- $f(x,y) = \frac{1}{2\pi \sigma_x \sigma_y \sqrt{1-\rho_{xy}^2}} \cdot \exp\left[-\frac{1}{2\cdot(1-\rho_{xy}^2)} \cdot \left\{ \left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho_{xy} \cdot \left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2 \right\} \right]$, $-\infty < x, y < \infty$

- $f_x(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$

$$= \frac{1}{2\pi \sigma_x \sigma_y \sqrt{1-\rho_{xy}^2}} \cdot \exp\left[-\frac{1}{2(1-\rho_{xy}^2)} \left(\frac{x-\mu_x}{\sigma_x}\right)^2 \right] \cdot \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2(1-\rho_{xy}^2)} \left\{ -2\rho_{xy}\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2 \right\} \right] dy$$

〃 〃 $= \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2(1-\rho_{xy}^2)} \left\{ \frac{y-\mu_y}{\sigma_y} - \rho_{xy} \cdot \frac{x-\mu_x}{\sigma_x} \right\}^2 \right] dy \cdot \exp\left[\frac{1}{2(1-\rho_{xy}^2)} \rho_{xy}^2 \cdot \left(\frac{x-\mu_x}{\sigma_x}\right)^2 \right]$

〃 〃 $= \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2} \left\{ \frac{y-\mu_y}{\sigma_y\sqrt{1-\rho_{xy}^2}} - \frac{\rho_{xy}}{\sqrt{1-\rho_{xy}^2}} \cdot \frac{x-\mu_x}{\sigma_x} \right\}^2 \right] dy \exp\left[\frac{1}{2(1-\rho_{xy}^2)} \rho_{xy}^2 \left(\frac{x-\mu_x}{\sigma_x}\right)^2 \right]$

↳ $= t$

Then $\frac{1}{\sigma_y\sqrt{1-\rho_{xy}^2}}\, dy = dt$

$Z \sim N(0,1)$

$f(z) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}z^2}$

↳ $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}z^2} dz = 1$

$= \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}t^2 \right] \cdot \times \sigma_y \sqrt{1-\rho_{xy}^2} \cdot dt$

$= \sqrt{2\pi} \cdot \sigma_y \cdot \sqrt{1-\rho_{xy}^2}$

$$= \frac{1}{2\pi \sigma_x \sigma_y \sqrt{1-\rho_{xy}^2}} \cdot \exp\left[-\frac{1}{2\cdot(1-\rho_{xy}^2)} \left(\frac{x-\mu_x}{\sigma_x}\right)^2 \right] \cdot \sqrt{2\pi} \cdot \sigma_y \sqrt{1-\rho_{xy}^2} \cdot \exp\left[\frac{1}{2(1-\rho_{xy}^2)} \rho_{xy}^2 \cdot \left(\frac{x-\mu_x}{\sigma_x}\right)^2 \right]$$

$$= \frac{1}{\sqrt{2\pi} \cdot \sigma_x} \cdot \exp\left[-\frac{1}{2\cdot(1-\rho_{xy}^2)} \left(\frac{x-\mu_x}{\sigma_x}\right)^2 \cdot (1-\rho_{xy}^2) \right]$$

$$= \frac{1}{\sqrt{2\pi}\, \sigma_x} \cdot \exp\left(-\frac{1}{2} \left(\frac{x-\mu_x}{\sigma_x}\right)^2 \right)$$