Part2: Rendering

6. Texture

Outline

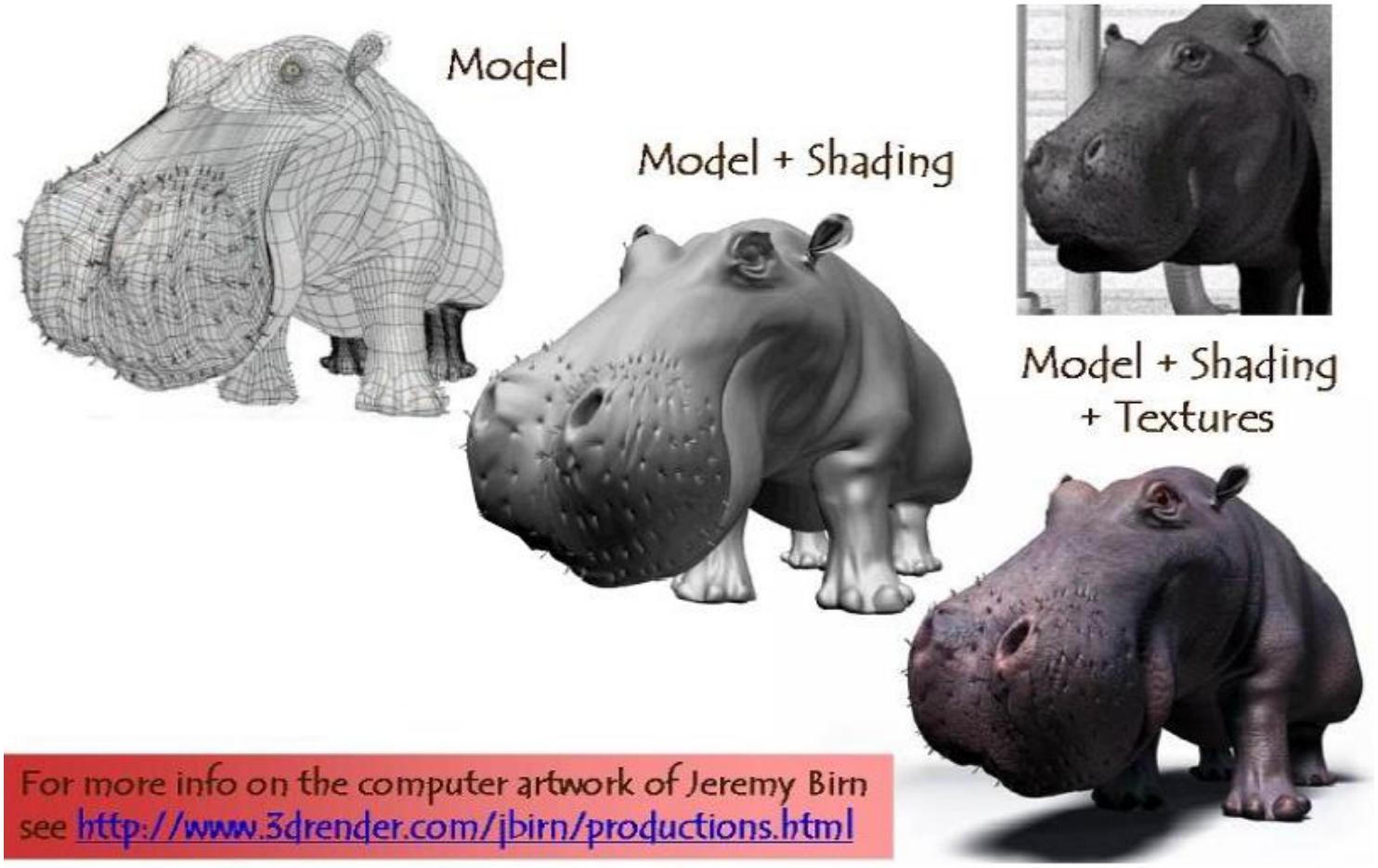
I. Texture Mapping

II. UV Texture Mapping

III. Parametric Texturing

1. Texture Mapping

- Effect of Textures



1. Texture Mapping

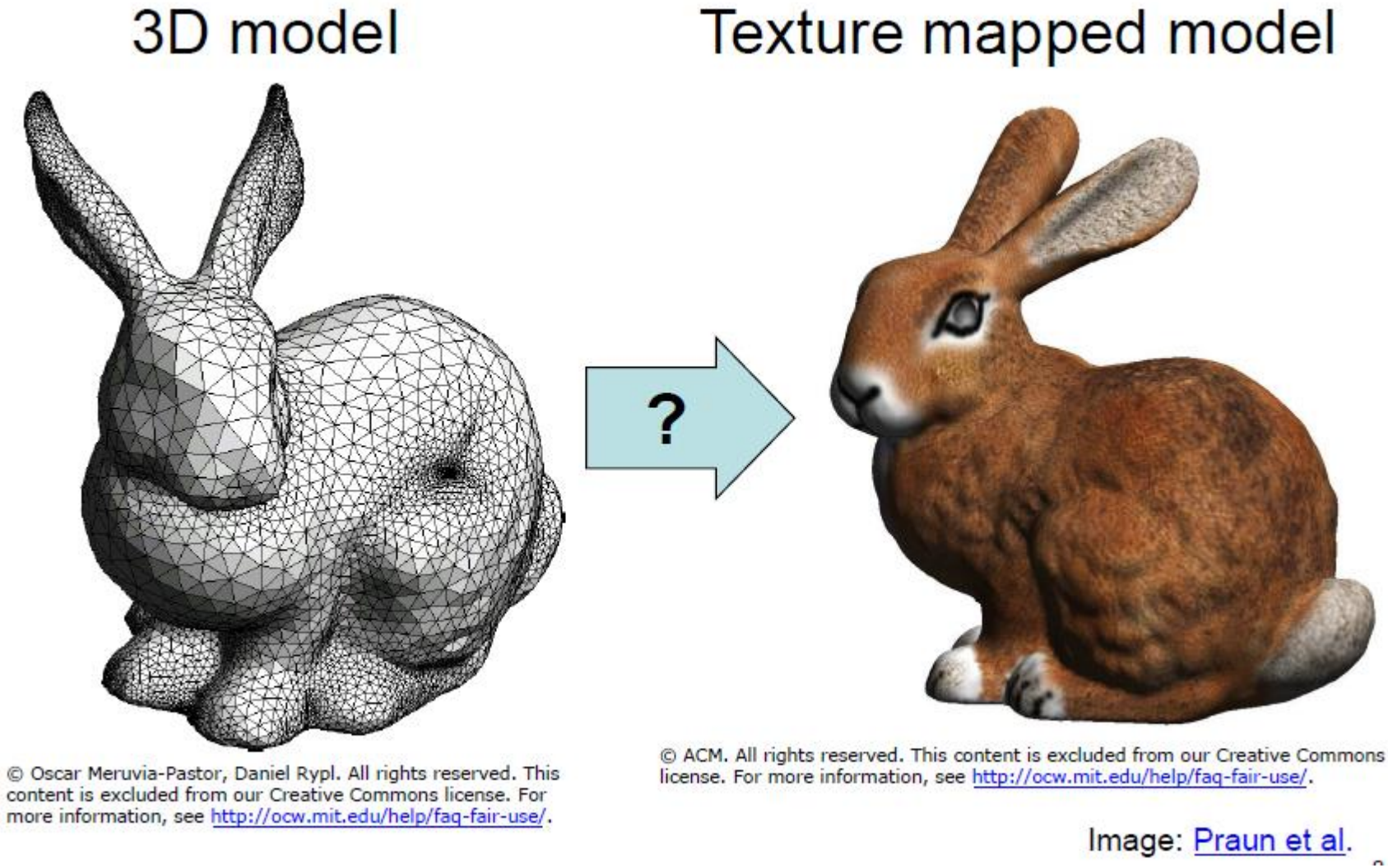
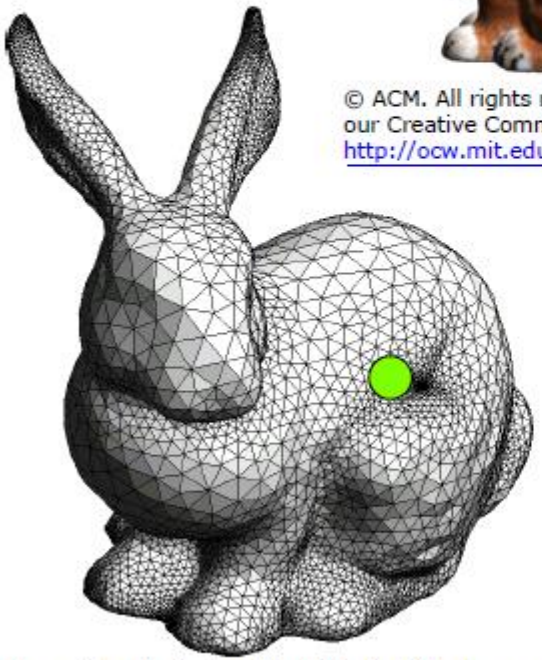


Image: Praun et al.

2 UV Texture Mapping

Texture mapped model

We need a function that associates each surface point with a 2D coordinate in the texture map



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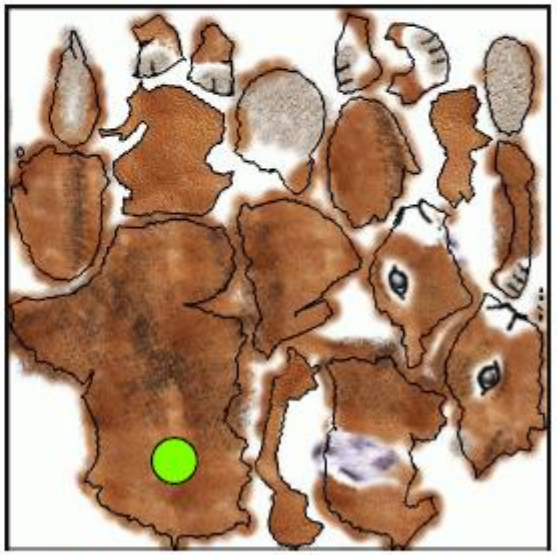
Texture map (2D image)

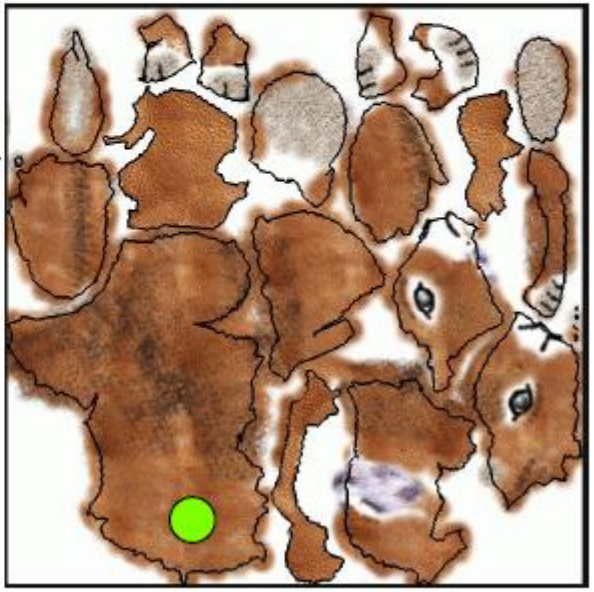


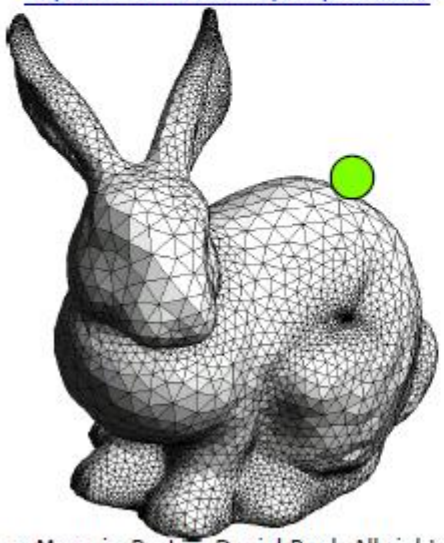
2.1 UV Texture Mapping

Texture mapped model

For each point rendered, look up color in texture map

Texture map (2D image)





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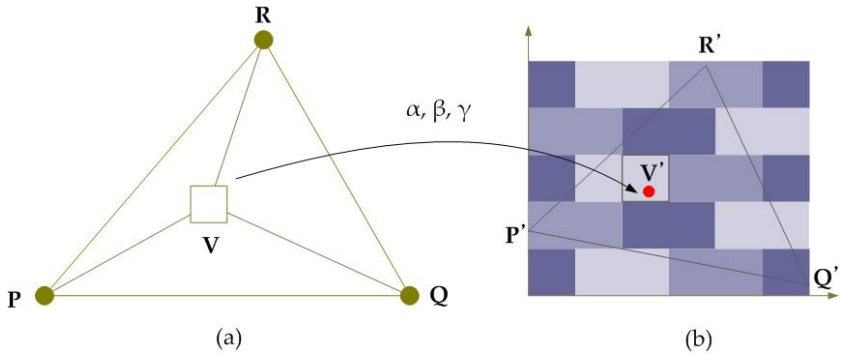
2.2 UV Coordinates

- 모든 Vertex P는 2D (u,v) texture coordinates 를 가짐
- UV는 texture map의 2D 위치임
- Barycentrics를 통해서 interpolation 값을 획득

$$V = \alpha P + \beta Q + \gamma R$$

$$V' = \alpha P' + \beta Q' + \gamma R'$$



2.3 Barycentric Coordinates

- 모든 Vertex P는 2D (u,v) texture coordinates 를 가짐
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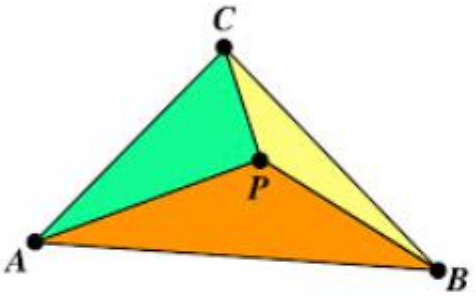
- Barycentrics를 통해서 interpolation 값을 획득



$$P = w_A \times A + w_B \times B + w_C \times C$$

$$w_A = \frac{\Delta PBC}{\Delta ABC} =$$

$$w_B = \frac{\Delta PCA}{\Delta ABC} =$$

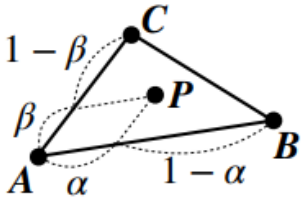
$$w_C = \frac{\Delta PAB}{\Delta ABC} =$$

inside condition

$$0 \le w_A, w_B, w_C \le 1 \qquad w_A + w_B + w_C = 1$$

http://wanochoi.com/lecture/BarycentricCoordinates.pdf

2.3 Barycentric Coordinates



$$AB = B - A$$
$$AC = C - A$$
$$Q = P - A$$

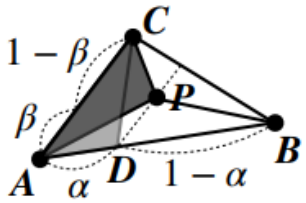
$$\alpha AB + \beta AC + A = P$$
$$\iff \alpha AB + \beta AC = P - A$$
$$\iff \alpha AB + \beta AC = Q$$
$$\iff \begin{matrix} \alpha AB.x + \beta AC.x = Q.x \\ \alpha AB.y + \beta AC.y = Q.y \end{matrix}$$
$$\iff \begin{bmatrix} AB.x & AC.x \\ AB.y & AC.y \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} Q.x \\ Q.y \end{bmatrix}$$
$$\iff \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} AB.x & AC.x \\ AB.y & AC.y \end{bmatrix}^{-1} \begin{bmatrix} Q.x \\ Q.y \end{bmatrix}$$
$$\iff \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{1}{det} \begin{bmatrix} AC.y & -AC.x \\ -AB.y & AB.x \end{bmatrix} \begin{bmatrix} Q.x \\ Q.y \end{bmatrix}$$
$$det = AB.x \times AC.y - AC.x \times AB.y$$
$$\iff \begin{matrix} \alpha = (AC.y \times Q.x - AC.x \times Q.y)/det \\ \beta = (AB.x \times Q.y - AB.y \times Q.x)/det \end{matrix}$$

It is inside in the triangle iff:

$$\beta \geq 0 \quad \& \quad \beta \leq 1 \quad \& \alpha \geq 0 \quad \& \quad \alpha + \beta \leq 1$$

Its barycentric coordinates:

$$w_A = 1 - \alpha - \beta$$
$$w_B = \alpha$$
$$w_C = \beta$$

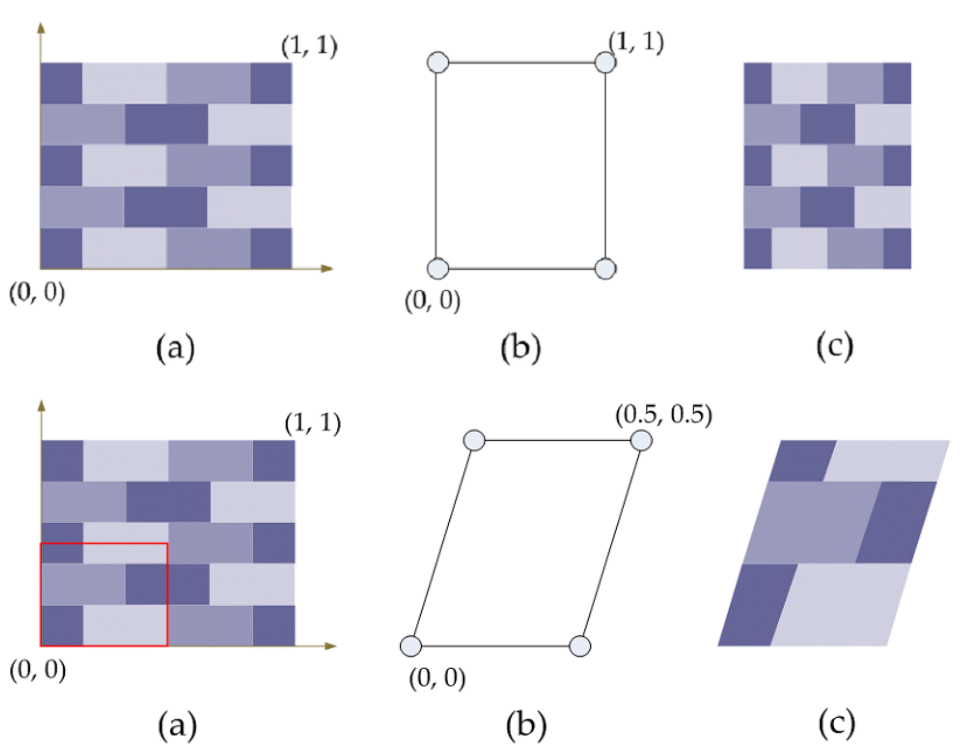


$$w_B = \frac{\Delta APC}{\Delta ABC} = \frac{\Delta ADC}{\Delta ABC} = \alpha$$

http://wanochoi.com/lecture/BarycentricCoordinates.pdf

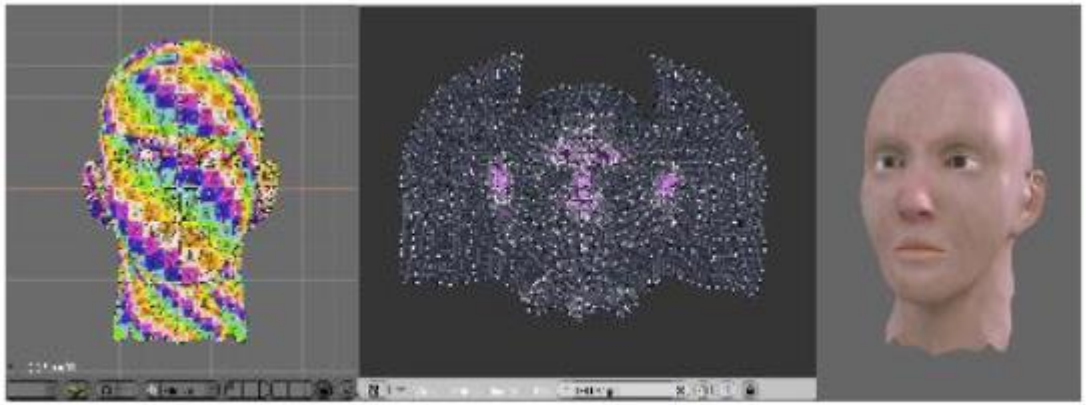
2.3 평면 다각형

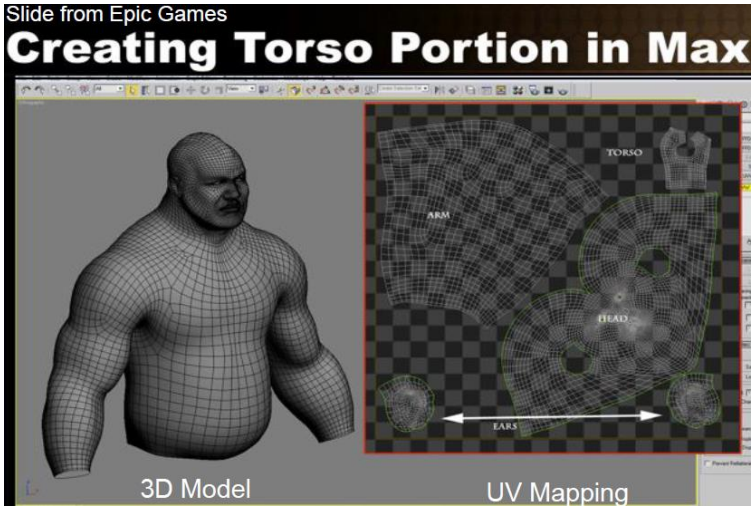
- 수작업 좌표 명시



2.4 곡면 UV Texture Mapping

- Goal : "flatten" 3D object onto 2D UV coordinates
- For each vertex, find coordinates U,V such that distortion is minimized



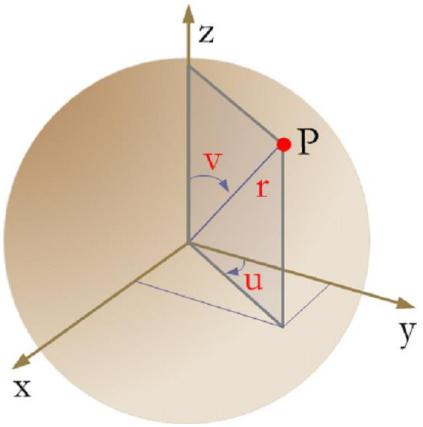


3. Parametric Texturing (파라미터 곡면)

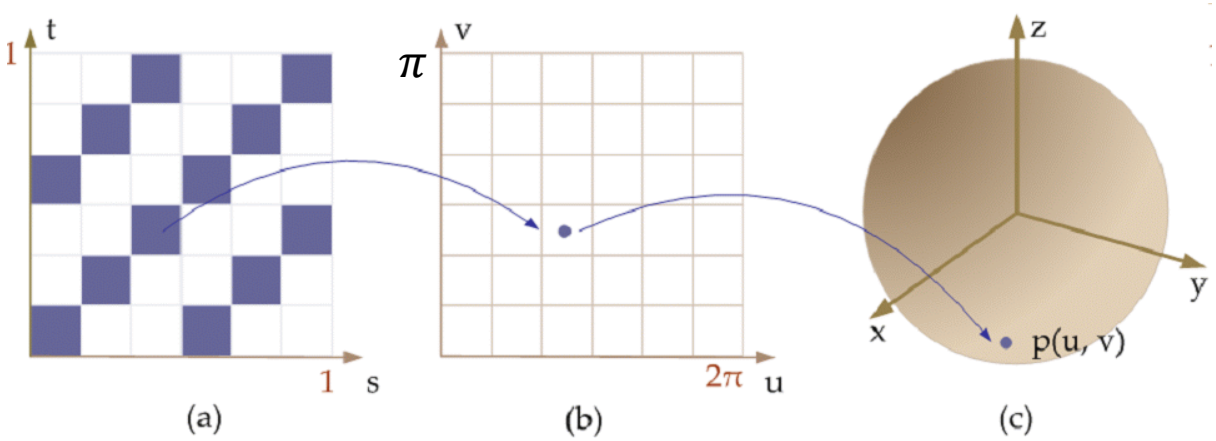
- 구와 같은 non-triangular geometry 에 적용
  vertex가 없기 때문에 UV를 정의 할 수 없음

3.1 원구

- 표면상의 점을 경도, 위도로 표현 가능



$$z = r\cos v$$
$$y = r\sin v\ \cos u$$
$$x = r\sin v\ \sin u$$

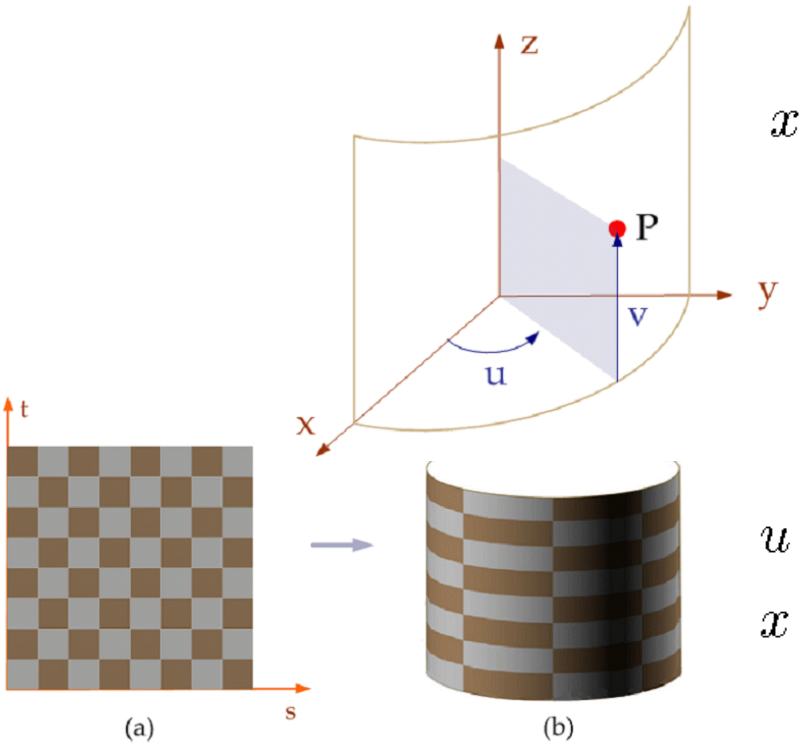


$$u = 2\pi s, \quad v = 2\pi t$$

$$z = r\cos\ \pi t$$
$$y = r\sin\ \pi t\ \cos 2\pi s$$
$$x = r\sin\ \pi t\ \sin 2\pi s$$

3.2 다각형 곡면

- 2 단계 매핑(2-Stage Mapping)
- 곡면을 매개변수로 표시할 수 없을 때
- S 매핑(S Mapping)에서는 텍스쳐를 원기둥, 육면체, 원구 등 중개면(仲介, Intermediate Surface)에 입힘.
- S 매핑의 예: 원기둥 중개면



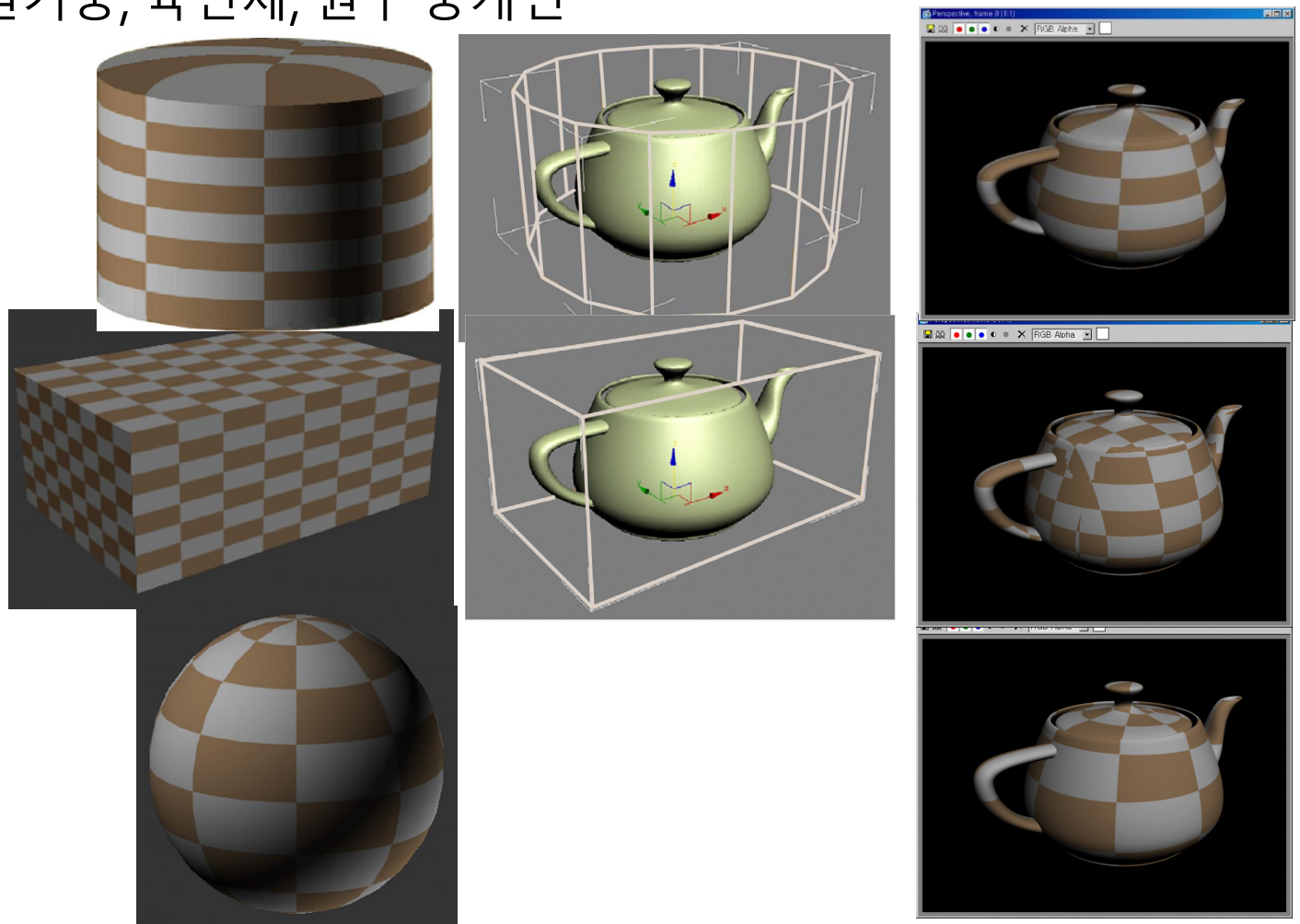
$$x = r\cos u,\ y = r\sin u,\ z = v$$

$$u = 2\pi s,\quad v = t$$

$$x = r\cos 2\pi s,\ y = r\sin 2\pi s,\ z = t$$

3.2 다각형 곡면

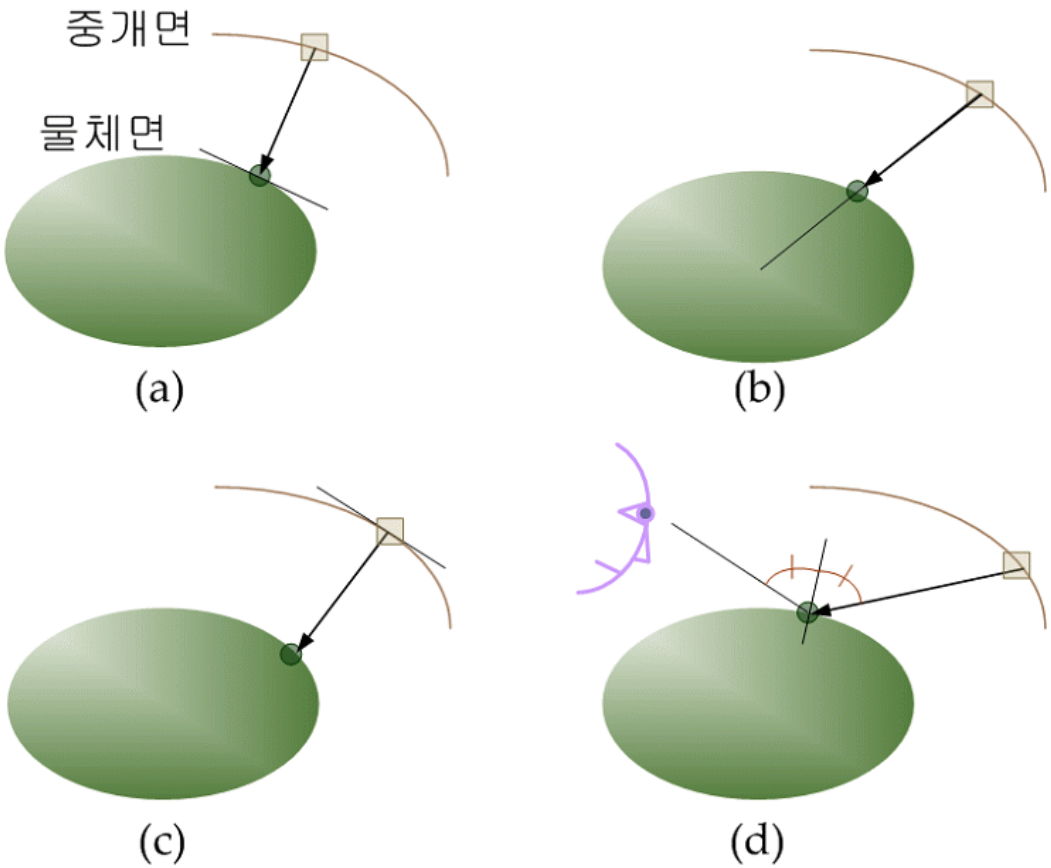
- O 매핑
- 물체를 중개면 내부에 넣고 물체면에 텍스쳐를 입힘.
- 원기둥, 육면체, 원구 중개면



3.2 다각형 곡면

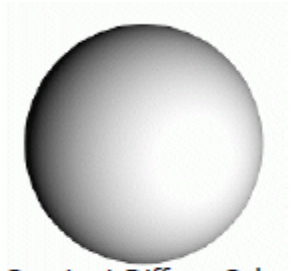
- ㅇ 매핑의 종류

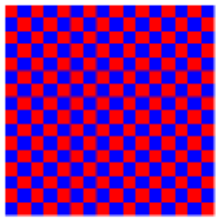
- 물체면 법선벡터, 물체 중심, 중개면 법선벡터, 시점 반사벡터

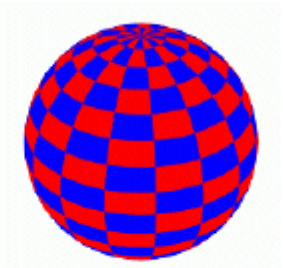


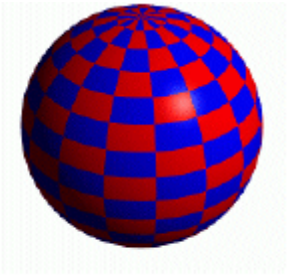
4. Texture mapping & Illumination

- Texture mapping can be used to alter some or all of the constants in the illumination equation


Constant Diffuse Color


Diffuse Texture Color


Texture used as Label


Texture used as Diffuse Color

5. Normal Mapping

- The normal vector is really important in conveying the small-scale surface detail
- For each shaded point, normal is given by a 2D image normalMap that stores the 3D normal

5. Normal Mapping

Final render

Diffuse texture k_d

Normal Map

5. Normal Mapping

- Model a detailed mesh
- •Generate a UV parameterization for the mesh
- –A UV mapping such that each 3D point has unique image coordinates in the 2D texture map
- –This is a difficult problem, but tools are available
- •E.g., the DirectX SDK has functionality to do this
- •Simplify the mesh (again, see DirectX SDK)
- •Overlay simplified and original model
- •For each point P on the simplified mesh, find closest point P′ on original model (ray casting)
- •Store the normal at P′ in the normal map. Done!

5. Normal Mapping

- You can store an object-space normal
- –Convenient if you have a unique parameterization
- •….but if you want to use a tiling
- normal map, this will not work
- –Must account for the curvature of the object!
- –Think of mapping this diffuse+normal map combination on a cylindrical tower
- •Solution: Tangent space normal map
- –Encode a "difference" from the geometric normal in a local coord. system