#### RLPBWT con bitvectors

#### Algorithm 1 Algoritmo per la costruzione di una colonna della RLPBWT con bitvectors

```
1: function BUILD(col, pref, div)
         c \leftarrow 0, \ u \leftarrow 0, \ v \leftarrow 0, \ u' \leftarrow 0, \ v' \leftarrow 0, \ curr_{lcs} \leftarrow 0, \ tmp_{thr} \leftarrow 0, \ tmp_{beg} \leftarrow 0
<del>-</del>3:
         start \leftarrow \top, \ beg_{run} \leftarrow \top, \ push_{zero} \leftarrow \bot, \ push_{one} \leftarrow \bot
4:
         for every k \in [0, height) do
5:
             if k = 0 \land col[pref[k]] = 1 then
6:
                  start \leftarrow \bot
7:
             if col[k] = 0 then
8:
                 c \leftarrow c + 1
9:
         runs \leftarrow [0..0]
                                                                                          \triangleright sparse bit
vector for runs of length height+1
10:
          thrs \leftarrow [0..0]
                                                                                       \triangleright sparse bit
vector for thresholds of length height
          zeros \leftarrow [0..0]

ones \leftarrow [0..0]
11:
                                                                                                      \triangleright sparse bit
vector for zeros of length c
12:
                                                                                          \triangleright sparse bitvector for ones of length height-c
          samples_{beg} \leftarrow [], \ samples_{end} \leftarrow [] if start then
13:
                                                                                                \triangleright couple of vectors for samples of length r
14:
15:
              push_{one} \leftarrow \top
16:
          else
17:
              push_{zero} \leftarrow \top
18:
          for every k \in [0, height) do
19:
               if beg_{run} then
20:
                   u \leftarrow u', \ v \leftarrow v', \ tmp_{beg} \leftarrow pref[k]
21:
                   beg_{run} \leftarrow \bot
22:
               if col[pref[k]] = 1 then
23:
                   v' \leftarrow v' + 1
24:
               else
25:
                   u' \leftarrow u' + 1
26:
               if k = 0 \lor col[pref[k]] \ne col[pref[k-1]] then
27:
                   curr_{lcs} \leftarrow div[i], \ tmp_{thr} \leftarrow i
28:
               if div[i] < curr_{lcs} then
29:
                   curr_{lcs} \leftarrow div[i], \ tmp_{thr} \leftarrow i
30:
               if k = height - 1 \lor col[pref[k]] \neq col[pref[k+1]] then
31:
                   runs[k] \leftarrow 1, \ thrs[tmp_{thr}] \leftarrow 1
32:
                   push(samples_{beg}, tmp_{beg})
33:
                   push(samples_{end}, pref[k])
34:
                   if push_{one} then
35:
                        if v \neq 0 then
36:
                            ones[k-1] = 1
37:
                        swap(push_{zero}, push_{one})
38:
                   \mathbf{else}
39:
                        if u \neq 0 then
40:
                            zeros[k-1] = 1
41:
                        swap(push_{zero}, push_{one})
42:
                   beg_{run} \leftarrow \top
43:
          if |zeros| \neq 0 then
44:
               zeros[|zeros|-1] \leftarrow 1
45:
          if |ones| \neq 0 then
46:
               ones[|ones|-1] \leftarrow 1
47:
          build\ rank/select\ for\ the\ four\ bit vectors
48:
          return (start, c, runs, zeros, ones, samples_{beg}, samples_{end}, div)
```

#### Algorithm 2 Algoritmo per estrazione simbolo da una run in una colonna

```
1: function GET_SYMBOL(s, r) 
ightharpoonup s = 	op 	ext{ iff column start with } 0, r 	ext{ run index}
2: if s then
3: if r 	ext{ mod } 2 = 0 then return 0 else return 1
4: else
5: if r 	ext{ mod } 2 = 0 then return 1 else return 0
```

# Algorithm 3 Algoritmo per uvtrick

```
1: function uvtrick(k, i)
                                                                                \triangleright k is column index, i row index
 2:
         if i = 0 then
 3:
             return (0, 0)
         run \leftarrow rank_h^k(i)
 4:
         if run = 0 then
 5:
 6:
             if start^k then
                 return (index, 0)
 7:
 8:
             else
 9:
                 return (0, index)
10:
         else if run = 1 then
              if start^k then
11:
                  return (select_h^k(run) + 1, i - (select_h^k(run) + 1))
12:
13:
                  return (i - (select_h^k(run) + 1), select_h^k(run) + 1)
14:
15:
         else
              if run \mod 2 = 0 then
16:
17:
                  pre_u \leftarrow select_u^k(\frac{run}{2}) + 1
                  pre_v \leftarrow select_v^{\tilde{k}}(\frac{r\tilde{u}n}{2}) + 1
18:
                  offset \leftarrow i - (se\tilde{l}ect_h^k(run) + 1)
19:
                  if start^k then
20:
21:
                      return (pre_u + offset, pre_v)
22:
                  else
                      return (pre_u, pre_v + offset)
23:
24:
              else
                  run_u \leftarrow \left(\frac{run}{2}\right) + 1
25:
                  run_v \leftarrow \frac{ru\tilde{n}}{2}
26:
                  if \neg start^{k^2} then
27:
28:
                      swap(run_u, run_v)
29:
                  pre_u \leftarrow select_u^k(run_u) + 1
30:
                  pre_v \leftarrow select_v^k(run_v) + 1
                  offset \leftarrow i - (select_h^k(run) + 1)
31:
                  if start^k then
32:
33:
                      return (pre_u, pre_v + offset)
34:
                  else
                      return (pre_u + offset, pre_v)
35:
```

#### Algorithm 4 Algoritmo per lf-mapping

```
1: function LF(k, i, s) \triangleright k is column index, i row index, s symbol 2: c \leftarrow rlpbwt[k].c 3: (u, v) \leftarrow uvtrick(k, i) 4: if s = 0 then 5: return u 6: else 7: return c + v
```

## ${\bf Algorithm}~{\bf 5}$ Algoritmo per lf-mapping inverso

```
1: function REVERSE_LF(k, i)
                                                                   \triangleright k is column index, i row index
         if k = 0 then
                                                                                                ⊳ by design
 2:
              return 0
 3:
         k \leftarrow k - 1
 4:
         c \leftarrow rlpbwt[k].c
 5:
         if i < c then
 6:
              if start^k then
 7:
                   run \leftarrow rank_u^k(i) \cdot 2
 8:
              else
 9:
                   run \leftarrow rank_{n}^{k}(i) \cdot 2 + 1
10:
              i_{run} \leftarrow 0
11:
              if run \neq 0 then
12:
                   i_{run} \leftarrow select^k_h(run) + 1
13:
              (prev_0, \_) \leftarrow uvtrick(k, i_{run})
14:
              return i_{run} + (i - prev_0)
15:
         else
16:
              if start^k then
17:
                   run \leftarrow rank_v^k(i) \cdot 2 + 1
18:
19:
              else
                   run \leftarrow rank_v^k(i) \cdot 2
20:
              i_{run} \leftarrow 0
21:
22:
              if run \neq 0 then
                   i_{run} \leftarrow select_h^k(run) + 1
23:
              (\_, prev_1) \leftarrow uvtrick(k, i_{run})
24:
              return i_{run} + (i - (c + prev_1))
25:
```

### **Algorithm 6** Algoritmo per match con aplotipo esterno con panel $width \times height$

```
1: function EXTERNAL_MATCHES(z)
                                                                                            \triangleright assuming |z| = rlpbwt.width
          f \leftarrow 0, \ f_{run} \leftarrow 0, \ f' \leftarrow 0
 2:
          g \leftarrow 0, \ g_{run} \leftarrow 0, \ g' \leftarrow 0
 3:
          e \leftarrow 0, l \leftarrow 0
 4:
 5:
          for every k \in [0, |z|) do
               f_{run} \leftarrow rank_h^k(f), \ g_{run} \leftarrow rank_h^k(g)
 6:
               f' \leftarrow lf(k, f, z[k]), g' \leftarrow lf(k, g, z[k])
 7:
 8:
               l \leftarrow g - f
               if f' < g' then
 9:
10:
                    f \leftarrow f', \ g \leftarrow g'
               else
11:
12:
                    if k \neq 0 then
13:
                         report matches in [e, k-1] with l haplotypes
                    if f' = |lcp^{k+1}| then
14:
15:
                         e \leftarrow k + 1
16:
                    else
                         e \leftarrow lcp^{k+1}[f']
17:
18:
                    if (z[e] = 0 \land f' > 0) \lor f' = height then
19:
                         f' \leftarrow g' - 1
20:
                         if e > 1 then
                              f_{rev} \leftarrow f', \ k' \leftarrow k+1
21:
                              while k' \neq e - 1 do
22:
                                  f_{rev} \leftarrow reverse\_lf(k', f_{rev}), k' \leftarrow k' - 1
23:
                             run \leftarrow rank_h^{k'}(f_{rev}), \ symb \leftarrow get\_symbol(start^{k'}, run)
24:
                             while e > 0 \land z[e-1] = symb \ \mathbf{do}
25:
26:
                                  f_{rev} \leftarrow reverse\_lf(e, f_{rev})
                                  run \leftarrow rank_h^{e-1}(f_{rev})
27:
28:
                                  symb \leftarrow get\_symbol(start^{e-1}, run)
                         while f' > 0 \wedge (k+1) - lcp^{k+1}[f] \le e do e \leftarrow e-1
29:
30:
                         f \leftarrow f', \ g \leftarrow g'
31:
                    else
32:
                         g' \leftarrow f' - 1
33:
                         if e \ge 1 then
                             f_{rev} \leftarrow f', \ k' \leftarrow k+1
34:
                             while k' \neq e - 1 do
35:
                                  f_{rev} \leftarrow reverse\_lf(k', f_{rev}), k' \leftarrow k' - 1
36:
                             run \leftarrow rank_h^{k'}(f_{rev}), \ symb \leftarrow get\_symbol(start^{k'}, run)
37:
                             while e > 0 \wedge z[e-1] = symb \ \mathbf{do}
38:
39:
                                  f_{rev} \leftarrow reverse\_lf(e, f_{rev})
                                  run \leftarrow rank_h^{e-1}(f_{rev})
40:
                                  symb \leftarrow get\_symbol(start^{e-1}, run)
41:
                         while e < height \land (k+1) - lcp^{k+1}[e] \le e do e \leftarrow e+1
42:
43:
                         f \leftarrow f', \ g \leftarrow g'
          if f < g then
44:
45:
               report matches in [e, |z|-1] with l haplotypes
46:
```

#### Algorithm 7 Algoritmo per match con matching-statistics (ms) e thresholds

```
1: function MATCHES_MS(z)
          ms_{row} \leftarrow [0..0], \ ms_{len} \leftarrow [0..0]
                                                                                                      \triangleright ms vectors with row and len of length |z|
3:
          curr_{row} \leftarrow rlpbwt[0].samples_{end}[|rlpbwt[0].samples_{end}|-1]
4:
          curr_{index} \leftarrow curr_{row}
         \begin{array}{l} curr_{run} \leftarrow rank_h^0(curr_{index}) \\ symb \leftarrow get\_symbol(start^0, curr_{run}) \end{array}
5:
6:
7:
                                                                                                                 ▶ build matching statistics row
          for every k \in [0, |z|) do
 8:
              if z[i] = symb then
9:
                   ms_{row}[k] \leftarrow curr_{row}
                    curr_{index} \leftarrow lf(k, curr_{index}, z[k])curr_{run} \leftarrow rank_h^{k+1}(curr_{index})
10:
11:
12:
                    symb \leftarrow get\_symbol(start^{k+1}, curr_{run})
13:
                    \begin{aligned} & curr_{thr} \leftarrow rank_t^k(curr_{index}), \ single_{run} \leftarrow \bot \\ & \text{if } samples_{beg}^k[curr_{run}] = samples_{end}^k[curr_{run}] \ \text{then} \end{aligned}
14:
15:
16:
                          single_{run} \leftarrow \top
                    \mathbf{if}\ |samples^k_{beg}| = 0\ \mathbf{then}
17:
                         ms_{row}[k] \leftarrow rlpbwt.height if k \neq |z| - 1 then
18:
19:
20:
                              curr_{row} \leftarrow rlpbwt[0].samples_{end}[|rlpbwt[0].samples_{end}|-1]
                              \begin{array}{l} curr_{index} \leftarrow curr_{row} \\ curr_{run} \leftarrow rank_h^{k+1}(curr_{index}) \end{array}
21:
22:
23:
                              symb \leftarrow get\_symbol(start^{k+1}, curr_{run})
24:
                    else if (curr_{run} \neq 0 \land \neg single_{run} \land curr_{run} = curr_{thr}) \lor curr_{run} = |samples_{beq}^k| - 1 then
25:
                         curr_{index} \leftarrow select_h^k(curr_{run})
26:
                         curr_{row} \leftarrow samples_{end}^{k}[curr_{run} - 1]
27:
                         ms_{row}[k] \leftarrow curr_{row}
                         if k \neq |z| - 1 then
28:
29:
                              curr_{index} \leftarrow lf(k, curr_{index}, z[k])
                              curr_{run} \leftarrow rank_h^{k+1}(curr_{index})
30:
                              symb \leftarrow get\_symbol(start^{k+1}, curr_{run})
31:
32:
                    else
33:
                         curr_{index} \leftarrow select_h^k(curr_{run} + 1) + 1
34:
                         curr_{row} \leftarrow samples_{beg}^{k}[curr_{run} + 1]
35:
                         ms_{row}[k] \leftarrow curr_{row}
36:
                         if k \neq |z| - 1 then
37:
                              curr_{index} \leftarrow lf(k, curr_{index}, z[k])
                              curr_{run} \leftarrow rank_h^{k+1}(curr_{index})
38:
39:
                              symb \leftarrow get\_symbol(start^{k+1}, curr_{run})
40:
           tmp_{index} \leftarrow 0
                                                                                                                  ▷ build matching statistics len
41:
           for every k \in [0, |ms_{row}|) do
42:
                if ms_{row}[k] = rlpbwt.height then
43:
                    ms_{len}[k] \leftarrow 0
44:
                _{
m else}
45:
                                                                    \triangleright ra is a data structure for random access over the originale panel
                    tmp_{index} \leftarrow i
46:
                     while tmp_{index} \ge 0 \land z[tmp_{index}] = ra(ms_{row}[k], tmp_{index}) do
47:
                         tmp_{index} \leftarrow tmp_{index} - 1
48:
                    ms_{len}[k] \leftarrow k - tmp_{index}
49:
           for every k \in [0, |ms_{row}|) do
                                                                                                         ▶ build matching statistics matches
                report match at ms_{row}[k] with ms_{len}[k] iff ms_{len}[k] > ms_{len}[k+1]
50:
51:
                or ms_{len}[k] = ms_{len}[k+j], j \ge 1 and \nexists j such that ms_{len}[k] < ms_{len}[k+j]
```