Week 5-1

Assignments: Textbook Excercises

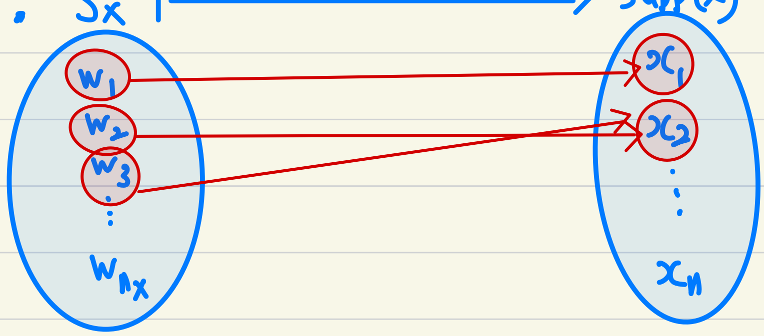
- # 2.1-2, 2.1-4, 2.1-6
- # 2.2-3, 2.2-5
- # 2.3-8, 2.3-9, 2.3-10
- # 2.4-5, 2.4-8
- # 2.5-5, 2.5-6

* 확률변수 X 와 Y의 독립 이란?

- let $S_X = \{w_1, w_2, \cdots, w_{n_X}\}$ is a sample space of X.

$Supp(X) = \{x_1, x_2, \cdots, x_n\}$ is a support of X

$X : S_X \longmapsto Supp(X)$



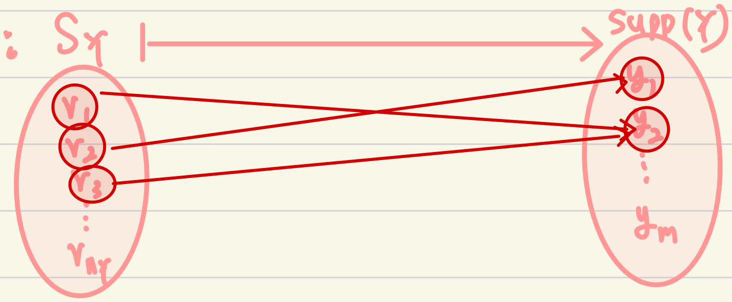
- $X(w_1) = x_1$
- $X(w_2) = x_2$, $X(w_3) = x_2$
...

- $P(X = x_1) = P(X(w) = x_1, w \in S_X) = P(w_1)$

- let $S_Y = \{v_1, v_2, \cdots, v_{n_Y}\}$ is a sample space of Y

$Supp(Y) = \{y_1, y_2, \cdots, y_m\}$ is a support of Y

$Y : S_Y \longmapsto Supp(Y)$



- $Y(v_1) = y_2$, $Y(v_3) = y_2$
- $Y(v_2) = y_1$ ...

- $P(Y = y_1) = P(Y(v) = y_1, v \in S_Y) = P(v_2)$
- $P(Y = y_2) = P(Y(v) = y_2, v \in S_Y) = P(v_1 \sqcup v_3)$

Recall

1.4 독립 사건 (Independent event)

Definition 1.4.1 (독립사건). *사건 A 와 B 에 대해*

$$P(A \cap B) = P(A)P(B) \tag{1.15}$$

$\Rightarrow$ 확률 변수의 X와 Y의 독립은 각 확률 변수의 표본공간에 속하는 모든 사건들이 서로 독립이다.

ex) $P(w_1 \cap r_2) = P(w_1) \cdot P(r_2)$

$\hookrightarrow P(X=x_1 \cap Y=y_1)$ $\quad \hookrightarrow P(X=x_1)$ $\quad \hookrightarrow P(Y=y_1)$

$\Rightarrow$ $P(X=x_i, Y=y_j) = P(X=x_i) \cdot P(Y=y_j)$

$\Rightarrow$ · $E(X+Y) = E(X) + E(Y)$, $\hookrightarrow$ $E(XY) = (EX)(EY)$

· $V(X+Y) = V(X) + V(Y)$.

· let X 와 Y 가 독립

$\Rightarrow$ 임의의 함수 $f(x)$ 와 $g(y)$ 도 독립.

ex) X^2 & $2Y$: 독립

e^{tX} & e^{tY} : 독립 ...

· let $Z = X+Y$, X 와 Y가 독립.

then $Ee^{tZ} = Ee^{t(X+Y)} = E\,e^{tX} \cdot e^{tY}$

$= Ee^{tX} \cdot Ee^{tY}$

$Ee^{tZ} \to M_Z(t)$

$Ee^{tX} \to M_X(t)$, $Ee^{tY} \to M_Y(t)$.

$X \models$ $Ee^{tX} = M_X(t)$; 적률생성 함수 ; mgf.

↳ $C_X(t) = \log\left[M_X(t)\right]$; 누율생성 함수 ; cgf.

$\models$

- $\left.\dfrac{d\,C_X(t)}{dt}\right|_{t=0} = E(X)$
- $\left.\dfrac{d^2 C_X(t)}{dt^2}\right|_{t=0} = Var(X).$

- $C_X(t) = \log\left[M_X(t)\right]$, $M_X(t)$의 Taylor expansion

$$= \log\left[\sum_{k=0}^{\infty} \frac{M_X^{(k)}(0)}{k!} t^k\right]$$

$$= \log\Bigg[1 + \underbrace{\sum_{k=1}^{\infty} \frac{M_X^{(k)}(0)}{k!} t^k}_{=\ \alpha\ =\ M_X^{(1)}(0)\cdot t + \frac{1}{2!} M_X^{(2)}(0)\cdot t^2 + \cdots}\Bigg]$$

cf. $\log(1+\alpha) = \alpha - \frac{1}{2}\alpha^2 + \frac{1}{3}\alpha^3 + \cdots$

$$= t^1\cdot\left(M_X^{(1)}(0)\right) + t^2\cdot\left(\frac{1}{2} M_X^{(2)}(0) - \frac{1}{2}\left[M_X^{(1)}(0)\right]^2\right) + \cdots$$

$$C_X(t) = \sum_{r=0}^{\infty} \frac{C_X^{(r)}(0)}{r!} t^r.$$

$$= t^1\cdot\left(C_X^{(1)}(0)\right) + t^2\left(\frac{1}{2!} C_X^{(2)}(0)\right)$$

$$C_X^{(1)}(0) = M_X^{(1)}(0)$$

$$C_X^{(2)}(0) = M_X^{(2)}(0) - \left[M_X^{(1)}(0)\right]^2$$

$X \sim Ber(p)$.

$f(x) = p^x \cdot (1-p)^{1-x}, \quad x = 0, 1$

- $M_X(t) = E e^{tX}$

$$= \sum_{x=0}^{1} e^{tx} \cdot p^x \cdot (1-p)^{1-x}$$

$$= \sum_{x=0}^{1} (p \cdot e^t)^x \cdot (1-p)^{1-x}$$

$$= [pe^t + 1-p]^1.$$

- $C_X(t) = \log[M_X(t)] = \log[pe^t + 1-p]$

- $C_X^{(1)}(t) = \left.\frac{pe^t}{pe^t + (1-p)}\right|_{t=0} \Rightarrow p = EX$

- $C_X^{(2)}(t) = \left.\frac{pe^t(pe^t + (1-p)) - (pe^t)^2}{[pe^t + (1-p)]^2}\right|_{t=0} \Rightarrow p - p^2 = p(1-p)$

$= Var(X)$

A : 독감 백신 맞았을때 A에서 생산된 사건

B : 〃 〃 〃 B 〃 〃

$P(A) = 0.4$, $P(B) = 0.6$

C : 효과X

$P(C|A) = 0.03$, $P(C|B) = 0.02$

$P(C) = \frac{8}{100} = 0.08$

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{P(C|A)P(A)}{P(C)}$$