## Seoul National University

M1522.000900 Data Structure

Fall 2017, Kang

Homework 1: Mathematical Preliminaries (Chapter 2)

Due: September 21, 02:00 PM

**Reminders**

* The points of this homework add up to 100.
* Like all homework, this has to be done individually.
* Lead T.A.: Beunguk Ahn ([beunguk.ahn@gmail.com](mailto:beunguk.ahn@gmail.com))
* Please type your answers in English. Illegible handwriting may get no points, at the discretion of the graders.
* If you have a question about assignments, please upload your question in eTL.
* If you want to use slipdays or consider late submission with penalties, please note that you are allowed one week to submit your assignment after the due date.

Remember that:

* Whenever you are making an assumption, please state it clearly

**Question 1**

For each question, answer whether on {0, 1, 2, 3} is an equivalence relation or not. Each question shows a set of all the pairs satisfying . [15 points]

1. {(0, 0), (1, 1), (2, 2), (3, 3)}
2. {(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)}
3. {(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)}
4. {(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2)}
5. {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)}

**Question 2**

The Tower of Hanoi is a game to move all the disks from the leftmost pole to the rightmost pole in the minimal number of moves (see a figure below). You can move a disk on top of the towers to other towers at a time. You cannot place a larger disk onto a smaller disk. Let be the number of disk movements to finish the game with disks. Answer the following questions. [25 points]

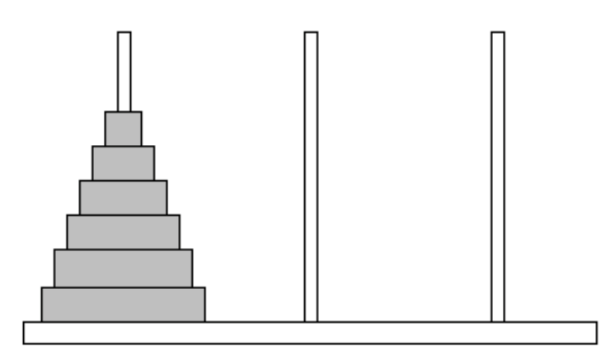
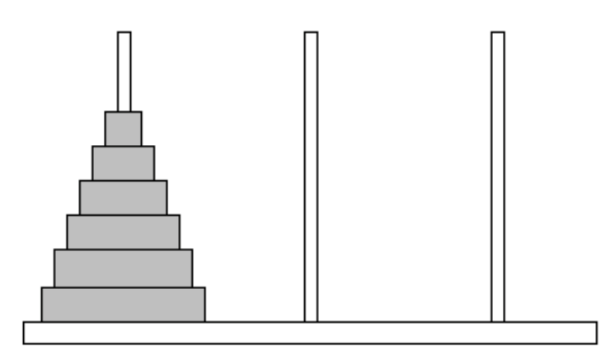


Figure . Tower of Hanoi game with 6 disks. The left figure is an initial state of disks and the right figure is the final state of disks.

1. What is value of ?
2. Write as a recurrence relation.
3. What is a closed form solution of ?

**Question 3**

Expand the following recurrence to help you find a closed-form solution, and then use induction to prove your answer is correct. [30 points]

for

(a) Derive closed-form solution.

(b) **Basis:** Show holds for .

**(c) Inductive step:** Show that the following recurrence holds: If holds for *,* then also holds for *+1*.

**Question 4**

Expand the following recurrence to help you find a closed-form solution, and then use induction to prove your answer is correct. [30 points]

for

(a) Derive closed-form solution.

(b) **Basis:** Show that holds for .

**(c) Inductive step**: Show that the following recurrence holds: If holds for , then also holds for .