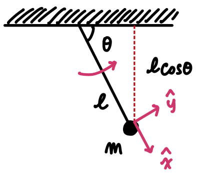
* 1-DOF Pendulum



• MOE with Lagrangian

$K_1 = \frac{1}{2} m V_1^T V_1 + \frac{1}{2} I' \omega_1^T{}' \omega_1$

$I = 0$

$\therefore K_1 = \frac{1}{2} m V_1^T V_1 \quad , V_1 = l \cdot \dot{\theta}\, \hat{y}$

$\Rightarrow K_1 = \frac{1}{2} m l^2 \dot{\theta}^2$

$U_1 = -mg \cdot l\cos\theta + U_{ref}$

$L = K - U \longrightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \tau$

$\Rightarrow L = \frac{1}{2} m l^2 \dot{\theta}^2 - mg \cdot l\cos\theta + U_{ref}$

$\Rightarrow \underbrace{ml^2\ddot{\theta} + mgl\sin\theta = \tau}_{\text{moe}}$

• Control

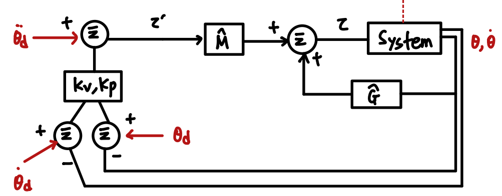
$M(\theta) = ml^2$

$V(\theta, \dot{\theta}) = 0$

$G(\theta) = mgl\sin\theta$

$\rightarrow \tau = M(\theta)\ddot{\theta} + G(\theta)$ 제어 입력

$\tau = M(\theta)\ddot{\theta} + G(\theta)$



① model Part → Linearization

$\tau = a\tau' + b$

$a = \hat{M}(\theta) \quad , b = \hat{G}(\theta)$

② Servo

$\tau' = \ddot{\theta}_d + k_v(\dot{\theta}_d - \dot{\theta}) + k_p(\theta_d - \theta)$

$\rightarrow \tau' = \ddot{\theta}$

$\therefore \ddot{E} + k_v\dot{E} + k_pE = 0$ 을 제어

→ $E = 0$이 되는 Behavior와 성능제어

Non-tracking...

→ $\theta_d \; \dot{\theta}_d \; \ddot{\theta}_d = 0$ 일 때?

$\Rightarrow \tau' = -k_v\dot{\theta} - k_p\theta$

$\Rightarrow \ddot{\theta} + k_v\dot{\theta} + k_p\theta = 0$

→ $\theta = 0$이 되는 Behavior와 성능제어

• Observe Natural Response

$\ddot{\theta} = \frac{1}{ml^2}\tau - \frac{g}{l}\sin\theta$ → $\ddot{\theta}(t)$ 를 구하고, 수치적분을 통해 $\theta(t), \dot{\theta}(t)$ 구한다!

→ Control 할 시 τ에 비선형항을 설계해서 입력

$\dot{\theta}_1 = \theta_2$

$\dot{\theta}_2 = \frac{1}{ml^2}\tau - \frac{g}{l}\sin\theta_1 \rightarrow \tau = ml^2 v + \frac{g}{l}\sin\theta_1$

$\Rightarrow \dot{\theta}_2 = v \quad \cdots \quad v = -k_v\dot{\theta} - k_p\theta$

$= -k_v\theta_2 - k_p\theta_1$

$\therefore \dot{\theta}_1 = \theta_2$

$\dot{\theta}_2 = -k_p\theta_1 - k_v\theta_2$

$\rightarrow A = \begin{bmatrix} 0 & 1 \\ -k_p & -k_v \end{bmatrix} \rightarrow \begin{vmatrix} \lambda & -1 \\ k_p & \lambda + k_v \end{vmatrix}$

$\Rightarrow \lambda(\lambda + k_v) + k_p = \lambda^2 + k_v\lambda + k_p = 0$ 의 해가 음의 실수가 되게 k_p, k_v 선택