

Week 6-1

$$a \vee b = \max(a, b)$$

$$a \wedge b = \min(a, b)$$

$$X \sim \text{HG}(N-D, D, n), \quad X \in \{ \underbrace{0 \vee n - (N-D)}, \underbrace{n \wedge D} \}$$

$$\mathbb{E} X(x-1)$$

$$= \sum_x x(x-1) \cdot \binom{D}{x} \cdot \binom{N-D}{n-x} / \binom{N}{n}$$

$$= \sum_x x(x-1) \cdot \frac{D!}{(D-x)! x!} \binom{N-D}{n-x} / \binom{N}{n}$$

$$= D(D-1) \sum_x \frac{(D-2)!}{[(D-2)-(x-2)]! (x-2)!} \binom{N-D}{n-x} / \binom{N}{n}$$

$$= D(D-1) \sum_x \binom{D-2}{x-2} \cdot \binom{N-2-(D-2)}{n-2-(x-2)} / \binom{N}{n}$$

$$= \binom{N-2}{n-2}$$

$$\uparrow \binom{D-2}{x-2} (1+t)^{D-2} \cdot (1+t)^{N-2-(D-2)} = (1+t)^{N-2}$$

$$= D(D-1) \cdot \frac{n(n-1)}{N(N-1)}$$

