연습문제 7.5

3-d)

13. $f(x) = xe^x \qquad f(0) = 0$

$f'(x) = xe^x + e^x \qquad f'(0) = 1$

$f''(x) = xe^x + 2e^x \qquad f''(0) = 2$

$f'''(x) = xe^x + 3e^x \qquad f'''(0) = 3$

$f^{(4)}(x) = xe^x + 4e^x \qquad f^{(4)}(0) = 4$

$$P_4(x) = 0 + x + \frac{2}{2!}x^2 + \frac{3}{3!}x^3 + \frac{4}{4!}x^4$$

$$= x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4$$

4-a)

19. $f(x) = \frac{1}{x} \qquad f(1) = 1$

$f'(x) = -\frac{1}{x^2} \qquad f'(1) = -1$

$f''(x) = \frac{2}{x^3} \qquad f''(1) = 2$

$f'''(x) = -\frac{6}{x^4} \qquad f'''(1) = -6$

$f^{(4)}(x) = \frac{24}{x^5} \qquad f^{(4)}(1) = 24$

$$P_4(x) = 1 - (x-1) + \frac{2}{2!}(x-1)^2 + \frac{-6}{3!}(x-1)^3 + \frac{24}{4!}(x-1)^4$$

$$= 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4$$

4-b)

21. $f(x) = \sqrt{x} \qquad f(1) = 1$

$f'(x) = \frac{1}{2\sqrt{x}} \qquad f'(1) = \frac{1}{2}$

$f''(x) = -\frac{1}{4x\sqrt{x}} \qquad f''(1) = -\frac{1}{4}$

$f'''(x) = \frac{3}{8x^2\sqrt{x}} \qquad f'''(1) = \frac{3}{8}$

$f^{(4)}(x) = -\frac{15}{16x^3\sqrt{x}} \qquad f^{(4)}(1) = -\frac{15}{16}$

$$P_4(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 - \frac{5}{128}(x-1)^4$$

10-b)

41. $f(x) = e^x$

$f^{(n+1)}(x) = e^x$

Max on $[0, 0.6]$ is $e^{0.6} \approx 1.8221$.

$$R_n \le \frac{1.8221}{(n+1)!}(0.6)^{n+1} < 0.001$$

By trial and error, $n = 5$.

n은 5이상

(e^0.6의 근삿값으로 어떤 값을 사용하느냐 여부에 따라 답이 달라질 수 있음)

14

Let f be an odd function and P_n be the n^{th} Maclaurin polynomial for f. Since f is odd, f' is even:

$$f'(-x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h} = \lim_{h \to 0} \frac{-f(x-h) + f(x)}{h} = \lim_{h \to 0} \frac{f(x+(-h)) - f(x)}{-h} = f'(x).$$

Similarly, f'' is odd, f''' is even, etc. Therefore, $f, f'', f^{(4)}$, etc. are all odd functions, which implies that $f(0) = f''(0) = \cdots = 0$. Hence, in the formula

$P_n(x) = f(0) + f'(0)x + \dfrac{f''(0)x^2}{2!} + \cdots$ all the coefficients of the even power of x are zero.

연습문제 7.6

3-b)

9. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{n+1}}{n+1}\cdot\frac{n}{(-1)^n x^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{nx}{n+1}\right| = |x|$$

Interval: $-1 < x < 1$

When $x = 1$, the alternating series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ converges.

When $x = -1$, the p-series $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges.

Therefore, the interval of convergence is $-1 < x \le 1$.

3-c)

11. $\sum_{n=0}^{\infty}\frac{x^n}{n!}$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)!}\cdot\frac{n!}{x^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x}{n+1}\right| = 0$$

The series converges for all x. Therefore, the interval of convergence is $-\infty < x < \infty$.

3-e)

15. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^n}{4^n}$

Since the series is geometric, it converges only if $|x/4| < 1$ or $-4 < x < 4$.

3-g)

19. $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+2}(x-1)^{n+2}}{n+2}\cdot\frac{n+1}{(-1)^{n+1}(x-1)^{n+1}}\right| = \lim_{n\to\infty}\left|\frac{(n+1)(x-1)}{n+2}\right| = |x-1|$$

$R = 1$

Center: $x = 1$

Interval: $-1 < x - 1 < 1$ or $0 < x < 2$

When $x = 0$, the series $\sum_{n=0}^{\infty}\frac{1}{n+1}$ diverges by the integral test.

When $x = 2$, the alternating series $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n+1}$ converges.

Therefore, the interval of convergence is $0 < x \le 2$.

7

39. (a) $f(x) = \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n, -2 < x < 2$ (Geometric)

(b) $f'(x) = \sum_{n=1}^{\infty} \left(\frac{n}{2}\right)\left(\frac{x}{2}\right)^{n-1}, -2 < x < 2$

(c) $f''(x) = \sum_{n=2}^{\infty} \left(\frac{n}{2}\right)\left(\frac{n-1}{2}\right)\left(\frac{x}{2}\right)^{n-2}, -2 < x < 2$

(d) $\int f(x)\,dx = \sum_{n=0}^{\infty} \frac{2}{n+1}\left(\frac{x}{2}\right)^{n+1}, -2 \le x < 2$

12-a)b)c)

61. (a) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, -\infty < x < \infty$

(See Exercise 29.)

$g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, -\infty < x < \infty$

(b) $f'(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = g(x)$

(c) $g'(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n-1)!} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}$

$= -\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = -f(x)$

(d) $f(x) = \sin x$ and $g(x) = \cos x$

연습문제7.7

2-b)

5. Writing $f(x)$ in the form $a/(1-r)$, we have

$$\frac{3}{2x-1} = \frac{-3}{1-2x} = \frac{a}{1-r}$$

which implies that $a = -3$ and $r = 2x$. Therefore, the power series for $f(x)$ is given by

$$\frac{3}{2x-1} = \sum_{n=0}^{\infty} ar^n = \sum_{n=0}^{\infty} (-3)(2x)^n$$

$$= -3\sum_{n=0}^{\infty}(2x)^n,\ |2x| < 1 \text{ or } -\frac{1}{2} < x < \frac{1}{2}.$$

2-f)

13. $\dfrac{2}{1-x^2} = \dfrac{1}{1-x} + \dfrac{1}{1+x}$

Writing $f(x)$ as a sum of two geometric series, we have

$$\frac{2}{1-x^2} = \sum_{n=0}^{\infty} x^n + \sum_{n=0}^{\infty}(-x)^n = \sum_{n=0}^{\infty}(1+(-1)^n)x^n = \sum_{n=0}^{\infty} 2x^{2n}.$$

The interval of convergence is $|x^2| < 1$ or $-1 < x < 1$ since $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{2x^{2n+2}}{2x^{2n}}\right| = |x^2|$.

(초항이 2이고 공비가 x^2인 무한등비급수의 합으로 보고 멱급수를 유도해도 됨)

3-b)

17. By taking the first derivative, we have $\dfrac{d}{dx}\left[\dfrac{1}{x+1}\right] = \dfrac{-1}{(x+1)^2}$. Therefore,

$$\frac{-1}{(x+1)^2} = \frac{d}{dx}\left[\sum_{n=0}^{\infty}(-1)^n x^n\right] = \sum_{n=1}^{\infty}(-1)^n n x^{n-1}$$

$$= \sum_{n=0}^{\infty}(-1)^{n+1}(n+1)x^n,\ -1 < x < 1.$$

3-c)

19. By integrating, we have $\displaystyle\int \frac{1}{x+1}\,dx = \ln(x+1)$. Therefore,

$$\ln(x+1) = \int\left[\sum_{n=0}^{\infty}(-1)^n x^n\right]dx = C + \sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{n+1},\ -1 < x \le 1.$$

To solve for C, let $x = 0$ and conclude that $C = 0$. Therefore,

$$\ln(x+1) = \sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{n+1},\ -1 < x \le 1.$$

3-e)

since, $\dfrac{1}{x+1} = \displaystyle\sum_{n=0}^{\infty}(-1)^n x^n$,

we have $\dfrac{1}{4x^2+1} = \displaystyle\sum_{n=0}^{\infty}(-1)^n(4x^2)^n = \sum_{n=0}^{\infty}(-1)^n 4^n x^{2n}$

$$= \sum_{n=0}^{\infty}(-1)^n(2x)^{2n},\ -\frac{1}{2} < x < \frac{1}{2}$$

11-c) arctanx의 매클로린 급수로부터

49. From Exercise 48, we have

$$\sum_{n=0}^{\infty}(-1)^n \frac{1}{2^{2n+1}(2n+1)} = \sum_{n=0}^{\infty}(-1)^n \frac{(1/2)^{2n+1}}{2n+1}$$

$$= \arctan\frac{1}{2} \approx 0.4636.$$