연습문제 7.5

3-d)

13. $f(x) = xe^x \qquad f(0) = 0$

$f'(x) = xe^x + e^x \qquad f'(0) = 1$

$f''(x) = xe^x + 2e^x \qquad f''(0) = 2$

$f'''(x) = xe^x + 3e^x \qquad f'''(0) = 3$

$f^{(4)}(x) = xe^x + 4e^x \qquad f^{(4)}(0) = 4$

$$P_4(x) = 0 + x + \frac{2}{2!}x^2 + \frac{3}{3!}x^3 + \frac{4}{4!}x^4$$

$$= x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4$$

4-a)

19. $f(x) = \frac{1}{x} \qquad f(1) = 1$

$f'(x) = -\frac{1}{x^2} \qquad f'(1) = -1$

$f''(x) = \frac{2}{x^3} \qquad f''(1) = 2$

$f'''(x) = -\frac{6}{x^4} \qquad f'''(1) = -6$

$f^{(4)}(x) = \frac{24}{x^5} \qquad f^{(4)}(1) = 24$

$$P_4(x) = 1 - (x-1) + \frac{2}{2!}(x-1)^2 + \frac{-6}{3!}(x-1)^3 + \frac{24}{4!}(x-1)^4$$

$$= 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4$$

4-b)

21. $f(x) = \sqrt{x} \qquad f(1) = 1$

$f'(x) = \frac{1}{2\sqrt{x}} \qquad f'(1) = \frac{1}{2}$

$f''(x) = -\frac{1}{4x\sqrt{x}} \qquad f''(1) = -\frac{1}{4}$

$f'''(x) = \frac{3}{8x^2\sqrt{x}} \qquad f'''(1) = \frac{3}{8}$

$f^{(4)}(x) = -\frac{15}{16x^3\sqrt{x}} \qquad f^{(4)}(1) = -\frac{15}{16}$

$$P_4(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 - \frac{5}{128}(x-1)^4$$

10-b)

41. $f(x) = e^x$

$f^{(n+1)}(x) = e^x$

Max on $[0, 0.6]$ is $e^{0.6} \approx 1.8221$.

$$R_n \le \frac{1.8221}{(n+1)!}(0.6)^{n+1} < 0.001$$

By trial and error, $n = 5$.

n은 5이상

(e^0.6의 근삿값으로 어떤 값을 사용하느냐 여부에 따라 답이 달라질 수 있음)

14

Let f be an odd function and P_n be the n^{th} Maclaurin polynomial for f. Since f is odd, f' is even:

$$f'(-x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h} = \lim_{h \to 0} \frac{-f(x-h) + f(x)}{h} = \lim_{h \to 0} \frac{f(x + (-h)) - f(x)}{-h} = f'(x).$$

Similarly, f'' is odd, f''' is even, etc. Therefore, $f, f'', f^{(4)}$, etc. are all odd functions, which implies that $f(0) = f''(0) = \cdots = 0$. Hence, in the formula

$P_n(x) = f(0) + f'(0)x + \dfrac{f''(0)x^2}{2!} + \cdots$ all the coefficients of the even power of x are zero.