

The *pointwise expansions* for C and D don't really yield any interesting results. When we do it for B , though, things start to happen.

$$P(B, C, D|A) = P(B)P(C|A, B)P(D|B)$$

Then, we have a nuisance variable in the first part of the probability, and so we can *sum it out*:

$$P(C, D|A) = P(C, D, b|A) + P(C, D, \neg b|A) \quad (1)$$

which is helpful because $P(C, D|A)$ is related to $P(C, D, A)$, which is in turn related to $P(C|D, A)$, all through formulae we know.

At this point we have three variables in our probability, corresponding to a three-dimensional array, but we're really only interested for the case where $A = \neg a$ and $D = d$, so we're looking for $P(C|\neg a \wedge d)$, and knowing that

$$P(c|\neg a \wedge d) + P(\neg c|\neg a \wedge d) = 1$$

and

$$P(C|\neg a \wedge d) = \frac{P(C, \neg a, d)}{P(\neg a, d)}$$

where $P(\neg a, d)$ is some constant value, we don't have to actually determine that value analytically, but can just normalize the vector on top ($P(C, \neg a, d)$) to get our result. From (1), and the same normalization process applied with $P(C, d|\neg a)$ and $P(C, d, \neg a)$, we can then find a solution to $P(C|\neg a \wedge d)$:

$$P(C, d|\neg a) = \frac{P(C, \neg a, d)}{P(\neg a)} \Rightarrow P(C, \neg a, d) = P(C, d|\neg a)P(\neg a)$$

$$P(C|\neg a \wedge d) = \beta \frac{P(C, \neg a, d)}{P(\neg a, d)} = \beta \frac{P(\neg a)}{P(\neg a, d)} P(C, d|\neg a) = \alpha P(C, d|\neg a)$$

where β normalizes the first expression, and α normalizes $P(C, d|\neg a)$. A little plug and chug, and then smooth sailing. I won't give away the ending, but *don't be surprised by a very clean answer*.