5.1-3 5

$X \sim gamma(3, 2)$

$$f_X(x) = \frac{1}{\Gamma(3)\cdot 2^3} x^2 e^{-\frac{x}{2}} = \frac{1}{16} x^2 e^{-\frac{3}{2}}, \quad 0 < x < \infty = \begin{cases} \frac{1}{16} x^2 e^{-\frac{x}{2}}, & 0 < x < \infty \\ 0, & o.w \end{cases}$$ √

$T : Y = \sqrt{X} = u(X) \quad \{x : 0 < x < \infty\}$

$\Updownarrow$ 1:1 대응 √

$T^{-1} : X = Y^2 = u^{-1}(Y) \quad \{y : 0 < y < \infty\}$

변수변환 기법에 의해, $f_Y(y) = f_X(u^{-1}(y)) \cdot \left| \frac{d}{dy} u^{-1}(y) \right|$

$$= \frac{1}{16} y^4 \cdot e^{-\frac{y^2}{2}} \cdot |2y| = \frac{1}{8} y^5 e^{-\frac{y^2}{2}}, \quad 0 < y < \infty$$

$$= \begin{cases} \frac{1}{8} y^5 e^{-\frac{y^2}{2}}, & 0 < y < \infty \\ 0, & o.w \end{cases}$$ √

$\therefore$ Y의 pdf $= f_Y(y) = \begin{cases} \frac{1}{8} y^5 e^{-\frac{y^2}{2}}, & 0 < y < \infty \\ 0, & o.w \end{cases}$

5.1-4 5

$$f_X(x) = \begin{cases} \theta x^{\theta-1}, & 0<x<1, \ 0<\theta<\infty \\ 0, & o.w \end{cases}$$

$T: \ Y = -2\theta \ln X = u(X) \qquad \{x : 0<x<1\}$

$\Updownarrow$ 1:1 대응 ✓

$T^{-1}: \ X = \exp[-\frac{Y}{2\theta}] = u^{-1}(Y) \qquad \{y : 0<y<\infty\}$

변수변환 기법에 의해, $f_Y(y) = f_X(u^{-1}(y)) \cdot \left| \frac{d}{dy} u^{-1}(y) \right| = \theta \cdot (e^{-\frac{y}{2\theta}})^{\theta-1} \cdot \left| e^{-\frac{y}{2\theta}} \cdot -\frac{1}{2\theta} \right|$

$= \frac{1}{2} e^{-\frac{y}{2}}, \quad 0<y<\infty$ ✓

$$f_Y(y) = \begin{cases} \frac{1}{2} e^{-\frac{y}{2}}, & 0<y<\infty \\ 0, & o.w \end{cases}$$

이므로 Y는 θ가 2인 지수분포이다. ✓

(Y는 평균이 2인 지수분포 = $Y \sim Exp(2)$)

5.1-7

$X \sim N(\mu, \sigma^2)$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], \quad -\infty < x < \infty$$

5 (a) $T: Y = e^X = u(X) \quad \{x : -\infty < x < \infty\}$

$\Updownarrow$ 1:1 대응 √

$T^{-1}: X = \ln Y = u^{-1}(Y) \quad \{y : 0 < y < \infty\}$

변수변환 기법에 의해, $g(y) = f_Y(y) = f_X(u^{-1}(y)) \left|\frac{d}{dy}u^{-1}(y)\right|$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(\ln y - \mu)^2}{2\sigma^2}\right] \cdot \left|\frac{1}{y}\right|, \quad 0 < y < \infty$$

$$\therefore g(y) = \begin{cases} \dfrac{1}{y\sqrt{2\pi\sigma^2}} \exp\left[-(\ln y - \mu)^2/2\sigma^2\right], & 0 < y < \infty \\ 0, & \text{o.w} \end{cases}$$ √

(b) 3

(I) $Y = e^{X}$ 이므로 $E(Y) = E(e^{X}) = M_X(1) = \exp(u + \frac{\sigma^2}{2})$

$\therefore E(Y) = \exp(u + \frac{\sigma^2}{2})$ ✓

(II) $Y^2 = e^{2X}$ $E(Y^2) = E(e^{2X}) = M_X(2) = \exp(2u + 2\sigma^2)$

$\therefore E(Y^2) = \exp(2u + 2\sigma^2)$ ✓

(III) $Var(Y) = E(Y^2) - [E(Y)]^2 = \exp(2u + 2\sigma^2) - \exp(2u + \sigma^2)$

$\therefore Var(Y) = \exp(2u + 2\sigma^2) - \exp(2u + \sigma^2)$ ✓

5.1-8

(a) $X \sim N(0,1)$ $\quad f_X(x) = \frac{1}{\sqrt{2\pi}} \exp[-\frac{x^2}{2}]$, $-\infty < x < \infty$

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$T: Y = X^2 \qquad \{x : -\infty < x < \infty\}$

1) $T^{-1} : X = +\sqrt{Y} \quad \{x : 0 < x < \infty\} \underset{1:1\ 대응}{\Longleftrightarrow} \{y : 0 < y < \infty\}$ ✓

2) $T^{-1} : X = -\sqrt{Y} \quad \{x : -\infty < x < 0\} \underset{1:1\ 대응}{\Longleftrightarrow} \{y : 0 < y < \infty\}$ ✓

변수변환 기법에 의해,

1) 에서 $f_Y(y) = \frac{1}{\sqrt{2\pi}} \exp[-\frac{y}{2}] \cdot \underline{\left|\frac{1}{2\sqrt{y}}\right|} = \frac{1}{2\sqrt{2\pi}\sqrt{y}} \cdot \exp[-\frac{y}{2}]$, $0 < y < \infty$ ✓

2) 에서 $f_Y(y) = \frac{1}{\sqrt{2\pi}} \exp[-\frac{y}{2}] \underline{\left|-\frac{1}{2\sqrt{y}}\right|} = \frac{1}{2\sqrt{2\pi}\sqrt{y}} \exp[-\frac{y}{2}]$, $0 < y < \infty$ ✓

$$\underline{1) + 2)} = f_Y(y) = \begin{cases} \frac{1}{\sqrt{2\pi y}} e^{-\frac{y}{2}}, & 0 < y < \infty \\ 0, & o.w \end{cases}$$ ✓

$$\therefore Y\text{의 pdf} : f_Y(y) = \begin{cases} \frac{1}{\sqrt{2\pi y}} \exp[-\frac{y}{2}], & 0 < y < \infty \\ 0, & o.w \end{cases}$$

(b)

$$f_X(x) = \begin{cases} \frac{3}{2}x^2, & -1<x<1 \\ 0, & o.w \end{cases}$$

$T: Y = X^2 \quad \{x: -1<x<1\}$

1) $T^{-1}: X = +\sqrt{Y} \quad \{x: 0<x<1\} \Longleftrightarrow \{y: 0<y<1\}$ ✓

1:1 대응

2) $T^{-1}: X = -\sqrt{Y} \quad \{x: -1<x<0\} \Longleftrightarrow \{y: 0<y<1\}$ ✓

1:1 대응

변수변환 기법에 의해,

1) 에서 $f_Y(y) = \frac{3}{2}y \cdot \left|\frac{1}{2\sqrt{y}}\right| = \frac{3}{4}\sqrt{y}, \quad 0<y<1$

2) 에서 $f_Y(y) = \frac{3}{2}y \cdot \left|-\frac{1}{2\sqrt{y}}\right| = \frac{3}{4}\sqrt{y}, \quad 0<y<1$

$$1) + 2) = f_Y(y) = \begin{cases} \frac{3}{2}\sqrt{y}, & 0<y<1 \\ 0, & o.w \end{cases}$$ ✓

Y의 pdf : $f_Y(y) = \begin{cases} \frac{3}{2}\sqrt{y}, & 0<y<1 \\ 0, & o.w \end{cases}$