Propositional Logic

Propositions (প্রশা= গথৈছন)

A proposition is a statement that can be either true or false.

- "Yoonjin has an Apple laptop."
- "Yoonjin is a professor."
- ▶ "3 == 2 + 1" T
- ▶ "3 == 2 + 2" F

Not propositions:

- ▶ "Are you Bob?" 의원
- ► "x == 7" 이지는
- ▶ "I am heavy." 기둥・19ት

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Propositional variables দুসাধুদ

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We use propositional variables to refer to propositions.

- ▶ Usually are lower case letters starting with p (i.e. p, q, r, etc.).
- ▶ A propositional variable can have one of two values: true (T) or false (F).

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A proposition can be

- A single variable: p
- ▶ An operation of multiple variables: $p \land (q \lor \neg r)$

· 일반다. 기이 위원생이도 악장은 악두었는 일반다 (제단구단) 일반나는 검독적 그자체다. 단등적합 반복, 대왕으로 베르게 하는다는 가장 기본적인 본국고제이 단위, 코덩이산이들을 반드는것

Introduction to Logical Operators

コンリダイント

Similar to algebraic operators + * - /

In the following examples,

- ▶ p = "Today is Friday."
- ▶ q = "Today is my birthday."

Logical operators: Not

A "not" operation switches (negates) the truth value.

Symbol: - or ~ (tilda)

 $\neg p \equiv$ "Today is not Friday."

р	$\neg p$
Т	F
F	Т

Logical operators: And

An "and" operation is true if both operands are true.

Symbol: ^

It's like the 'A' in And.

 $p \land q \equiv$ "Today is Friday and today is my birthday."

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p	q	p∧q
Т	Т	Т
Т	F	F
F	Т	F
F	F	F

Logical operators: Or

An "or" operation is true if either operands are true.

Symbol: V

 $p \lor q \equiv$ "Today is Friday or today is my birthday (or possibly both)."

р	q	p∨q
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F

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Logical operators: Exclusive Or

XOR

An exclusive or operation is true if one of the operands THEFT are true, but false if both are true. ZOH F

Symbol:

+) 컨튜터시스테이 필수적인 논리회로 (덧셈기)

Often called XOR

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$$p \oplus q \equiv (p \vee q) \wedge \neg (p \wedge q)$$

р	q	p⊕q
Т	Т	F
Т	F	Т
F	Т	Т
F	F	F

 $p \oplus q \equiv$ "Today is Sunday or

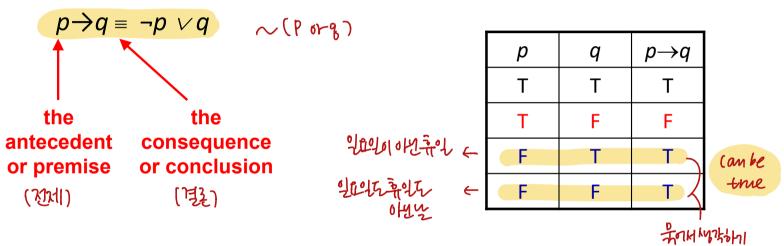
today is a weekday, but not both."

- weekday: Monday ~ Saturday を Saturday

A conditional means "if p then q" ગામ સ્પાયન તામ

Symbol: > PCQZYYXXX

 $p \rightarrow q \equiv$ "If today is Sunday, then today is a holiday."



Such a conditional statement is also called an implication.

Note that if p is false, then the conditional can be true regardless of whether q is true or false.

CAUTION!

Be careful when translating English sentences into Propositions.

For example, unreasonable example

- I state: $p \rightarrow q$ = "If today is Friday, then today is my birthday."
- Consider all possibilities

р	q	$p \rightarrow q$
Т	Т	?
Т	F	?
F	Т	?
F	F	?

Reasonable example

Let p = "he is 5 years old." and q = "he is a child."

I state: $p \rightarrow q \equiv$ "If he is 5 years old, then he is a child."

Consider all possibilities

р	q	$p \rightarrow q$
Т	Т	Т
Т	F	F
F	Т	Т
F	F	Т

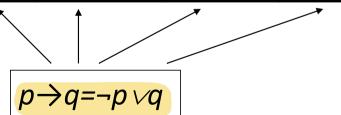
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Alternate ways of stating a conditional:

- p implies q
- ▶ If p, q▶ p only if q
- ▶ p is sufficient for q
- **q** if *p*
- ▶ q whenever p
- q is necessary for p

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				Conditional	Inverse	Converse	Contrapositive
p	q	$\neg p$	$\neg q$	$p{ ightarrow}q$	$\neg p \rightarrow \neg q$	$q{ ightarrow} p$	$\neg q \rightarrow \neg p$
Т	Т	F	F	Т	Т	Т	T
T	F	F	Т	F	Т	Т	F
F	Т	Т	F	Т	F	F	Т
F	F	Т	Т	Т	Т	Т	Т

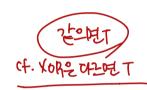


= XNOR

A bi-conditional means "p if and only if q"

Symbol: 智特な

Alternatively, it means "(if p then q) and (if q then p)"



Note that a bi-conditional has the opposite truth values of the Exclusive Or

Also called Exclusive Nor

p	q	$p \leftrightarrow q$
Т	Т	Т
Т	F	F
F	Т	F
F	F	Т

Let p = "Today is Saturday" and q = "Tomorrow is Sunday."

Then $p \leftrightarrow q$ means

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"Today is Saturday if and only if tomorrow is Sunday"

Alternatively, it means "If today is Saturday, then tomorrow is Sunday and if tomorrow is Sunday then today is Saturday"

p	q	p↔q
Τ	Τ	Т
Т	F	F
F	Т	F
F	F	Т

Boolean operators summary

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		not	not	and	or	xor	conditional	Bi-conditional
р	q	$\neg p$	$\neg q$	p∧q	p∨q	p⊕q	$p{ ightarrow}q$	p↔q
Т	Т	F	F	Т	Т	F	Т	Т
Т	F	F	Т	F	Т	Т	F	F
F	Т	Т	F	F	Т	Т	Т	F
F	F	Т	Т	F	F	F	Т	Т

Learn what they mean, don't just memorize the table!

Precedence of operators

Just as in algebra, operators have precedence.

ightharpoonup 4+3*2 = 4+(3*2), not (4+3)*2

Precedence order (from highest to lowest):

$$\neg \land \lor \rightarrow \longleftrightarrow$$

▶ The first three are the most important.

This means that $p \lor q \land \neg r \to s \longleftrightarrow t$ (가서니같은. 아니다) yields: $((p \lor (q \land (\neg r))) \to s) \longleftrightarrow (t)$ 살개움장하지 전달성이었어서

– (Not) is *always* performed before any other operation.

Propositional Equivalence

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Tautology, Contradiction, Equivalence

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- Tautology: a statement that's always true.
 - p ∨ ¬ p will always be true
 p or ~p
- Contradiction: a statement that's always false.
 - p ∧ ¬ p will always be false
 p and ~p
- A logical equivalence means that the two sides always have the same truth values.
 - ▶ Symbol is =

Logical Equivalences

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$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity Laws	$(p \lor q) \lor r \equiv p \lor (q \lor r)$ $(p \land q) \land r \equiv p \land (q \land r)$	Associative Laws
$p \lor T \equiv T$ $p \land F \equiv F$	Domination Laws	$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	Distributive Laws
$p \lor p \equiv p$ $p \land p \equiv p$	Idempotent Laws	$\neg (p \land q) \equiv \neg p \lor \neg q$ $\neg (p \lor q) \equiv \neg p \land \neg q$	De Morgan's Laws
¬(¬ p) ≡ p	Double negation law	$p \lor (p \lor q) \equiv p$ $p \land (p \lor q) \equiv p$	Absorption Laws
$p \lor q \equiv q \lor p$ $p \land q \equiv q \land p$	Commutative Laws	$p \lor \neg p \equiv T : Tautology$ $p \land \neg p \equiv F : Contradiction$	Negation Laws
p→q ≡ ¬p∨q	Definition of Implication	$p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$	Definition of Bi-conditional

How to prove equivalence?

For example, Let's prove the below proposition.

 $(p \rightarrow r) \lor (q \rightarrow r) \equiv (p \land q) \rightarrow r$

Two methods:

- Using truth tables
 - X Not good for long formula
 - In this course, only allowed if specifically stated!
- Using the logical equivalences
 - X The preferred method

Truth Table Solution

$$(p \rightarrow r) \lor (q \rightarrow r) \equiv (p \land q) \rightarrow r$$

р	q	r	p→r	q →r	$(p \rightarrow r) \lor (q \rightarrow r)$	p∧q	(p∧q) > r
Т	Т	Т	Т	Т	Т	Т	Т
Т	Т	F	F	F	F	Т	F
Т	F	Т	Т	Т	Т	F	Т
Т	F	F	F	Т	Т	F	Т
F	Т	Т	Т	Т	Т	F	Т
F	Т	F	Т	F	Т	F	Т
F	F	Т	Т	Т	Т	F	Т
F	F	F	Т	Т	Т	F	Т

Proof using Logical Equivalence

(Proof)

$$(p \rightarrow r) \lor (q \rightarrow r)$$

$$\equiv (\neg p \lor r) \lor (\neg q \lor r)$$

$$\equiv \neg p \lor (r \lor \neg q) \lor r$$

$$\equiv \neg p \lor (\neg q \lor r) \lor r$$

$$\equiv \neg p \lor (\neg q \lor r) \lor r$$

$$\equiv (\neg p \lor \neg q) \lor (r \lor r)$$

$$\equiv (\neg p \lor \neg q) \lor (r \lor r)$$

$$\equiv \neg (p \land q) \lor r$$

$$\equiv (p \land q) \rightarrow r$$
Definition of implication
$$(p \land q) \rightarrow r$$

Example

Show that $(p \land q) \rightarrow (p \lor q)$ is a Tautology. (Proof) $(p \land q) \rightarrow (p \lor q) \equiv T$ Definition of implication | Pzpnhu hzwiten $\equiv \neg (p \land q) \lor (p \lor q)$ $\equiv (\neg p \lor \neg q) \lor (p \lor q)$ De Morgan $\equiv \neg p \lor (\neg q \lor p) \lor q$ **Associative** $\equiv \neg p \lor (p \lor \neg q) \lor q$ Commutative $\equiv (\neg p \lor p) \lor (\neg q \lor q)$ **Associative** $\equiv T \vee T$ Negation $\equiv T$