$X \sim B(n, p)$

↳ $M_X(t) = E e^{tX}$

<다항 분포의 mgf 유도>

- $X = (X_1, \cdots, X_k) \sim Multi(n, (p_1 \cdots p_k))$

↓

$$E e^{t_1 X_1 + t_2 X_2 + \cdots + t_{k-1} X_{k-1}}$$

$$= \sum_{x_1, x_2 \cdots, x_{k-1}} \left(e^{t_1 x_1 + t_2 x_2 + \cdots t_{k-1} x_{k-1}} \right) \times \text{const} \times p_1^{x_1} \times p_2^{x_2} \times \cdots \times p_k^{x_k}$$

$$\left(p_k = 1 - \sum_{j=1}^{k-1} p_j \right.$$

$$\left. x_k = n - \sum_{j=1}^{k-1} x_j \right.$$

$$= \sum_{x_1 \cdots x_{k-1}} \text{const} \times (p_1 e^{t_1})^{x_1} \times (p_2 e^{t_2})^{x_2} \times \cdots \times (p_{k-1} e^{t_{k-1}})^{x_{k-1}} \times p_k^{x_k}$$

$$= (p_1 e^{t_1} + p_2 e^{t_2} + \cdots + p_{k-1} e^{t_{k-1}} + p_k)^n$$