과제3

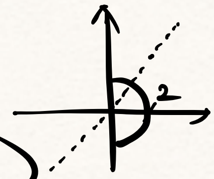
#1.3

6(d)

$y = \sqrt{4-x^2}$ $(x \geq 0)$

$x^2 = 4-y^2$

$x = \sqrt{4-y^2}$ $(y \geq 0)$



11 (1)

$(f^{-1} \circ g^{-1})(1)$

$= f^{-1}(1)$

$= 32$

(2)

$(f^{-1} \cdot f^{-1})(6)$

$= f^{-1}(72)$

$= 75 \times 8 = 600$

#1.4

$x = f^{-1}(y)$ 형태

15. $y = \ln(x + \sqrt{x^2+1})$

☆ 범위대시

$x = \ln(\underline{y + \sqrt{y^2+1}})$

>0

x, y 범위.

#1.6

13. $\lim_{x \to 0} \frac{\sin x(1-\cos x)}{2x^2}$

$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1-\cos x}{x} \cdot \frac{1}{2}$

$= 1 \cdot 0 \cdot \frac{1}{2} = 0$

#1.7

24. $f(x) = \frac{\sqrt{x+c^2}-c}{x}$, $c>0$

f가 $x=0$에서 연속이려면.

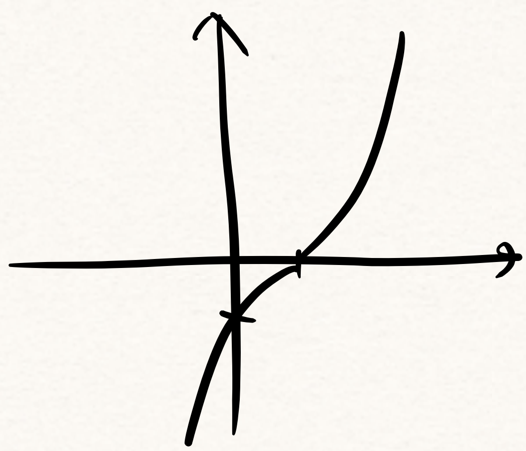
$\lim_{x \to 0} \frac{\sqrt{x+c^2}-c}{x}$

$= \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+c^2}+c)} = \frac{1}{2c}$

#2.1

14(b)

$$f(x) = \begin{cases} (x-1)^3, & x \leq 1 \\ (x-1)^2, & x > 1 \end{cases}$$



$$f'(x) = \begin{cases} 3(x-1)^2 & (x \leq 1) \\ 2(x-1) & (x > 1) \end{cases}$$

미분가능

$$\begin{cases} 1+ \to 0 \\ 1- \to 0 \end{cases}$$

15.

$$f(x)=\begin{cases} x^2+1 & (x\le 2) \\ 4x-3 & (x>2)\end{cases}$$

$$f'(x)=\begin{cases} 2x & (x\le 2) \\ 4 & (x>2)\end{cases}$$

미분가능 $\begin{cases} 2+ \to 4 \\ 2- \to 4 \end{cases}$

2.2

3(b)

$$f(x)=\sqrt{x}-6\sqrt[3]{x}$$

$$=x^{\frac{1}{2}}-6x^{\frac{1}{3}}$$

$$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}-2x^{-\frac{2}{3}}$$

$$=\frac{1}{2\sqrt{x}}-\frac{2}{\sqrt[3]{x^2}}$$

18. (1,0)에서 직선 $y=x-1$에 접하고 (0,1)을 지나는 포물선 $y=ax^2+bx+c$

$c=1$

$y=ax^2+bx+1$

$y'=2ax+b$

$a+b+1=0$

$2a+b=1$ (기울기)

$a=2,\ b=-3$

$y=2x^2-3x+1$

2.3

3(c)

$$f(x)=\frac{2x+5}{\sqrt{x}}=2\sqrt{x}+\frac{5}{\sqrt{x}}=2x^{\frac{1}{2}}+5x^{-\frac{1}{2}}$$

$$f'(x)=x^{-\frac{1}{2}}-\frac{5}{2}x^{-\frac{3}{2}}$$

$$=\frac{1}{\sqrt{x}}-\frac{5}{2}\frac{1}{x\sqrt{x}}=\frac{2x-5}{2x\sqrt{x}}$$

$$\frac{2}{\sqrt{x}}-\frac{2x+5}{2\sqrt{x}x}=\frac{2x\ \cancel{4+2x}-5}{2x\sqrt{x}}$$

9. $2y+x=6$에 평행한 함수 $f(x)=\frac{x+1}{x-1}$의 접선.

$$y=\frac{x+1}{x-1}=(x+1)(x-1)^{-1}$$

$$y'=(x-1)^{-1}+(x+1)(-1)(x-1)^{-2}$$

$$=\frac{x-1-x-1}{(x-1)^2}=\frac{-2}{(x-1)^2}$$

$$\frac{-2}{(x-1)^2}=-\frac{1}{2}$$

$(x-1)^2=4$

$x=-1,3$

$y=-\frac{1}{2}x$

$y=-\frac{1}{2}x+\frac{7}{2}$

#2.4

14(b)

$f(x) = \cos(x^2)$ $(0, 1)$

$f'(x) = -\sin(x^2) \cdot 2x$

$f''(x) = -\cos(x^2)\, 2x \cdot 2x$
$\quad - \sin(x^2) \cdot 2$

$f''(0) = 0$ 그냥 x값만 대입하면 되는건가?

#2.5

5(b) 쌍엽선? $(1,1)$에서 접선

$(x^2+y^2)^2 = 4x^2y$

$x^4 + 2x^2y^2 - 4x^2y + y^4 = 0$

$4x^3 + 4xy^2 + 2x^2 \cdot 2y\frac{dy}{dx} - 8xy - 4x^2\frac{dy}{dx}$
$+ 4y^3\frac{dy}{dx} = 0$

$\cancel{4+4} + \cancel{4\frac{dy}{dx}} - \cancel{8} - \cancel{4\frac{dy}{dx}} + 4\frac{dy}{dx} = 0$

$4\frac{dy}{dx} = 0$

→ ~~접선 $y=1$~~ ?

#2.6

1(a) $f(x) = x^3 + 2x - 1$ $(f^{-1})'(2)$ 구하기

$(f^{-1})'(y_0) = \frac{1}{f'(x_0)}$

y_0을 함숫값으로 갖는 점에서의 도함수의 역수

$f(c) = 2$, $c = 1$

$f'(x) = 3x^2 + 2$

$f'(1) = 5$

$\frac{1}{5}$

1(b)

$f(x) = \sin x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$f'(x) = \cos x$

$f(c) = \frac{1}{2}$, $c = \frac{\pi}{6}$ → $\frac{1}{f'(\frac{\pi}{6})} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

6.

$y = 2\arcsin x$ 의 $(\frac{1}{2}, \frac{\pi}{3})$에서의 접선

$y' = \frac{2}{\sqrt{1-x^2}}$

↳ $y = \frac{2}{\sqrt{1-\frac{1}{4}}}(x - \frac{1}{2}) + \frac{\pi}{3}$

$= \frac{4\sqrt{3}}{3}x - \frac{2\sqrt{3}}{3} + \frac{\pi}{3}$

#3.1

4(c) $y = 3x^{\frac{2}{3}} - 2x$, $[-1, 1]$에서 최대, 최소

$y' = \cancel{3} \cdot \frac{2}{\cancel{3}} x^{-\frac{1}{3}} - 2$

여기서 0.
↑

$2x^{-\frac{1}{3}} = \cancel{x}$

$x^{-\frac{1}{3}} = 1$, $x^{\frac{1}{3}} = 1$, $x = 1$

$f(-1) = 3 \cdot (-1)^{\frac{2}{3}} - 2(-1)$
$= 3 + 2 = 5$

$f(1) = 1$

~~$f(\frac{1}{27}) = 3 \cdot (\frac{1}{27})^{\frac{2}{3}} - 2 \cdot \frac{1}{27}$~~

~~$= 3 \cdot \frac{1}{9} = \frac{1}{3} - \frac{2}{27} = \frac{7}{27}$~~

→ 최대 5, 최소 ~~$\frac{7}{27}$~~ 1