연습문제 12.1

2-a)

11. $\int_0^1 \int_0^2 (x+y)\,dy\,dx = \int_0^1 \left[xy + \frac{1}{2}y^2\right]_0^2 dx = \int_0^1 (2x+2)\,dx = \left[x^2 + 2x\right]_0^1 = 3$

2-b)

13. $\int_0^\pi \int_0^{\sin x} (1+\cos x)\,dy\,dx = \int_0^\pi \left[(y + y\cos x)\right]_0^{\sin x} dx = \int_0^\pi [\sin x + \sin x \cos x]\,dx = \left[-\cos x + \frac{1}{2}\sin^2 x\right]_0^\pi = 1 + 1 = 2$

3-a)

23. $\int_1^\infty \int_0^{1/x} y\,dy\,dx = \int_1^\infty \left[\frac{y^2}{2}\right]_0^{1/x} dx = \frac{1}{2}\int_1^\infty \frac{1}{x^2}\,dx = \left[-\frac{1}{2x}\right]_1^\infty = 0 + \frac{1}{2} = \frac{1}{2}$

8-a)

51. $\int_0^2 \int_x^2 x\sqrt{1+y^3}\,dy\,dx = \int_0^2 \int_0^y x\sqrt{1+y^3}\,dx\,dy = \int_0^2 \left[\sqrt{1+y^3}\cdot\frac{x^2}{2}\right]_0^y dy$

$= \frac{1}{2}\int_0^2 \sqrt{1+y^3}\,y^2\,dy = \left[\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{2}{3}(1+y^3)^{3/2}\right]_0^2 = \frac{1}{9}(27) - \frac{1}{9}(1) = \frac{26}{9}$

8-b)

53. $\int_0^1 \int_y^1 \sin(x^2)\,dx\,dy = \int_0^1 \int_0^x \sin(x^2)\,dy\,dx = \int_0^1 \left[y\sin(x^2)\right]_0^x dx$

$= \int_0^1 x\sin(x^2)\,dx = \left[-\frac{1}{2}\cos(x^2)\right]_0^1 = -\frac{1}{2}\cos 1 + \frac{1}{2}(1) = \frac{1}{2}(1 - \cos 1) \approx 0.2298$

연습문제 12.2

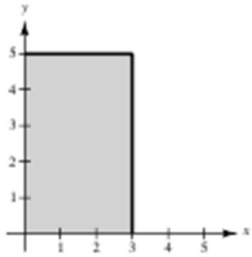
3-a)

11. $\displaystyle\int_0^5\int_0^3 xy\,dx\,dy = \int_0^3\int_0^5 xy\,dy\,dx$

$$= \int_0^3 \left[\frac{1}{2}xy^2\right]_0^5 dx$$

$$= \frac{25}{2}\int_0^3 x\,dx$$

$$= \left[\frac{25}{4}x^2\right]_0^3 = \frac{225}{4}$$



3-c)

15. $\displaystyle\int_3^4\int_{4-y}^{\sqrt{4-y}} -2y\,dx\,dy = \int_0^1\int_{4-x}^{4-x^2} -2y\,dy\,dx$

$$= -2\int_0^1 \left[\frac{1}{2}y^2\right]_{4-x}^{4-x^2} dx$$

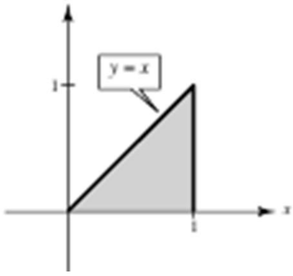
$$= -\int_0^1 \left[(4-x^2)^2 - (4-x)^2\right] dx = -\frac{6}{5}$$

5-a)

27. $\displaystyle V = \int_0^1\int_0^x xy\,dy\,dx$

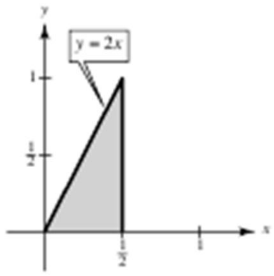
$$= \int_0^1 \left[\frac{1}{2}xy^2\right]_0^x dx = \frac{1}{2}\int_0^1 x^3\,dx$$

$$= \left[\frac{1}{8}x^4\right]_0^1 = \frac{1}{8}$$



9-a)

43. $$\int_0^1 \int_{y/2}^{1/2} e^{-x^2}\,dx\,dy = \int_0^{1/2}\int_0^{2x} e^{-x^2}\,dy\,dx$$
$$= \int_0^{1/2} 2xe^{-x^2}\,dx$$
$$= \Big[-e^{-x^2}\Big]_0^{1/2}$$
$$= -e^{-1/4} + 1$$
$$= 1 - e^{-1/4} \approx 0.221$$



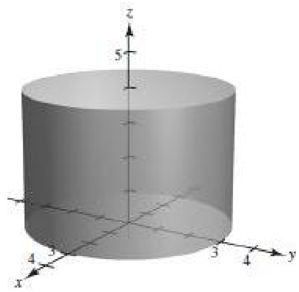
연습문제 12.6

2-a)

7. $\displaystyle\int_0^4\int_0^{4-x}\int_0^{4-x-y} dz\,dy\,dx$

5-b)

19. $Q = \{(x, y, z):\ x^2 + y^2 \le 9, 0 \le z \le 4\}$



$$\iiint_Q xyz\,dV = \int_0^4\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} xyz\,dy\,dx\,dz$$

$$= \int_0^4\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} xyz\,dx\,dy\,dz$$

$$= \int_{-3}^{3}\int_0^4\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} xyz\,dx\,dz\,dy$$

$$= \int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_0^4 xyz\,dz\,dx\,dy$$

$$= \int_{-3}^{3}\int_0^4\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} xyz\,dy\,dz\,dx$$

$$= \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_0^4 xyz\,dz\,dy\,dx\ (=0)$$

15)

47. $\displaystyle\frac{14}{15} = \int_0^1\int_0^{3-a-y^2}\int_a^{4-x-y^2} dz\,dx\,dy$

$$= \int_0^1\int_0^{3-a-y^2} (4 - x - y^2 - a)\,dx\,dy$$

$$= \int_0^1 \left[(4 - y^2 - a)x - \frac{x^2}{2}\right]_0^{3-a-y^2} dy$$

$$= \int_0^1 \left[(4 - y^2 - a)(3 - a - y^2) - \frac{(3 - a - y^2)}{2}\right] dy$$

$$= \frac{94}{15} - \frac{11a}{3} + \frac{1}{2}a^2$$

Hence, $3a^2 - 22a + 32 = 0$

$$(a - 2)(3a - 16) = 0$$

$$a = 2, \frac{16}{3}.$$