David Lee
Lee2173
Prof Sacks
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Homework #3

1. No, ¬BestFriend(Mary, Sophia) does not follow from the facts Mary != Katie and BestFriend(Katie, Sophia). This is because we don't know that each person only has one best friend. Some axioms to make sure it follows needs to make sure that ∃!BestFriend(Katie,Sophia) so that if BestFriend(Katie, Sophia), then ¬BestFriend(Mary, Sophia) when Mary != Katie.

2.
   a. $\exists x Parent(Roger,x) \wedge Male(x)$
   b. $\exists ! x Parent(Roger,x) \wedge Male(x)$
   c. $\exists ! x Parent(Roger,x) \Rightarrow Male(x)$
   d. $\exists ! x Parent(Roger,x) \wedge Parent(Amy,x)$
   e. $\exists x Parent(Roger,x) \Rightarrow Parent(Amy,x)$
3.
   a. {x/A, y/B. z/B}
   b. Does not exist.
   c. {x/John, y/John}
   d. Does not exist
4.
   a. A: For all natural numbers x, there exists some natural number y where x is greater than or equal to y.
      B: For some natural number y, all natural numbers x is greater than or equal to y.
   b. Yes
   c. Yes
   d. Yes
   e. No
   f. $\forall x \exists y (x > y) \Rightarrow \exists y \forall x (x > y)$

      $\neg(\exists y \forall x (x > y)) \vee \forall x \exists y (x > y)$

      $\forall y \exists x \neg(x > y) \vee \forall x \exists y (x > y)$

      $\forall y \neg(A > y) \vee \forall x (x > B)$

      $\neg(A > y) \vee (x > B)$

      {x/A,y/B}

g. $\exists y \forall x (x > y) \Rightarrow \forall x \exists y (x > y)$

$\neg(\forall x \exists y (x > y)) \vee \exists y \forall x (x > y)$

$\exists x \forall y \neg (x > y) \vee \exists y \forall x (x > y)$

$\neg (f(y) > y) \vee (x > f(x))$

Not true, because x binds to f(y) and y binds to f(x) which points to each other.