STA 200B **Homework 6** Winter 2017

Textbook: 9.2, 9.17, 9.30 (a) & (c) (you may use the result in (b) as you have already shown it in HW 1.3), 9.58 (a)

Bonus Problem: 9.28 - You need not do this problem but if you submit a complete and correct solution you will get a bonus of 15 points. There will be no partial credit for this problem.

Additional problems:

1. The central limit theorem can be used to find an approximate c.i.. Let $X_1, \ldots, X_n$ be a random sample from a Poisson distribution with mean λ.

   (a) Show how to derive an approximate c.i. at level 99% for λ.

   (b) Find the sample size needed in order for the 99% c.i. to have length no more than 0.2 using the information that you know $\lambda \leq 10$.

2. Let $X_1, \ldots, X_n$ be a random sample from $N(\mu_1, \sigma_1^2)$ and $Y_1, \ldots, Y_m$ be a random sample from $N(\mu_2, \sigma_2^2)$.

   (a) Assume that σ_1 and σ_2 are known, derive the best 90% c.i. for $\mu_1 - \mu_2$.

   (b) As in (a) but this time both σ_1 and σ_2 are unknown. You may assume that $\sigma_1 = \sigma_2$.

   Note: If $\sigma_1 \neq \sigma_2$ the problem is much harder and is called the Behrens-Fisher problem.

   (c) As in (b) but your goal here is to test, at the 10% level of significance, $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$. Describe how you could perform such a test using a c.i.

   (d) As in (c) except that the alternative hypothesis is $H_1 : \mu_1 > \mu_2$.

3. Let $X_1, \ldots, X_n$ be a random sample from a uniform distribution $U(0, \theta)$ and $X_{(n)}$ denote the largest order statistic.

   (a) Show that $X_{(n)}/\theta$ is a pivot.

   (b) Find a level γ confidence interval for θ.

   (c) Derive the 95% confidence interval θ. You should be able to find the c.i. explicitly by first finding the quantiles of $X_{(n)}/\theta$.