기초통계학1 과제 4장 - 2016133 이유진

#4.1

1) $0.20+0.25+k+0.30=1$, $k=0.25$

$E(X)=1\times0.20+2\times0.25+4\times0.25+8\times0.30=4.1$

2) $k\times0.5^0+k\times0.5^1+k\times0.5^2=1$, $k=\frac{4}{7}=0.571$

$E(X)=0\times\frac{4}{7}\times0.5^0+1\times\frac{4}{7}\times0.5^1+2\times\frac{4}{7}\times0.5^2=\frac{4}{7}=0.571$

3) $\int_0^1 kx\,dx=\left[\frac{k}{2}x^2\right]_0^1=\frac{k}{2}=1$, $k=2$

$E(X)=\int_0^1 x\cdot2x\,dx=\frac{2}{3}=0.667$

4) $\int_0^\infty ke^{-x}dx=1$, $k=1$

$E(X)=\int_0^\infty x\cdot e^{-x}dx=1$

#4.5

1) $\Omega=\{R, FR, FFR, FFFR, FFFF\}$

$P(R)=0.4$ / $P(FR)=0.6\times0.4=0.24$ / $P(FFR)=0.6^2\times0.4=0.144$

$P(FFFR)=0.6^3\times0.4=0.0864$ / $P(FFFF)=0.6^4=0.1296$

2) ①

y \ x	1	2	3	4	$f_Y(y)$
0	0	0	0	0.1296	0.1296
1	0.4	0.24	0.144	0.0864	0.8704
$f_X(x)$	0.4	0.24	0.144	0.216	1

② 독립이 아니다. X, Y가 독립이려면 모든 x, y에 대하여 $f(x,y)=f_X(x)f_Y(y)$를 만족해야한다.

$x=1, y=0$일때 $0\neq0.4\times0.1296$

③ $E(X+Y)=E(X)+E(Y)$

$=(1\times0.4+2\times0.24+3\times0.144+4\times0.216)+(0\times0.1296+1\times0.8704)$

$=2.176+0.8704$

$=3.0464$

④ $Var(Y) = E(Y^2) - E(Y)^2$

$= (0^2 \times 0.1296 + 1^2 \times 0.8704) - (0 \times 0.1296 + 1 \times 0.8704)^2$

$= 0.8704 - 0.8704^2$

$= 0.113$

4.6

1)

y	0	1	2	3	합
$f_Y(y)$	0.125	0.1875	0.1875	0.5	1

↑ 0.5^3: YYY

↑ 3×0.5^4: YYDY, YDYY, DYYY

↑ 6×0.5^5: YYDDY, YDYDY, YDDYY, DYYDY, DYDYY, DDYYY

↑ $0.5^3 + 3 \times 0.5^4 + 6 \times 0.5^5$: DDD, DDYD, DYDD, YDDD, DDYYD, DYDYD, YDDYD, DYYDD, YDYDD, YYDDD

2) $E(Y) = 0 \times 0.125 + 1 \times 0.1875 + 2 \times 0.1875 + 3 \times 0.5 = 2.0625$

$Var(Y) = E(Y^2) - E(Y)^2$

$= (0^2 \times 0.125 + 1^2 \times 0.1875 + 2^2 \times 0.1875 + 3^2 \times 0.5) - 2.0625^2$

$= 1.184$

3)

y \ x	3	4	5	$f_Y(y)$
0	0.125	0	0	0.125
1	0	0.1875	0	0.1875
2	0	0	0.1875	0.1875
3	0.125	0.1875	0.1875	0.5
$f_X(x)$	0.25	0.375	0.375	1

독립이 아니다. X, Y가 독립이려면 모든 x, y에 대해 $f(x,y) = f_X(x) f_Y(y)$를 만족해야한다.

$x=3, y=0$일때 $0.125 \neq 0.25 \times 0.125$

4) 경기를 세번하는경우 : DDD, LLL

$$P(DDD) + P(LLL) = 0.5 \times 0.6 \times 0.6 + 0.5 \times 0.6 \times 0.6$$

$$= 0.36$$

4.7

1)

$y \backslash x$	1	2	3	$f_Y(y)$
1	0.3	0.6	0	0.9
3	0	0	0.1	0.1
$f_X(x)$	0.3	0.6	0.1	1

$$f(1,1) = \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{2}{5} \times \frac{1}{4} \times \frac{3}{3}$$

$$f(2,1) = \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} + \frac{2}{5} \times \frac{3}{4} \times \frac{2}{3}$$

2) 독립이 아니다. X, Y가 독립이려면 모든 x, y에 대해 $f(x,y) = f_X(x) f_Y(y)$를 만족해야한다.

$x=1, y=1$일때 $0.3 \neq 0.3 \times 0.9$

3) $Cov(X,Y) = E(XY) - E(X)E(Y)$

$$= \sum_x \sum_y xy f(x,y) - \mu_x \mu_y$$

$$= (1 \times 1 \times 0.3 + 2 \times 1 \times 0.6 + 3 \times 3 \times 0.1) - \underbrace{(1 \times 0.3 + 2 \times 0.6 + 3 \times 0.1)}_{E(X) = 1.8} \underbrace{(1 \times 0.9 + 3 \times 0.1)}_{E(Y) = 1.2}$$

$$= 2.4 - 2.16 = 0.24$$

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$$

$$= \frac{0.24}{\sqrt{(1^2 \times 0.3 + 2^2 \times 0.6 + 3^2 \times 0.1) - 1.8^2}\sqrt{(1^2 \times 0.9 + 3^2 \times 0.1) - 1.2^2}}$$

$$= \frac{0.24}{\sqrt{0.36}\sqrt{0.36}} = 0.667$$