

# RLPBWT con bitvectors

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**Algorithm 1** Algoritmo per la costruzione di una colonna della RLPBWT con bitvectors

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1: function BUILD(col, pref, div)
2:    $c \leftarrow 0$ ,  $u \leftarrow 0$ ,  $v \leftarrow 0$ ,  $u' \leftarrow 0$ ,  $v' \leftarrow 0$ ,  $curr_{lcs} \leftarrow 0$ ,  $tmp_{thr} \leftarrow 0$ ,  $tmp_{beg} \leftarrow 0$ 
3:    $start \leftarrow \top$ ,  $beg_{run} \leftarrow \top$ ,  $push_{zero} \leftarrow \perp$ ,  $push_{one} \leftarrow \perp$ 
4:   for every  $k \in [0, height)$  do
5:     if  $k = 0 \wedge col[pref[k]] = 1$  then
6:        $start \leftarrow \perp$ 
7:     if  $col[k] = 0$  then
8:        $c \leftarrow c + 1$ 
9:    $runs \leftarrow [0..0]$  ▷ sparse bitvector for runs of length  $height + 1$ 
10:   $thrs \leftarrow [0..0]$  ▷ sparse bitvector for thresholds of length  $height$ 
11:   $zeros \leftarrow [0..0]$  ▷ sparse bitvector for zeros of length  $c$ 
12:   $ones \leftarrow [0..0]$  ▷ sparse bitvector for ones of length  $height - c$ 
13:   $samples_{beg} \leftarrow []$ ,  $samples_{end} \leftarrow []$  ▷ couple of vectors for samples of length  $r$ 
14:  if  $start$  then
15:     $push_{one} \leftarrow \top$ 
16:  else
17:     $push_{zero} \leftarrow \top$ 
18:  for every  $k \in [0, height)$  do
19:    if  $beg_{run}$  then
20:       $u \leftarrow u'$ ,  $v \leftarrow v'$ ,  $tmp_{beg} \leftarrow pref[k]$ 
21:       $beg_{run} \leftarrow \perp$ 
22:    if  $col[pref[k]] = 1$  then
23:       $v' \leftarrow v' + 1$ 
24:    else
25:       $u' \leftarrow u' + 1$ 
26:    if  $k = 0 \vee col[pref[k]] \neq col[pref[k - 1]]$  then
27:       $curr_{lcs} \leftarrow div[i]$ ,  $tmp_{thr} \leftarrow i$ 
28:    if  $div[i] < curr_{lcs}$  then
29:       $curr_{lcs} \leftarrow div[i]$ ,  $tmp_{thr} \leftarrow i$ 
30:    if  $k = height - 1 \vee col[pref[k]] \neq col[pref[k + 1]]$  then
31:       $runs[k] \leftarrow 1$ ,  $thrs[tmp_{thr}] \leftarrow 1$ 
32:       $push(samples_{beg}, tmp_{beg})$ 
33:       $push(samples_{end}, pref[k])$ 
34:      if  $push_{one}$  then
35:        if  $v \neq 0$  then
36:           $ones[k - 1] = 1$ 
37:         $swap(push_{zero}, push_{one})$ 
38:      else
39:        if  $u \neq 0$  then
40:           $zeros[k - 1] = 1$ 
41:         $swap(push_{zero}, push_{one})$ 
42:       $beg_{run} \leftarrow \top$ 
43:    if  $|zeros| \neq 0$  then
44:       $zeros[|zeros| - 1] \leftarrow 1$ 
45:    if  $|ones| \neq 0$  then
46:       $ones[|ones| - 1] \leftarrow 1$ 
47:    build rank/select for the four bitvectors
48:  return ( $start$ ,  $c$ ,  $runs$ ,  $zeros$ ,  $ones$ ,  $samples_{beg}$ ,  $samples_{end}$ ,  $div$ )

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**Algorithm 2** Algoritmo per estrazione simbolo da una run in una colonna

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1: function GET_SYMBOL( $s, r$ )  $\triangleright s = \top$  iff column start with 0,  $r$  run index
2:   if  $s$  then
3:     if  $r \bmod 2 = 0$  then return 0 else return 1
4:   else
5:     if  $r \bmod 2 = 0$  then return 1 else return 0
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**Algorithm 3** Algoritmo per uvtrick

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1: function UVTRICK( $k, i$ )  $\triangleright k$  is column index,  $i$  row index
2:   if  $i = 0$  then
3:     return (0, 0)
4:    $run \leftarrow rank_h^k(i)$ 
5:   if  $run = 0$  then
6:     if  $start^k$  then
7:       return ( $index, 0$ )
8:     else
9:       return (0,  $index$ )
10:  else if  $run = 1$  then
11:    if  $start^k$  then
12:      return ( $select_h^k(run) + 1, i - (select_h^k(run) + 1)$ )
13:    else
14:      return ( $i - (select_h^k(run) + 1), select_h^k(run) + 1$ )
15:  else
16:    if  $run \bmod 2 = 0$  then
17:       $pre_u \leftarrow select_u^k(\frac{run}{2}) + 1$ 
18:       $pre_v \leftarrow select_v^k(\frac{run}{2}) + 1$ 
19:       $offset \leftarrow i - (select_h^k(run) + 1)$ 
20:      if  $start^k$  then
21:        return ( $pre_u + offset, pre_v$ )
22:      else
23:        return ( $pre_u, pre_v + offset$ )
24:    else
25:       $run_u \leftarrow (\frac{run}{2}) + 1$ 
26:       $run_v \leftarrow \frac{run}{2}$ 
27:      if  $\neg start^k$  then
28:         $swap(run_u, run_v)$ 
29:       $pre_u \leftarrow select_u^k(run_u) + 1$ 
30:       $pre_v \leftarrow select_v^k(run_v) + 1$ 
31:       $offset \leftarrow i - (select_h^k(run) + 1)$ 
32:      if  $start^k$  then
33:        return ( $pre_u, pre_v + offset$ )
34:      else
35:        return ( $pre_u + offset, pre_v$ )
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**Algorithm 4** Algoritmo per lf-mapping

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1: function LF( $k, i, s$ )  $\triangleright k$  is column index,  $i$  row index,  $s$  symbol
2:    $c \leftarrow rlpbwt[k].c$ 
3:    $(u, v) \leftarrow uvtrick(k, i)$ 
4:   if  $s = 0$  then
5:     return  $u$ 
6:   else
7:     return  $c + v$ 
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**Algorithm 5** Algoritmo per lf-mapping inverso

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1: function REVERSE_LF( $k, i$ )  $\triangleright k$  is column index,  $i$  row index
2:   if  $k = 0$  then  $\triangleright$  by design
3:     return 0
4:    $k \leftarrow k - 1$ 
5:    $c \leftarrow rlpbwt[k].c$ 
6:   if  $i < c$  then
7:     if  $start^k$  then
8:        $run \leftarrow rank_u^k(i) \cdot 2$ 
9:     else
10:       $run \leftarrow rank_u^k(i) \cdot 2 + 1$ 
11:      $i_{run} \leftarrow 0$ 
12:     if  $run \neq 0$  then
13:        $i_{run} \leftarrow select_h^k(run) + 1$ 
14:      $(prev_0, \_) \leftarrow uvtrick(k, i_{run})$ 
15:     return  $i_{run} + (i - prev_0)$ 
16:   else
17:     if  $start^k$  then
18:        $run \leftarrow rank_v^k(i) \cdot 2 + 1$ 
19:     else
20:        $run \leftarrow rank_v^k(i) \cdot 2$ 
21:      $i_{run} \leftarrow 0$ 
22:     if  $run \neq 0$  then
23:        $i_{run} \leftarrow select_h^k(run) + 1$ 
24:      $(\_, prev_1) \leftarrow uvtrick(k, i_{run})$ 
25:     return  $i_{run} + (i - (c + prev_1))$ 
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**Algorithm 6** Algoritmo per match con aplotipo esterno con panel  $width \times height$ 


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1: function EXTERNAL_MATCHES( $z$ ) ▷ assuming  $|z| = rlpbwt.width$ 
2:    $f \leftarrow 0, f_{run} \leftarrow 0, f' \leftarrow 0$ 
3:    $g \leftarrow 0, g_{run} \leftarrow 0, g' \leftarrow 0$ 
4:    $e \leftarrow 0, l \leftarrow 0$ 
5:   for every  $k \in [0, |z|)$  do
6:      $f_{run} \leftarrow rank_h^k(f), g_{run} \leftarrow rank_h^k(g)$ 
7:      $f' \leftarrow lf(k, f, z[k]), g' \leftarrow lf(k, g, z[k])$ 
8:      $l \leftarrow g - f$ 
9:     if  $f' < g'$  then
10:       $f \leftarrow f', g \leftarrow g'$ 
11:     else
12:       if  $k \neq 0$  then
13:         report matches in  $[e, k - 1]$  with  $l$  haplotypes
14:       if  $f' = |lcp^{k+1}|$  then
15:          $e \leftarrow k + 1$ 
16:       else
17:          $e \leftarrow lcp^{k+1}[f']$ 
18:       if  $(z[e] = 0 \wedge f' > 0) \vee f' = height$  then
19:          $f' \leftarrow g' - 1$ 
20:         if  $e \geq 1$  then
21:            $f_{rev} \leftarrow f', k' \leftarrow k + 1$ 
22:           while  $k' \neq e - 1$  do
23:              $f_{rev} \leftarrow reverse\_lf(k', f_{rev}), k' \leftarrow k' - 1$ 
24:            $run \leftarrow rank_h^{k'}(f_{rev}), symb \leftarrow get\_symbol(start^{k'}, run)$ 
25:           while  $e > 0 \wedge z[e - 1] = symb$  do
26:              $f_{rev} \leftarrow reverse\_lf(e, f_{rev})$ 
27:              $run \leftarrow rank_h^{e-1}(f_{rev})$ 
28:              $symb \leftarrow get\_symbol(start^{e-1}, run)$ 
29:           while  $f' > 0 \wedge (k + 1) - lcp^{k+1}[f] \leq e$  do  $e \leftarrow e - 1$ 
30:            $f \leftarrow f', g \leftarrow g'$ 
31:         else
32:            $g' \leftarrow f' - 1$ 
33:           if  $e \geq 1$  then
34:              $f_{rev} \leftarrow f', k' \leftarrow k + 1$ 
35:             while  $k' \neq e - 1$  do
36:                $f_{rev} \leftarrow reverse\_lf(k', f_{rev}), k' \leftarrow k' - 1$ 
37:              $run \leftarrow rank_h^{k'}(f_{rev}), symb \leftarrow get\_symbol(start^{k'}, run)$ 
38:             while  $e > 0 \wedge z[e - 1] = symb$  do
39:                $f_{rev} \leftarrow reverse\_lf(e, f_{rev})$ 
40:                $run \leftarrow rank_h^{e-1}(f_{rev})$ 
41:                $symb \leftarrow get\_symbol(start^{e-1}, run)$ 
42:             while  $e < height \wedge (k + 1) - lcp^{k+1}[e] \leq e$  do  $e \leftarrow e + 1$ 
43:              $f \leftarrow f', g \leftarrow g'$ 
44:       if  $f < g$  then
45:          $l \leftarrow g - f$ 
46:       report matches in  $[e, |z| - 1]$  with  $l$  haplotypes

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**Algorithm 7** Algoritmo per match con matching-statistics (ms) e thresholds

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1: function MATCHES_MS( $z$ )
2:    $ms_{row} \leftarrow [0..0]$ ,  $ms_{len} \leftarrow [0..0]$  ▷ ms vectors with row and len of length  $|z|$ 
3:    $curr_{row} \leftarrow rlpbwt[0].samples_{end}[|rlpbwt[0].samples_{end}| - 1]$ 
4:    $curr_{index} \leftarrow curr_{row}$ 
5:    $curr_{run} \leftarrow rank_h^0(curr_{index})$ 
6:    $symb \leftarrow get\_symbol(start^0, curr_{run})$  ▷ build matching statistics row
7:   for every  $k \in [0, |z|)$  do
8:     if  $z[k] = symb$  then
9:        $ms_{row}[k] \leftarrow curr_{row}$ 
10:       $curr_{index} \leftarrow lf(k, curr_{index}, z[k])$ 
11:       $curr_{run} \leftarrow rank_h^{k+1}(curr_{index})$ 
12:       $symb \leftarrow get\_symbol(start^{k+1}, curr_{run})$ 
13:     else
14:        $curr_{thr} \leftarrow rank_t^k(curr_{index})$ ,  $single_{run} \leftarrow \perp$ 
15:       if  $samples_{beg}^k[curr_{run}] = samples_{end}^k[curr_{run}]$  then
16:          $single_{run} \leftarrow \top$ 
17:       if  $|samples_{beg}^k| = 0$  then
18:          $ms_{row}[k] \leftarrow rlpbwt.height$ 
19:          $curr_{row} \leftarrow rlpbwt[0].samples_{end}[|rlpbwt[0].samples_{end}| - 1]$ 
20:          $curr_{index} \leftarrow curr_{row}$ 
21:          $curr_{run} \leftarrow rank_h^{k+1}(curr_{index})$ 
22:          $symb \leftarrow get\_symbol(start^{k+1}, curr_{run})$ 
23:       else if  $(curr_{run} \neq 0 \wedge \neg single_{run} \wedge curr_{run} = curr_{thr}) \vee curr_{run} = |samples_{beg}^k| - 1$  then
24:          $curr_{index} \leftarrow select_h^k(curr_{run})$ 
25:          $curr_{row} \leftarrow samples_{end}^k[curr_{run} - 1]$ 
26:          $ms_{row}[k] \leftarrow curr_{row}$ 
27:          $curr_{index} \leftarrow lf(k, curr_{index}, z[k])$ 
28:          $curr_{run} \leftarrow rank_h^{k+1}(curr_{index})$ 
29:          $symb \leftarrow get\_symbol(start^{k+1}, curr_{run})$ 
30:       else
31:          $curr_{index} \leftarrow select_h^k(curr_{run} + 1) + 1$ 
32:          $curr_{row} \leftarrow samples_{beg}^k[curr_{run} + 1]$ 
33:          $ms_{row}[k] \leftarrow curr_{row}$ 
34:          $curr_{index} \leftarrow lf(k, curr_{index}, z[k])$ 
35:          $curr_{run} \leftarrow rank_h^{k+1}(curr_{index})$ 
36:          $symb \leftarrow get\_symbol(start^{k+1}, curr_{run})$ 
37:    $tmp_{index} \leftarrow 0$  ▷ build matching statistics len
38:   for every  $k \in [0, |ms_{row}|)$  do
39:     if  $ms_{row}[k] = rlpbwt.height$  then
40:        $ms_{len}[k] \leftarrow 0$ 
41:     else
42:        $tmp_{index} \leftarrow i$  ▷ ra is a data structure for random access over the originale panel
43:       while  $tmp_{index} \geq 0 \wedge z[tmp_{index}] = ra(ms_{row}[k], tmp_{index})$  do
44:          $tmp_{index} \leftarrow tmp_{index} - 1$ 
45:        $ms_{len}[k] \leftarrow k - tmp_{index}$ 
46:   for every  $k \in [0, |ms_{row}|)$  do ▷ build matching statistics matches
47:     report match at  $ms_{row}[k]$  with  $ms_{len}[k]$  iff  $ms_{len}[k] > ms_{len}[k + 1]$ 
48:     or  $ms_{len}[k] = ms_{len}[k + j]$ ,  $j \geq 1$  and  $\nexists j$  such that  $ms_{len}[k] < ms_{len}[k + j]$ 

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