STA 200B **Homework 5** Winter 2017

Textbook: 7.41, 7.44, 7.60

Additional problems:

1. For the case study in Handout 3. Find the MLE for the two parameters α and β if you adopt the Gamma distribution assumption. You need to derive the MLE on your own through a numerical method and submit your code. Carry out your analysis in the following steps:

   (i) Start with the method of moments estimate as the initial value of your algorithm and report both the values of your estimate after just one-step of iteration and after a complete iteration using the convergence criterion that the relative difference between two consecutive estimates is less than 0.001. Compare the results and comment on it.

   (ii) Start with a different initial value, e.g. the MoM estimator jittered by a random number from N(0, 1) (i.e. add a random number from N(0, 1) to the moment estimate), and compare the results of the one-step estimator with the one after a full iteration using the same criterion as in (i). report how many iterations it takes for your algorithm to converge and If your algorithm fails to converge, attach the error message (it may or may not be a coding error).

   Very Important - Do not use any readymade package to obtain the MLE.

   [Hint] For a simpler solution, use the profile likelihood method as described in class and Exercise 7.2.

2. Suppose that $X_1, \ldots, X_n$ is a random sample from a Binomial distribution $Bin(5, \theta)$ and the prior distribution of θ is a certain specified beta distribution $Beta(\alpha, \beta)$.

   (a) Derive the form of the Bayes estimator of θ w.r.t. the square error loss function.

   (b) Find the bias and variance of the Bayes estimator in (a) and compare it with the MLE. Which one is better?

   (c) Is the Bayes estimator in (a) minimal sufficient? Justify your answer.

   (d) Derive the Bayes estimate for a Beta prior with $\alpha = 0.5$ and $\beta = 3$ if you observe $X_1 = 4, X_2 = 2, X_3 = 1, X_4 = 4$.

   (e) As in (a) but change the loss function to the absolute error loss function. What can you say about the minimum sufficiency of this Bayes estimator and find the Bayes estimate.

3. HW 3.4.

4. HW 3.5.

5. Let $X_1, \ldots, X_n$ is a random sample from a Gamma distribution $Gamma(\alpha, \beta)$, where the value of β is known. Is the sample mean an admissible estimator of the mean of this distribution when the square error loss function is used? Justify your answer.

6. Suppose that $X_1, \ldots, X_n$ is a random sample from an exponential distribution with mean θ.

(a) Find the Cramer-Rao bound for unbiased estimators of θ.

(b) Construct an efficient estimator of θ using a result in Theorem 3.7.2 (C-R Inequality).

(c) Determine the variance of this estimator in two ways, one of them through the Fisher information. You can choose the second way but it must be different from the approach of Fisher information or C-R bound.

(d) Derive a complete sufficient statistic and use the Lehmann-Scheffe Theorem to show that your estimate in (b) is the UMVUE.