STA 200B **Homework 7** Winter 2017

Textbook: 9.14, HW 5.1

Additional problems:

1. Let $X_1, \ldots, X_n$ be a random sample from $N(\mu_1, \sigma_1^2)$ and $Y_1, \ldots, Y_m$ be a random sample from $N(\mu_2, \sigma_2^2)$.

   (a) Assume that μ_1 and μ_2 are known, derive a 95% c.i. for the ratio σ_1^2/σ_2^2.

   (b) As in (a) except that both μ_1 and μ_2 are unknown.

   (c) As in (b) but find the a 95% c.i. for the ratio σ_1/σ_2.

2. In the June 1986 issue of Consumer Reports, some data on the calorie content of beef hotdogs is given. We provide here the numbers of calories in 20 hot dog brands.

   152, 111, 186, 181, 176, 141, 149, 153, 184, 190, 190, 157, 158, 131, 139, 149, 175, 135, 148, 132.

   Assume that these numbers are the observed values from a random sample of $N(\mu, \sigma^2)$, both parameters are unknown.

   (a) Find the shortest 90% C.I. for μ.

   (b) Use the c.i. in (a) to test the hypothesis that $H_0 : \mu = 150$ vs. $H_1 : \mu \neq 150$ at the 10% level of significance.

   (c) Find the p-value of your test in (b).

   (d) You suspect that the average calorie level for hot dogs exceeds 150. Test this hypothesis at the 5% level using a proper confidence set. State your null and alternative hypotheses clearly and your conclusion.

   (e) Find the p-value of your test in (d).

   (f) What type of error might you have committed in (d)? Explain briefly.

3. Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$.

   (a) Find the smallest n you need in order for the 90% c.i. for μ to have length less than $\frac{\sigma}{3}$ assuming that σ is known.

   (b) As in (a) but σ is unknown. Show that n now depends on the sample variance S^2 so you cannot guarantee an explicit n anymore.

   (c) As in (b) but show instead what you could find is the smallest n that can guarantee with a certain probability, say 95%, that the 90% c.i. for μ will have length less than $\frac{\sigma}{3}$. You need not find the actual value of n because it requires a numerical solution, so just provide a formula.

4. Let $X_1, \ldots, X_n$ be a random sample from a uniform distribution $U(0, \theta)$ and $X_{(n)}$ denote the largest order statistic.

   (a) You showed in HW 6 that $X_{(n)}/\theta$ is a pivot. Use this to find the shortest level γ c.i. for θ.

(b) Let θ have a prior distribution that is Pareto (α, β) with p.d.f. $f(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \alpha < x < \infty, \alpha > 0, \beta > 0$.

Find the shortest level γ Bayes credible interval for θ.

5. Let $X_1, \ldots, X_n$ be a random sample from $Exp(\lambda)$ with p.d.f. $f(x) = \lambda e^{-\lambda x}, x > 0$.

   (a) Is the MLE of the median a consistent estimator? Justify your answer.

   (b) Let λ be a r.v. with prior distribution $Gamma(\alpha, \beta)$ and p.d.f. $\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$.

   Is the Bayes estimator w.r.t. the square error loss function a consistent estimator of λ? Justify your answer.