RLPBWT

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Outline

1. RLPBWT
2. Example

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Some definitions

The permutation, panel M, $n \times m$

In *RLPBWT* we have a permutation π_j, $\forall\ 1 \leq j \leq m$ that stably sorts the bits of the j-th column of the PBWT.
This permutation can be stored in space proportional to the number of runs in the j-th column of the PBWT

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The positions in the columns of the PBWT of the bits in the i-th row of M are:

$$i, \pi_1(i), \pi_2(\pi_1(i)), \ldots, \pi_{m-1}(\cdots(\pi_2(\pi_1(i)))\cdots)$$

Extracting the bits of the i-th row of M reduces to iteratively applying the π_{m-1} permutations, corresponding to iteratively apply LF in a standard BWT

The compressed data structure

The tables

- a set of m tables in which the m-th table stores only the positions of the run-heads in the m-th column and a bool to check the first symbol: 0 or 1
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The quadruple

1. the position p of the i-th run-head in the j-th column of the PBWT
2. the permutation $\pi_j(p)$
3. the index of the run containing bit $\pi_j(p)$ in the $(j+1)$-st column of the PBWT
4. the threshold, that's the index of the minimum *LCP value* (current column minus divergence array value) in the run

Row extraction

First step

We start by finding the row of the first table that starts with the position p of the head of the run containing bit i in first column of the PBWT, computing:

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looking up the row for the run containing bit $\pi_1(p)$ in the the second table and scanning down the table until we find the row for the run containing bit $\pi_1(i)$

Next step

We continue repeating this procedure for each column

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The Panel

Panel

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	0	1	0	0	1	1	1	1	1	0	0	1	0	0	1	0	0	1
0	1	0	0	0	0	1	1	1	1	1	0	0	1	1	1	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1	1	1	0	0	0	1	0	1	0
1	0	0	1	1	0	1	0	1	0	0	0	1	1	1	0	0	0	1	0
0	1	1	0	1	1	1	1	1	0	0	1	0	0	1	1	1	1	0	0
1	1	0	0	1	0	1	0	1	0	1	0	1	0	0	0	1	1	1	1
0	0	0	1	0	1	1	1	1	1	1	1	0	0	1	0	0	0	1	1

PBWT Matrix

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	0	1	0	0	1	0	0	1	1	0	1	1	1	0	1	0	1	1
0	0	0	1	1	0	0	1	1	0	0	1	1	1	1	0	1	0	1	0
0	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	1	0	1	1
1	0	0	0	0	0	1	0	1	1	1	1	0	0	0	1	0	1	1	0
0	1	1	1	0	0	1	0	1	1	1	0	0	1	1	0	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1	0	1	0	0	1	1	0	1	0
0	1	0	0	0	0	1	1	1	0	1	1	0	0	1	0	0	1	0	1

Prefix and Divergence Arrays

Prefix Arrays

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	1	2	2	1	1	1	2	2	2	5	3	3	1	4	5	5	6	6	0
1	2	6	6	5	2	2	1	5	5	3	4	5	0	6	2	2	3	3	4
2	4	3	3	4	6	0	0	3	3	4	5	1	4	5	0	0	1	1	6
3	6	1	1	2	0	5	5	1	1	2	2	0	6	2	4	6	5	2	3
4	0	4	0	6	5	3	3	0	0	1	1	4	3	1	6	3	2	0	1
5	3	0	5	3	4	6	6	6	6	0	0	2	5	0	1	4	0	5	2
6	5	5	4	0	3	4	4	4	4	6	6	6	2	3	3	1	4	4	5

LCP Arrays: current k minus the original Durbin's divergence arrays

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	3	1	2	0	1	0	6	3	1	9	3	3	4	3	4	1
0	1	1	2	1	5	4	3	4	5	2	0	2	1	1	1	2	1	2	0
0	1	0	1	0	3	1	2	0	1	0	1	8	2	2	0	1	0	1	5
0	0	2	2	4	0	2	3	4	5	1	2	0	0	0	4	2	5	4	3
0	1	1	3	3	2	0	1	2	3	6	7	1	2	10	1	0	3	0	2
0	1	2	0	2	1	1	2	3	4	4	5	3	1	1	2	2	1	2	1

Run-Length PBWT I

Some tables: *p, perm, next perm, threshold*

$$
\begin{array}{cccc}
0 & 4 & 4 & 0\\
1 & 0 & 0 & 1\\
3 & 5 & 5 & 3\\
4 & 2 & 2 & 4\\
5 & 6 & 6 & 5\\
6 & 3 & 3 & 6
\end{array}
\Longrightarrow
\begin{array}{cccc}
0 & 3 & 0 & 0\\
1 & 0 & 0 & 1\\
2 & 4 & 1 & 2\\
3 & 1 & 0 & 3\\
4 & 5 & 2 & 4\\
5 & 2 & 0 & 5\\
6 & 6 & 2 & 6
\end{array}
\Longrightarrow
\begin{array}{cccc}
0 & 0 & 0 & 0\\
4 & 6 & 3 & 4\\
5 & 4 & 2 & 5
\end{array}
\Longrightarrow
\begin{array}{cccc}
0 & 3 & 2 & 0\\
3 & 0 & 0 & 3\\
4 & 6 & 4 & 4\\
5 & 1 & 1 & 6
\end{array}
\Longrightarrow
\begin{array}{cccc}
0 & 0 & 0 & 0\\
1 & 4 & 2 & 2\\
3 & 1 & 0 & 3\\
5 & 6 & 4 & 5\\
6 & 3 & 2 & 6
\end{array}
$$

$$
\begin{array}{cccc}
0 & 0 & 0 & 0\\
2 & 5 & 2 & 2\\
3 & 2 & 2 & 4\\
5 & 6 & 2 & 5\\
6 & 4 & 2 & 6
\end{array}
\Longrightarrow
\begin{array}{cccc}
0 & 1 & 1 & 0\\
1 & 0 & 0 & 1\\
2 & 2 & 1 & 5
\end{array}
\Longrightarrow
\begin{array}{cccc}
0 & 0 & 0 & 0\\
1 & 3 & 1 & 1\\
3 & 1 & 1 & 3\\
5 & 5 & 1 & 5
\end{array}
\Longrightarrow
\begin{array}{cccc}
0 & 0 & 0 & 0\\
1 & 1 & 1 & 3
\end{array}
\Longrightarrow
\begin{array}{cccc}
0 & 3 & 2 & 0\\
1 & 0 & 0 & 1\\
3 & 4 & 2 & 3\\
6 & 2 & 1 & 6
\end{array}
$$

Run-Length PBWT II

$$
\begin{array}{cccc} 0 & 2 & 2 & 0 \\ 1 & 0 & 0 & 2 \\ 3 & 3 & 3 & 3 \end{array}
\Longrightarrow
\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 1 & 4 & 1 & 1 \\ 2 & 1 & 0 & 2 \\ 3 & 5 & 2 & 3 \\ 4 & 2 & 1 & 4 \\ 6 & 6 & 3 & 6 \end{array}
\Longrightarrow
\begin{array}{cccc} 0 & 4 & 2 & 0 \\ 2 & 0 & 0 & 4 \\ 5 & 6 & 3 & 5 \\ 6 & 3 & 1 & 6 \end{array}
\Longrightarrow
\begin{array}{cccc} 0 & 4 & 2 & 0 \\ 2 & 0 & 0 & 2 \\ 4 & 6 & 4 & 4 \\ 5 & 2 & 1 & 6 \end{array}
\Longrightarrow
\begin{array}{cccc} 0 & 3 & 1 & 0 \\ 2 & 0 & 0 & 2 \\ 4 & 5 & 3 & 4 \\ 5 & 2 & 0 & 5 \\ 6 & 6 & 4 & 6 \end{array}
$$

$$
\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 3 & 5 & 2 & 3 \\ 4 & 3 & 1 & 4 \\ 5 & 6 & 3 & 5 \\ 6 & 4 & 1 & 6 \end{array}
\Longrightarrow
\begin{array}{cccc} 0 & 3 & 1 & 0 \\ 3 & 0 & 0 & 3 \\ 5 & 6 & 3 & 5 \\ 6 & 2 & 0 & 6 \end{array}
\Longrightarrow
\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 3 & 5 & 2 & 3 \\ 4 & 3 & 0 & 5 \\ 6 & 6 & 3 & 6 \end{array}
\Longrightarrow
\begin{array}{cccc} 0 & 2 & 2 & 0 \\ 4 & 0 & 0 & 4 \\ 5 & 6 & 4 & 5 \\ 6 & 1 & 1 & 6 \end{array}
\Longrightarrow
\begin{array}{cccc} 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 2 & 5 & 0 & 2 \\ 3 & 1 & 0 & 5 \\ 6 & 6 & 0 & 6 \end{array}
$$

Match with external haplotype I

First case, bits matches at column j-th

- we are looking at d-th bit of the k-th run, that come from the i-th row of the panel
- if this bit match the next bit of the pattern we can go to column $j+1$ and we figure out which bit to look at in that column
- the next bit we look at is still from row i-th

Match with external haplotype II

Second case, bits doesn't matches at column j-th

- we are looking at d-th bit of the k-th run and that bit doesn't match the next bit in the pattern
- we look at the threshold for the k-th run:
  - if d is at most the threshold (check this "at most") than we move to the last bit of the $(k-1)$-st run in the j-th column and then we proceed as in *case 1*
  - if d is greater than the threshold than we move to the first bit of the $(k-1)$-st run in the j-th column and then we proceed as in *case 1*