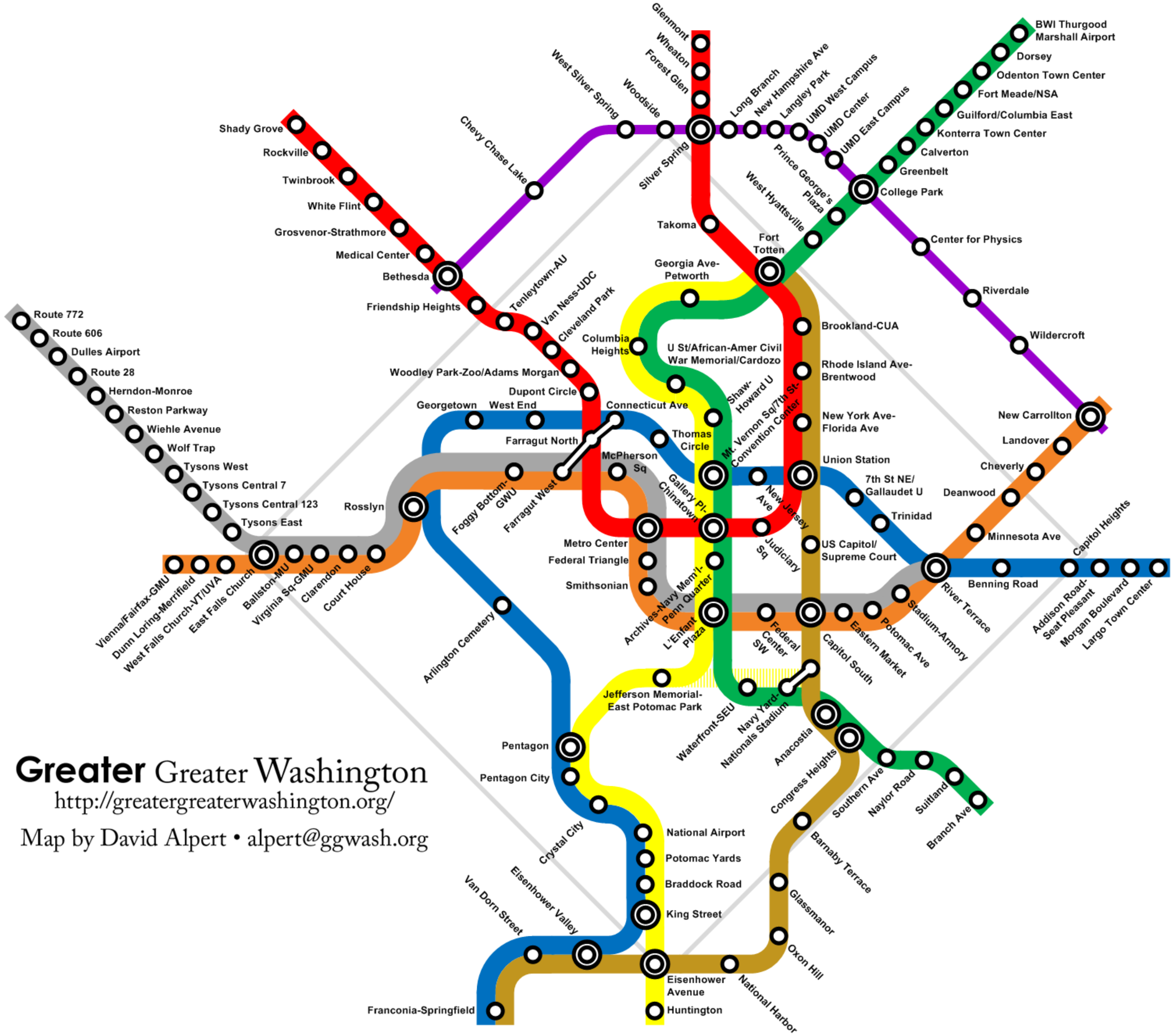
# Graphs – An Introduction

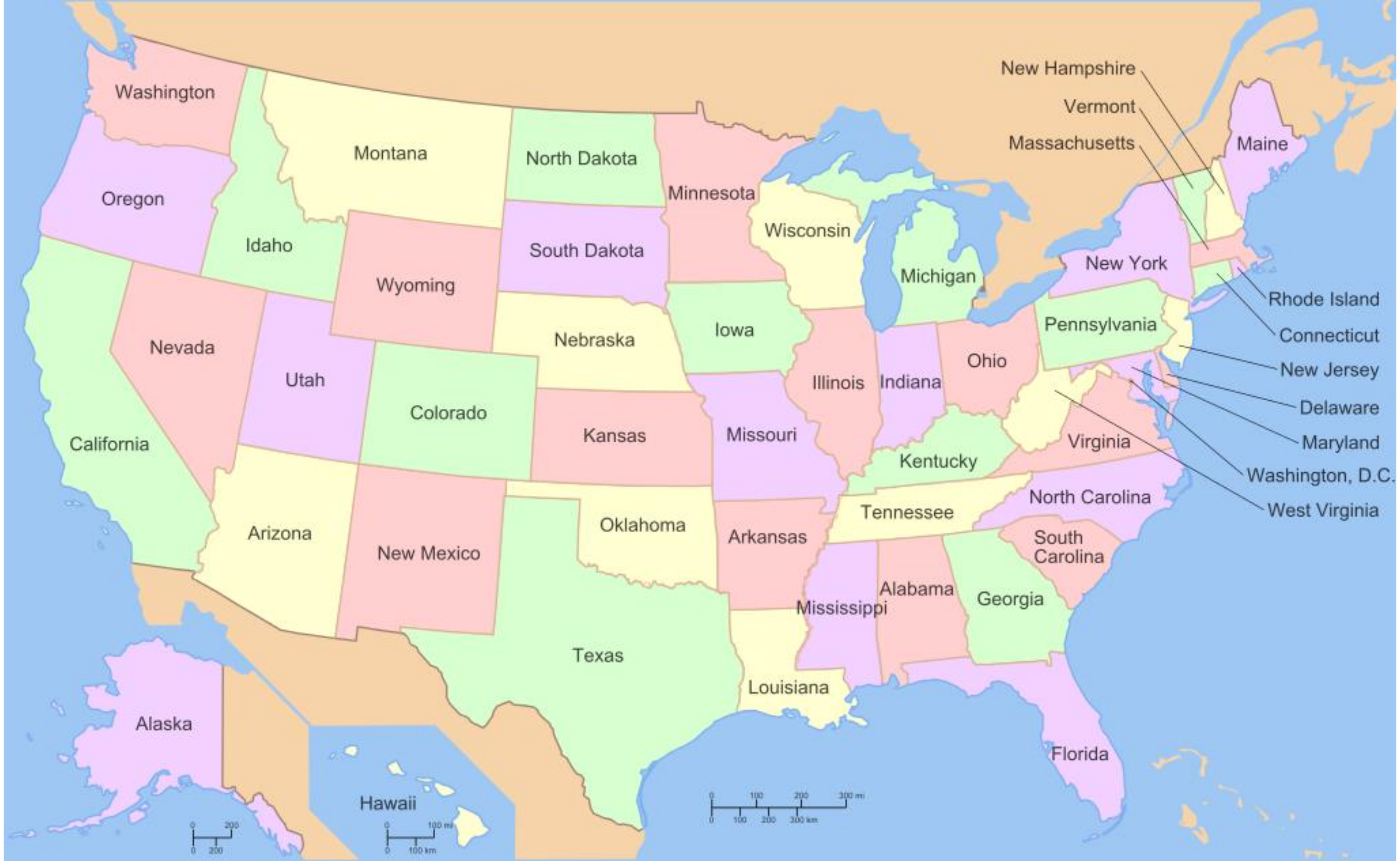
# GRAPHS


http://www.facebook.com/notes/facebook-engineering/visualizing-friendships/469716398919

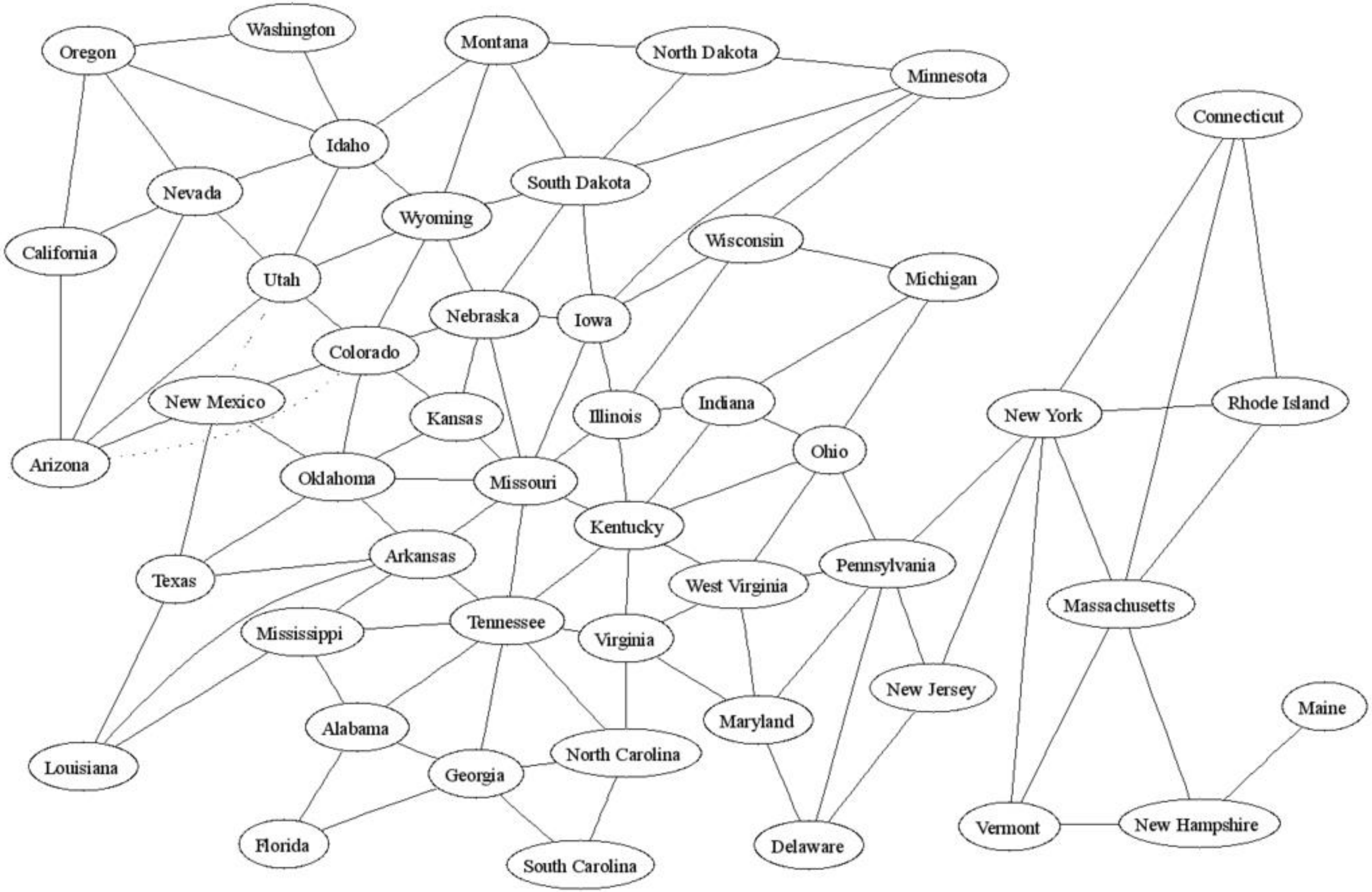
# WASHINGTON, D.C. METRO SYSTEM MAP/GRAPH



# AIRLINE ROUTE MAP/GRAPH (U.S.)




Source: http://en.wikipedia.org/wiki/File:Map_of_USA_with_state_names_2.svg

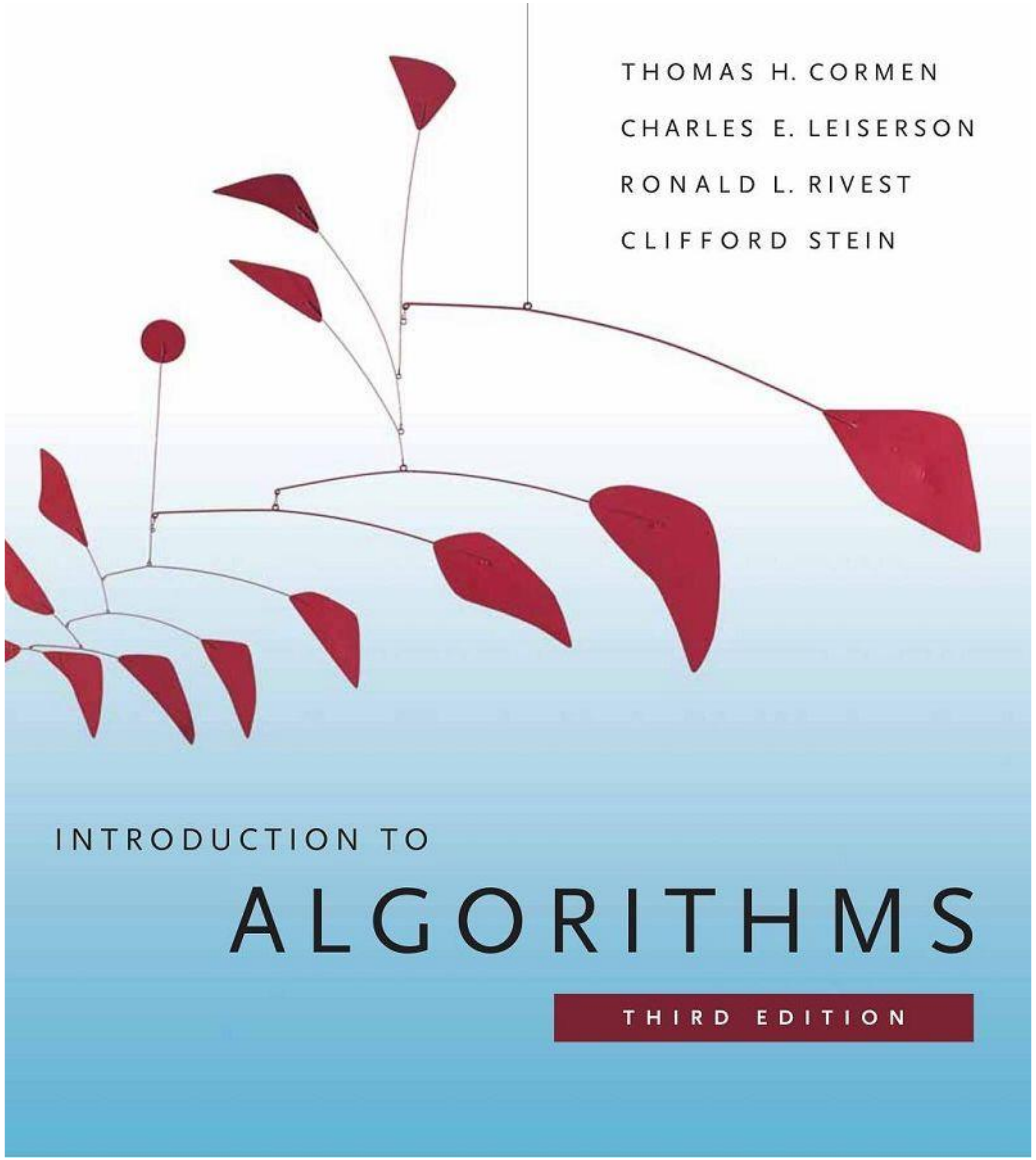



Source: http://en.wikipedia.org/wiki/File:UnitedStatesGraphViz.png

# GRAPH APPLICATIONS OVERVIEW

SEDGWICK 4.1

| Graph | Vertex | Edge |
| --- | --- | --- |
| communication | telephone, computer | fiber optic cable |
| circuit | gate, register, processor | wire |
| mechanical | joint | rod, beam, spring |
| financial | stock, currency | transactions |
| transportation | street intersection, airport | highway, airway route |
| internet | class C network | connection |
| game | board position | legal move |
| social | relationship person, actor | friendship, movie cast |
| neural network | neuron | synapse |
| protein | network protein | protein-protein interaction |
| chemical compound | molecule | bond |


THOMAS H. CORMEN
CHARLES E. LEISERSON
RONALD L. RIVEST
CLIFFORD STEIN
INTRODUCTION TO
ALGORITHMS
THIRD EDITION

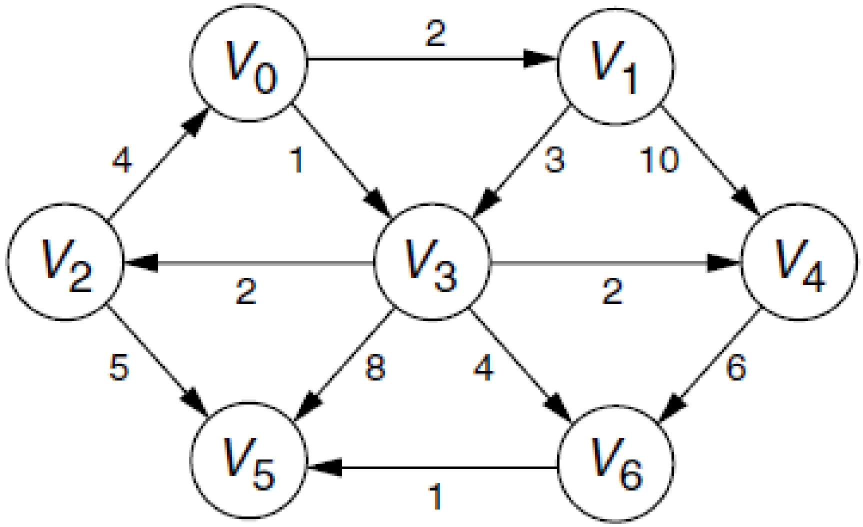
# GRAPHS

- Graph consists of a set of vertices and a set of edges that connect those vertices.
  - Denoted G(V,E)
  - Edges a.k.a. arcs, Vertices a.k.a. nodes
  - Cardinality is the number of elements in a set
    - $|V|$ = number of vertices
    - $|E|$ = number of edges

# TERMS: TYPES OF GRAPHS

- Directed vs. Undirected
  - having or not having directed edges
- Weighted vs. Unweighted
  - having or not having edge weights
- Cyclic vs. Acyclic
  - having or not having a cycle
- DAGs
  - directed acyclic graphs

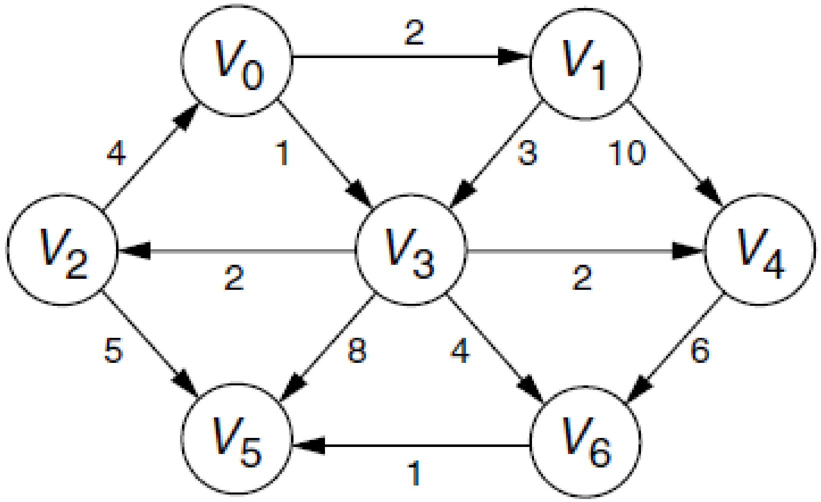
- The example on the right is a Directed Graph, or Digraph, with weighted edges and cycles



# GRAPHS
(WEISS, FIG. 14.1)

- The graph has seven vertices: $V = \{V_0, V_1, V_2, V_3, V_4, V_5, V_6\}$
- The graph has 12 edges:

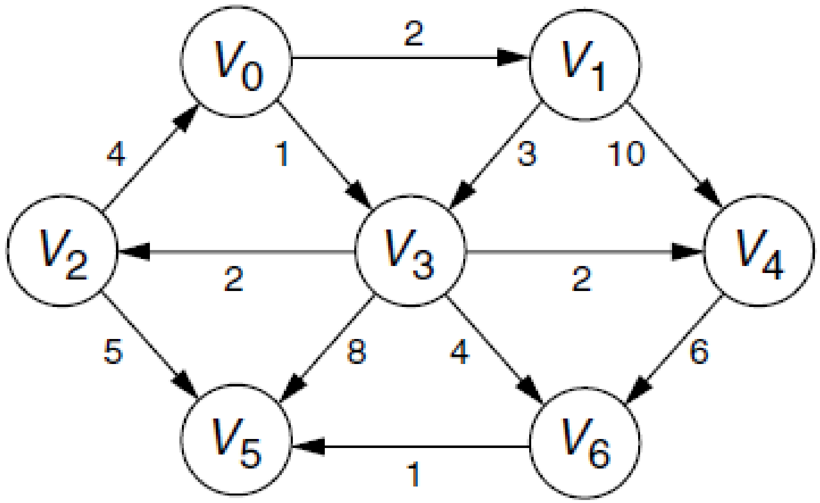
$$E = \begin{Bmatrix} (V_0, V_1, 2), (V_0, V_3, 1), (V_1, V_3, 3), (V_1, V_4, 10) \\ (V_3, V_4, 2), (V_3, V_6, 4), (V_3, V_5, 8), (V_3, V_2, 2) \\ (V_2, V_0, 4), (V_2, V_5, 5), (V_4, V_6, 6), (V_6, V_5, 1) \end{Bmatrix}$$



# GRAPHS

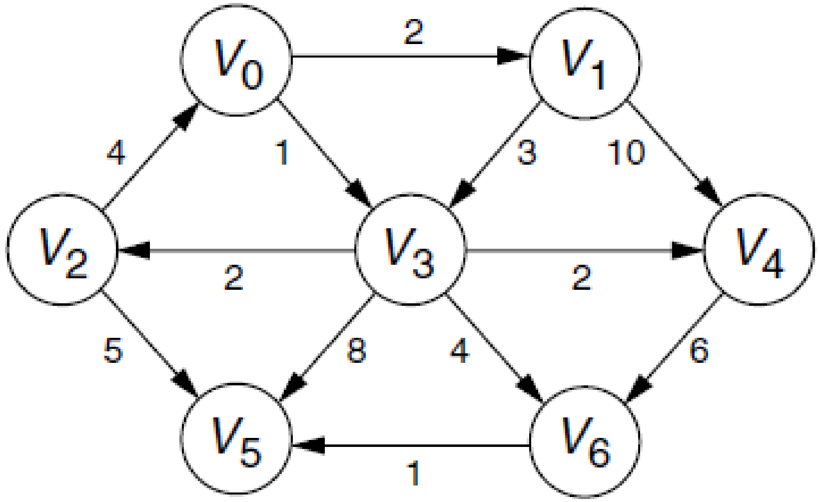
(WEISS, FIG. 14.1)

- A path in a graph is a sequence of vertices connected by edges
- Example: ($V_0$, $V_3$, $V_5$) is a path from node $V_0$ to $V_5$



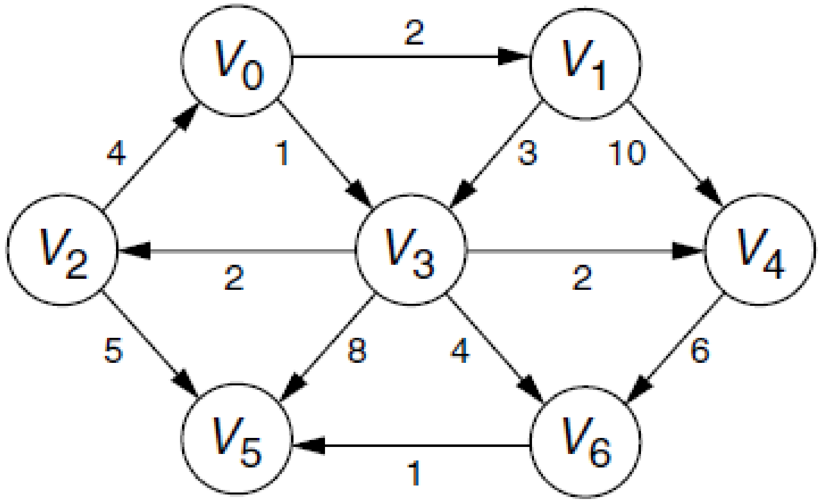
# GRAPHS
(WEISS, FIG. 14.1)

- |V| = 7, |E| = 12
- The degree of a vertex is the number of edges incident to it
- In a digraph, vertices v and w are adjacent iff (v, w) ∈ E
- The vertices v, w are an ordered pair, its edge has direction
- Example: ($V_2$, $V_0$) ∈ E, but ($V_5$, $V_1$) is not



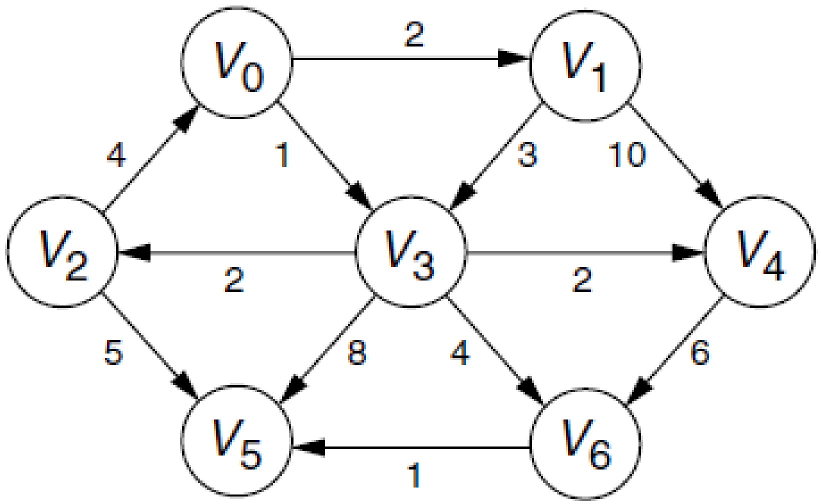
# GRAPHS
(WEISS, FIG. 14.1)

- If the edges have no weights, the unweighted path length is the number of edges along the path
- If they have weights, the weighted path length is the sum of the edge weights on the path
- Example: the unweighted length of ($V_0$, $V_3$, $V_5$) is 2, but the weighted length of ($V_0$, $V_3$, $V_5$) is 1 + 8 = 9



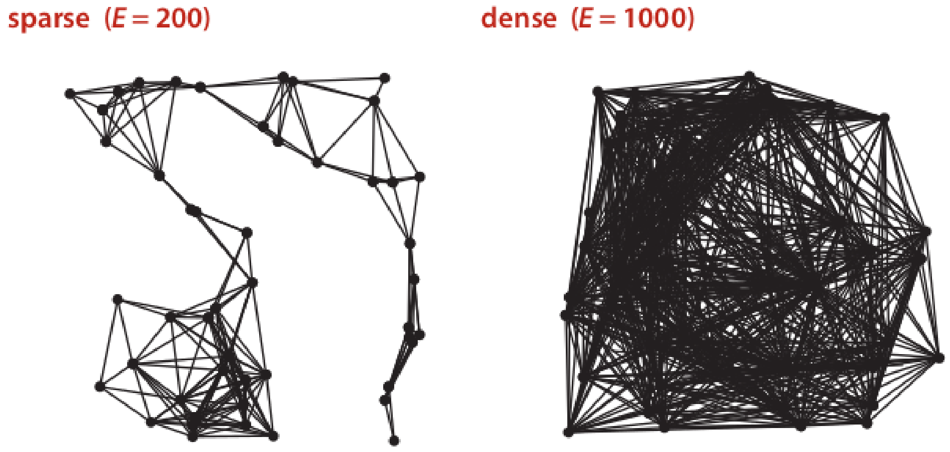
# GRAPHS
(WEISS, FIG. 14.1)

- A simple path is one in which all vertices are distinct, except the first and last can be the same
- A cycle in a directed graph is a path that begins and ends at the same vertex and has at least one edge
- A cycle can be simple or not
- Example: a cycle: $(V_2, V_0, V_3, V_2)$



# TERMS: TYPES OF GRAPHS

- When most of the possible edges are present, $|E| = \Theta(|V|^2)$, and the graph is considered dense
- Most applications have relatively few edges
- Examples: transportation systems, computer networks
- When there are few edges, $|E| = \Theta(|V|)$, and the graph is sparse



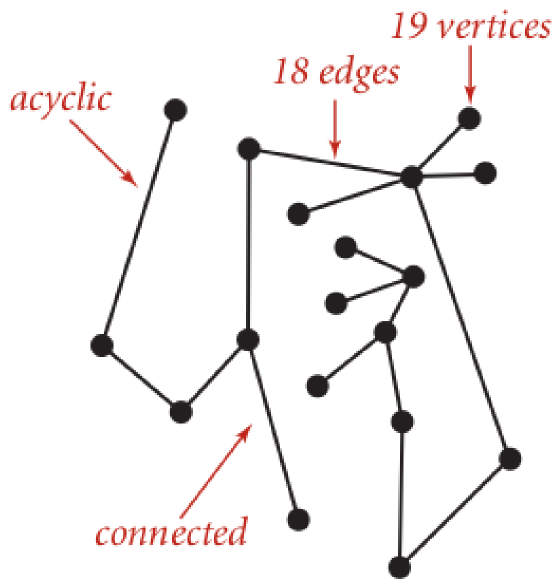

Two graphs ($V = 50$)

# TREES

- A Graph is connected if there is a path from every vertex to every other vertex.
- A Graph that is not connected consists of a set of connected components
- A tree is an acyclic, connected graph
- A graph is a tree iff has the following properties
  - G has $|V| - 1$ edges and no cycles
  - G has $|V| - 1$ edges and is connected
  - G is connected and removing any edge disconnects
  - G is acyclic and adding any edge creates a cycle
  - Exactly one simple path connects each pair vertices

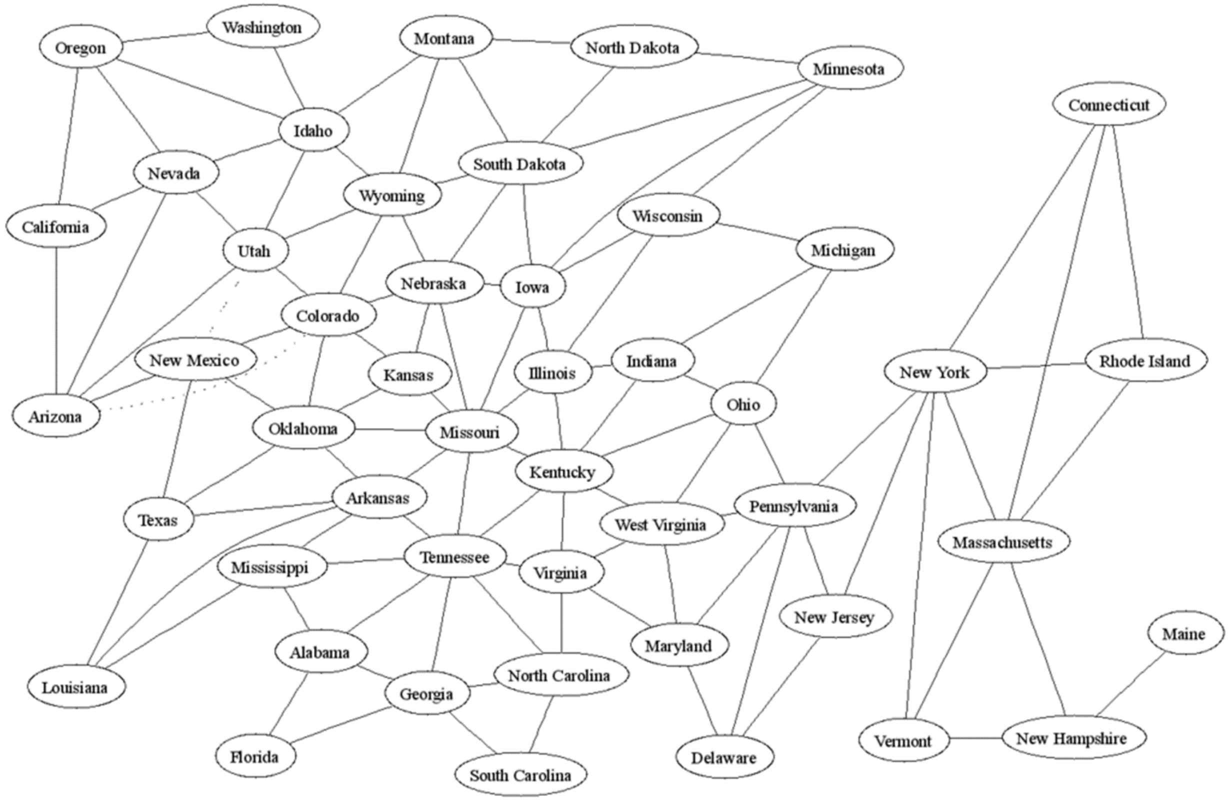
# TREE EXAMPLE

Is this definition consistent with our previous definition of trees?



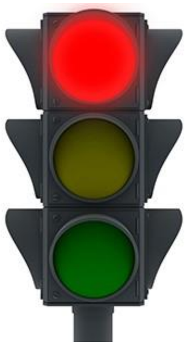

A tree

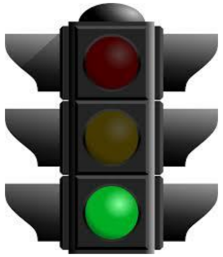
# WHAT KINDS OF THINGS WOULD WE LIKE TO KNOW FROM A GRAPH?




- Are Nebraska and Oklahoma adjacent to each other?
- How can you get from California to Virginia?
- What is the fewest number of states you can travel through to get from Oregon to Maine?
- What is the population of Texas?

# End of Graphs – An Introduction





Please proceed to the next course activity now or at a later time