for the technology for firm B will be:

$$9522 - 8450 = 1072$$

Answer to Cement (2)

With both acquiring the innovation,

$$Q_2 = 191 - \frac{Q_1}{2}$$

$$Q_2 = (181 - Q_1)2$$

$$Q_1 = 114, Q_2 = 134$$

$$P = 225 - \frac{Q_1 + Q_2}{2} = 101$$

$$Profit_1 = (P - 44)Q_1 - 600 = 5898$$

$$Profit_2 = (P - 34)Q_2 - 600 = 8378$$

The game matrix is presented below:

Firm 1 \ Firm 2	innovate	don't innovate
innovate	(5898, 8378)	(6362, 7938)
don't innovate	(5618, 8922)	(6050, 8450)

(The values of the left bottom and the top right box are from the calculated profit of (1) but subtract 600 from the one who innovates.)

One can easily see that the Nash equilibrium is both deciding to acquire the innovation.

Answer to Shipbuilding (1)

$$\begin{aligned}
 17.8 &= a - 0.42 * 19 \\
 a &= 25.78, b = 0.42 \\
 q_1 &= 19 * 0.24 \\
 q_2 &= 19 * 0.08 \\
 q_3 &= 19 * 0.68 \\
 \text{for China} \\
 MR &= P' * q_1 + P = a - b(2q_1 + q_2 + q_3) = mc_1 \\
 mc_1 &= 9928/625 = 15.88 \\
 \text{for Europe} \\
 MR &= P' * q_2 + P = a - b(q_1 + 2q_2 + q_3) = mc_2 \\
 mc_2 &= 10726/625 = 17.16 \\
 \text{for Japan} \\
 MR &= P' * q_3 + P = a - b(q_1 + q_2 + 2q_3) = mc_3 \\
 mc_3 &= 15467/1250 = 12.37 \\
 \text{Marginalcost for (China, Europe, Japan)} &= (15.88, 17.16, 12.37)
 \end{aligned}$$

Answer to Shipbuilding (2)

- Assume in 2006 Europe and Japan knew that the marginal cost of China had decreased and China knew that Europe and Japan knew this, let $q_1 = 50k$, $q_2 = 5k$, $q_3 = 45k$:

$$\begin{aligned}
 \text{for China} \\
 MR &= P' * q_1 + P = a - b(2q_1 + q_2 + q_3) = a - b(150k) = mc_1 - z \\
 \text{for Europe} \\
 MR &= P' * q_2 + P = a - b(q_1 + 2 * q_2 + q_3) = a - b(105k) = mc_2 \\
 k &= 171/875 = 0.195 \\
 \text{for Japan} \\
 MR &= P' * q_3 + P = a - b(q_1 + q_2 + 2 * q_3) = a - b(145k) = mc_3 \\
 k &= 0.215
 \end{aligned}$$

Since k calculated in Europe and Japan is different, we let \bar{k} be the mean of the two = $(0.215+0.195)/2 = 0.205$, and plug into the equation for china to compute z, which yields $z = 3.015$

- Assume in 2006 Europe and Japan didn't know that the marginal cost of China had decreased and China knew that Europe and Japan didn't know this:

Denote China's marginal cost in 2006 mc'_1 . $q_2 = 1.52$ and $q_3 = 12.92$ stay the same because Europe and Japan didn't know that the marginal cost of China had decreased. Meanwhile, China knew that q_2 and q_3 will remain unchanged, so it maximizes its profit given $q_2 = 1.52$ and $q_3 = 12.92$. From the data given, $q_1 : q_2 = 10 : 1$ and $q_1 : q_3 = 10 : 9$. We obtain

$14.355 \leq q_1 \leq 15.2$. q_1 maximizes China's profit given q_2 and q_3 :

$$\begin{aligned} mc'_1 &= MR_1 = a - b(q_2 + q_3) - 2bq_1 \\ mc'_1 &= 25.78 - 0.42(1.52 + 12.92) - 2 * 0.42 * q_1 \end{aligned}$$

Hence, $6.947 \leq mc'_1 \leq 7.657$ and $8.228 \leq z \leq 8.938$.

- The situation in which China didn't know whether or not Japan and Europe knew that its marginal cost had changed needs to be dealt with a game theory model with incomplete information, and the result heavily depends on China's belief on whether or not Japan and Europe knew that its marginal cost had changed. The analysis becomes much more complicated and since we don't think this is the main point of this course, we do not derive the answer under this situation.

Answer to Shipbuilding (3)

- Before 2006

$(q_1, q_2, q_3) = (4.56, 1.52, 12.92)$:

Consumer Surplus

$$(25.78 - 17.8) * 19/2 = 75.81$$

China's Profit

$$4.56 * (17.8 - 15.885) = 8.732$$

Europe's Profit

$$1.52 * (17.8 - 17.162) = 0.97$$

Japan's Profit

$$12.92 * (17.8 - 12.374) = 70.104$$

- Assume in 2006 Europe and Japan knew that the marginal cost of China had decreased and China knew that Europe and Japan knew this: assume marginal cost is constant,

$$q = 100 * \bar{k} = 20.5, p = 25.78 - 0.42 * q = 17.17$$

for China

$$profit = (17.17 - 15.88) * (50 * 0.205) = 13.2225$$

for Europe

$$profit = (17.17 - 17.16) * (5 * 0.0205) = 0.001$$

for Japan

$$profit = (17.17 - 12.37) * (45 * 0.205) = 44.28$$

$$Consumer\ surplus = (a - p) * q/2 = (25.78 - 17.17) * 20.5/2 = 88.2525$$

As a result, consumer is gained, china's profit is gained, and both Europe and Japan are hurt after China's subsidies.

- Assume in 2006 Europe and Japan didn't know that the marginal cost of China had decreased and China knew that Europe and Japan didn't know this:

Let us take $q_1 = (14.355 + 15.2)/2 = 14.78$ and $mc'_1 = (6.947 + 7.657)/2 = 7.257$ in 2006.

$$P = 25.78 - 0.42(14.78 + 1.52 + 12.92) = 13.5076.$$

Consumer Surplus

$$(25.78 - 13.5076) * 29.2 * (1/2) = 179.177$$

China's Profit

$$14.78 * (13.508 - 7.257) = 92.37$$

Europe's Profit

$$1.52 * (13.508 - 17.162) = -5.55$$

Japan's Profit

$$12.92 * (13.508 - 12.374) = 14.651$$

Hence, China and consumers gained, while Europe and Japan were hurt.