* Ch 5 kinematics / Jacobian

- Forward · Inverse kinematics
  - Joint 변수 → Hand의 위치 방향
  - Hand의 위치 방향 → Joint 변수

  Dynamics 제외! Robot의 형상을 파악!

  - Forward : ① D-H로 변수파악
    - ② ${}^{n}_{0}T$ 구하기
    - ③ 필요하면 $\vec{\omega}$를 RPY로!
  - Inverse : Solution 이용

- Jacobian
  - ① Joint의 속도 ⟷ Hand 속도
  - ② Joint의 토크 ⟷ Hand의 힘

①
$$\begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \\ \dot\theta_4 \\ \dot\theta_5 \\ \dot\theta_6 \end{bmatrix} \leftrightarrow \begin{bmatrix} \dot x \\ \dot y \\ \dot z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \text{ or } \begin{bmatrix} \dot x \\ \dot y \\ \dot z \\ \dot\theta \\ \dot\phi \\ \dot\psi \end{bmatrix} \longrightarrow \begin{bmatrix} x \\ y \\ z \\ \theta \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} h_1(\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6) \\ h_2(\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6) \\ h_3(\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6) \\ h_4(\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6) \\ h_5(\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6) \\ h_6(\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6) \end{bmatrix}$$

$$Y = h(\vec\theta)$$

$$\dot Y = \frac{h(\vec\theta)}{d\vec\theta}\cdot\frac{d\vec\theta}{dt} \Rightarrow \dot Y = J\dot\theta$$

$$\begin{bmatrix} \dot x \\ \dot y \\ \dot z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \underbrace{J}_{\text{Jacobian}} \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \\ \dot\theta_4 \\ \dot\theta_5 \\ \dot\theta_6 \end{bmatrix}$$

Virtual Work (Work … $\Sigma\vec F\cdot d\vec r$, $\Sigma\vec M\cdot d\vec\theta$)

② $F\cdot\Delta\vec x = \tau\cdot\Delta\vec\theta$ 내적

$\Rightarrow F^T\cdot\Delta\vec x = \tau^T\cdot\Delta\vec\theta$ ⟶ $\Delta\vec x = J\cdot\Delta\vec\theta$

$\Rightarrow F^T\cdot J\cdot\Delta\vec\theta = \tau^T\cdot\Delta\vec\theta$

$\Rightarrow F^T\cdot J = \tau^T$

$\Rightarrow F\cdot J^T = \tau \qquad \therefore\ \tau = J^T F$

⊕ $\vec\omega \rightarrow \dot\theta, \dot\phi, \dot\psi$

$J_\omega = C\cdot J_\theta$, XYZ Euler Angle

$$\begin{bmatrix} \omega_x \\ \omega_Y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot\theta \\ 0 \\ 0 \end{bmatrix} + R_x(\theta)\begin{bmatrix} 0 \\ \dot\phi \\ 0 \end{bmatrix} + R_x(\theta)\cdot R_Y(\phi)\begin{bmatrix} 0 \\ 0 \\ \dot\psi \end{bmatrix} \longrightarrow \begin{bmatrix} \omega_x \\ \omega_Y \\ \omega_z \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & -\sin\phi \\ 0 & \cos\theta & \sin\theta\cos\phi \\ 0 & -\sin\theta & \cos\theta\cos\phi \end{bmatrix}}_{C}\begin{bmatrix} \dot\theta \\ \dot\phi \\ \dot\psi \end{bmatrix}$$

$\therefore J_\theta = C^{-1} J_\omega$

- How to get J

① Forward kinematic로 구한 x, y, z 미분 → J_x Get … J_ω는 못구한다!

② Propagation 이용 → J_x, J_ω Get

③ Back Propagation 이용 → J_x^T, J_ω^T Get

⊕ J의 행수 : Link 수
〃 열수 : Joint Var 수

⟶ V만으로 J가 구해지며, ω의 추가는 J의 확장이다.