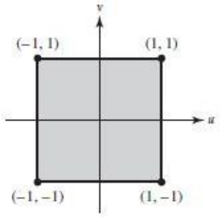
연습문제 12.8

3-a)

11. $x = \frac{1}{2}(u + v)$

$y = \frac{1}{2}(u - v)$



$$\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v} = \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = -\frac{1}{2}$$

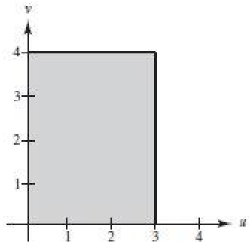
$$\iint_R 4(x^2 + y^2)\, dA = \int_{-1}^{1}\int_{-1}^{1} 4\left[\frac{1}{4}(u + v)^2 + \frac{1}{4}(u - v)^2\right]\left(\frac{1}{2}\right) dv\, du$$

$$= \int_{-1}^{1}\int_{-1}^{1} (u^2 + v^2)\, dv\, du = \int_{-1}^{1} 2\left(u^2 + \frac{1}{3}\right) du = \left[2\left(\frac{u^3}{3} + \frac{u}{3}\right)\right]_{-1}^{1} = \frac{8}{3}$$

3-b)

13. $x = u + v$

$y = u$

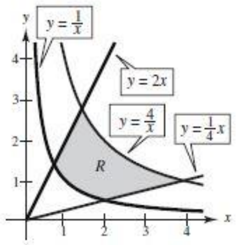


$$\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v} = (1)(0) - (1)(1) = -1$$

$$\iint_R y(x - y)\, dA = \int_0^3\int_0^4 uv(1)\, dv\, du = \int_0^3 8u\, du = 36$$

3-c)

15. $\iint_R e^{-xy/2}\, dA$

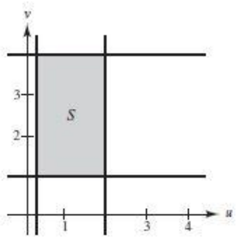


R: $y = \frac{x}{4}, y = 2x, y = \frac{1}{x}, y = \frac{4}{x}$

$x = \sqrt{v/u}, y = \sqrt{uv} \implies u = \frac{y}{x}, v = xy$

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} -\frac{1}{2}\frac{v^{1/2}}{u^{3/2}} & \frac{1}{2}\frac{1}{u^{1/2}v^{1/2}} \\ \frac{1}{2}\frac{v^{1/2}}{u^{1/2}} & \frac{1}{2}\frac{u^{1/2}}{v^{1/2}} \end{vmatrix} = -\frac{1}{4}\left(\frac{1}{u} + \frac{1}{u}\right) = -\frac{1}{2u}$$

Transformed Region:



$y = \frac{1}{x} \implies yx = 1 \implies v = 1$

$y = \frac{4}{x} \implies ux = 4 \implies v = 4$

$y = 2x \implies \frac{y}{x} = 2 \implies u = 2$

$y = \frac{x}{4} \implies \frac{y}{x} = \frac{1}{4} \implies u = \frac{1}{4}$

$$\iint_R e^{-xy/2}\, dA = \int_{1/4}^{2}\int_{1}^{4} e^{-v/2}\left(\frac{1}{2u}\right) dv\, du = -\int_{1/4}^{2}\left[\frac{e^{-v/2}}{u}\right]_1^4 du = -\int_{1/4}^{2}(e^{-2} - e^{-1/2})\frac{1}{u}\, du$$

$$= -\left[(e^{-2} - e^{-1/2})\ln u\right]_{1/4}^{2} = -(e^{-2} - e^{-1/2})\left(\ln 2 - \ln\frac{1}{4}\right) = (e^{-1/2} - e^{-2})\ln 8 \approx 0.9798$$

4-a)

17. $u = x + y = 4, \qquad v = x - y = 0$

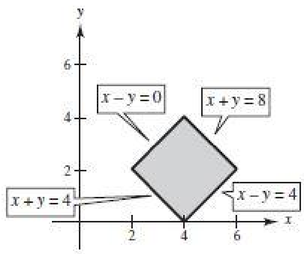
$u = x + y = 8, \qquad v = x - y = 4$

$x = \frac{1}{2}(u + v) \qquad y = \frac{1}{2}(u - v)$

$$\frac{\partial(x, y)}{\partial(u, v)} = -\frac{1}{2}$$

$$\iint_R (x + y)e^{x-y}\, dA = \int_4^8 \int_0^4 ue^v\left(\frac{1}{2}\right) dv\, du$$

$$= \frac{1}{2}\int_4^8 u(e^4 - 1)\, du = \left[\frac{1}{4}u^2(e^4 - 1)\right]_4^8 = 12(e^4 - 1)$$

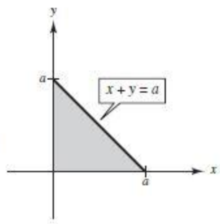


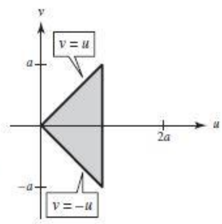
4-c)

21. $u = x + y,\ v = x - y,\ x = \frac{1}{2}(u + v),\ y = \frac{1}{2}(u - v)$

$$\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v} = -\frac{1}{2}$$

$$\iint_R \sqrt{x + y}\, dA = \int_0^a \int_{-u}^{u} \sqrt{u}\left(\frac{1}{2}\right) dv\, du = \int_0^a u\sqrt{u}\, du = \left[\frac{2}{5}u^{5/2}\right]_0^a = \frac{2}{5}a^{5/2}$$





$R = \{(x, y) : x \ge 0,\ y \ge 0,\ 0 \le x + y \le a\}$

$x = \frac{1}{2}(u + v), \quad y = \frac{1}{2}(u - v)$

x, y 가 만족해야하는 부등식에 u,v로 표현한식 대입,,

$\Rightarrow \quad \frac{1}{2}(u + v) \ge 0,\ \frac{1}{2}(u - v) \ge 0,\ 0 \le u \le a$

$\Leftrightarrow \quad u + v \ge 0,\ u - v \ge 0,\ 0 \le u \le a$

$\Leftrightarrow \quad \underbrace{v \ge -u, \quad v \le u}_{\text{v의 범위};\ -u \le v \le u},\ \underbrace{0 \le u \le a}_{\text{u의 범위}}$

연습문제 12.3

3-a)

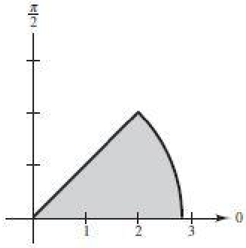
11. $\displaystyle\int_0^a\int_0^{\sqrt{a^2-y^2}} y\,dx\,dy = \int_0^{\pi/2}\int_0^a r^2\sin\theta\,dr\,d\theta = \frac{a^3}{3}\int_0^{\pi/2}\sin\theta\,d\theta = \left[\frac{a^3}{3}(-\cos\theta)\right]_0^{\pi/2} = \frac{a^3}{3}$

3-b)

13. $\displaystyle\int_0^3\int_0^{\sqrt{9-x^2}} (x^2+y^2)^{3/2}\,dy\,dx = \int_0^{\pi/2}\int_0^3 r^4\,dr\,d\theta = \frac{243}{5}\int_0^{\pi/2} d\theta = \frac{243\pi}{10}$

4)

17. $$\int_0^2\int_0^x \sqrt{x^2+y^2}\,dy\,dx + \int_2^{2\sqrt{2}}\int_0^{\sqrt{8-x^2}} \sqrt{x^2+y^2}\,dy\,dx = \int_0^{\pi/4}\int_0^{2\sqrt{2}} r^2\,dr\,d\theta$$
$$= \int_0^{\pi/4}\frac{16\sqrt{2}}{3}\,d\theta$$
$$= \frac{4\sqrt{2}\pi}{3}$$

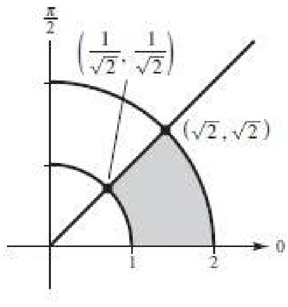


5-a)

19. $$\int_0^2\int_0^{\sqrt{4-x^2}} (x+y)\,dy\,dx = \int_0^{\pi/2}\int_0^2 (r\cos\theta + r\sin\theta)r\,dr\,d\theta = \int_0^{\pi/2}\int_0^2 (\cos\theta+\sin\theta)r^2\,dr\,d\theta$$
$$= \frac{8}{3}\int_0^{\pi/2}(\cos\theta+\sin\theta)\,d\theta = \left[\frac{8}{3}(\sin\theta-\cos\theta)\right]_0^{\pi/2} = \frac{16}{3}$$

5-b)

21. $$\int_0^{1/\sqrt{2}}\int_{\sqrt{1-y^2}}^{\sqrt{4-y^2}} \arctan\frac{y}{x}\,dx\,dy + \int_{1/\sqrt{2}}^{\sqrt{2}}\int_y^{\sqrt{4-y^2}} \arctan\frac{y}{x}\,dx\,dy$$
$$= \int_0^{\pi/4}\int_1^2 \theta r\,dr\,d\theta$$
$$= \int_0^{\pi/4}\frac{3}{2}\theta\,d\theta = \left[\frac{3\theta^2}{4}\right]_0^{\pi/4} = \frac{3\pi^2}{64}$$



12)

45. (a) $\displaystyle I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dA = 4\int_0^{\pi/2}\int_0^{\infty} e^{-r^2/2}r\,dr\,d\theta = 4\int_0^{\pi/2}\left[-e^{-r^2/2}\right]_0^{\infty} d\theta = 4\int_0^{\pi/2} d\theta = 2\pi$

(b) Therefore, $I = \sqrt{2\pi}$.