A simple mechanism for higher-order correlations in integrate-and-fire neurons

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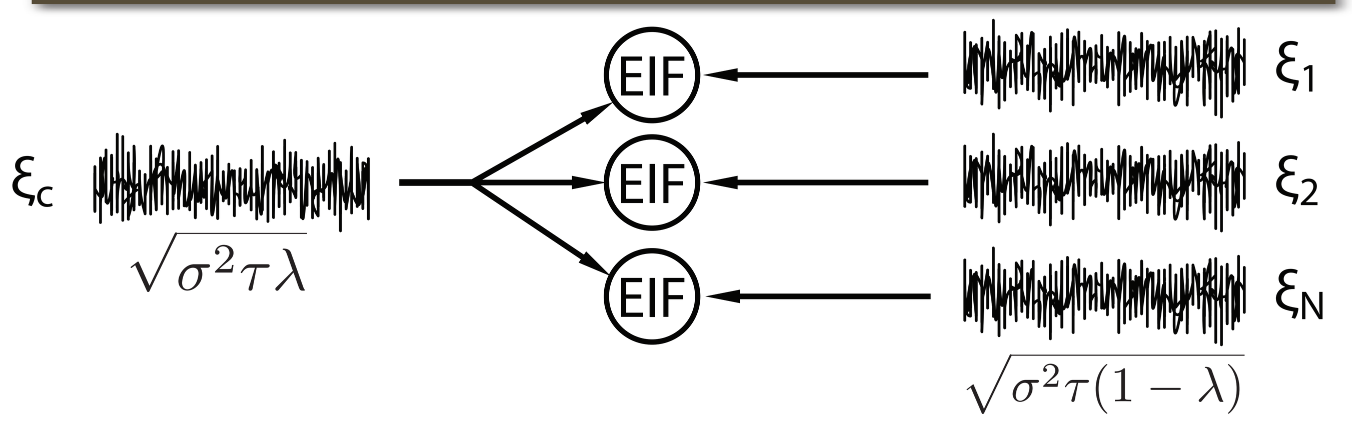
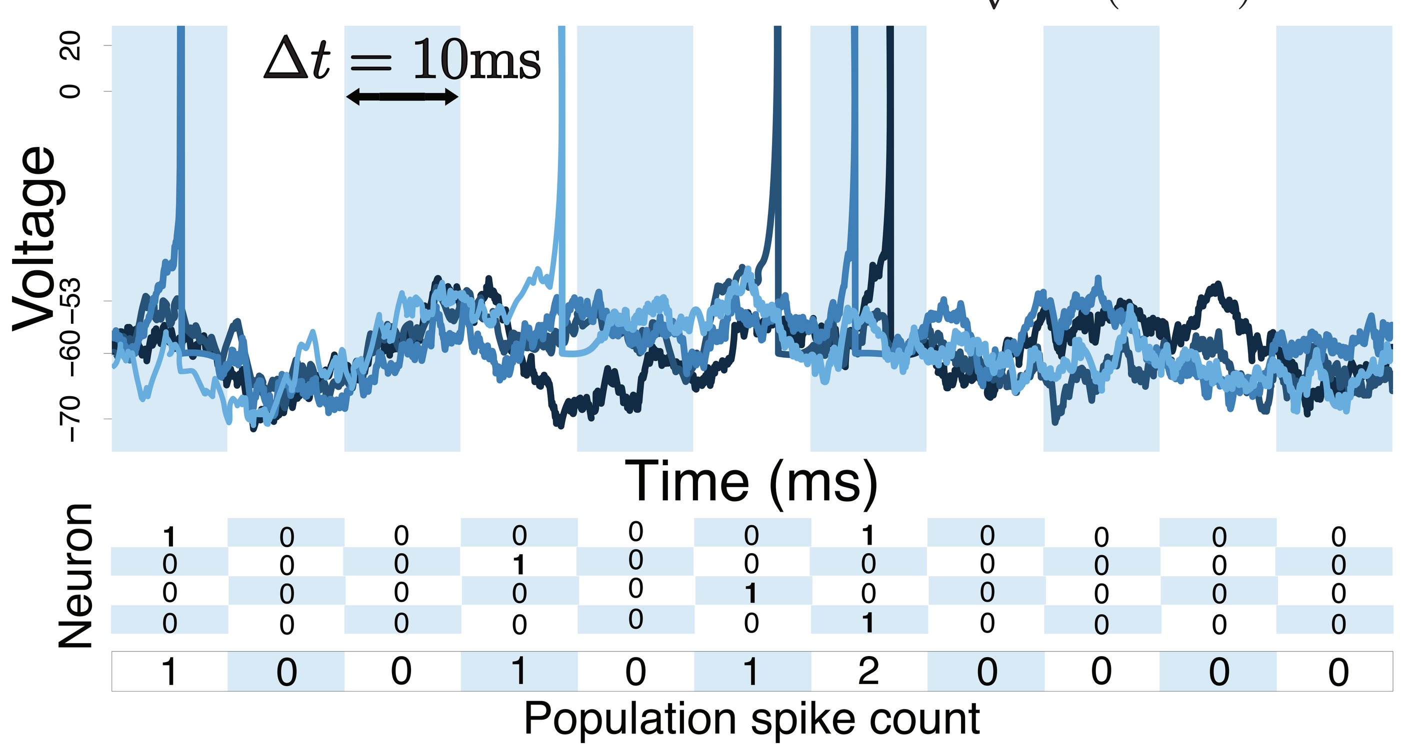
Introduction

- This study asks whether **common input** to **exponential integrate-and-fire** neurons gives rise to **higher-order correlations.**
- A tractable reduction of the **EIF model** - the **linear-nonlinear cascade** (LNL) [3], [4], is used to provide an analytic description of our results.
- The **Dichotomized Gaussian** model provides an excellent description of the exponential IF setup [1], [2].
- Finally we compare the LNL cascade to the Dichotomized Gaussian model.

Exponential IF neurons; common input



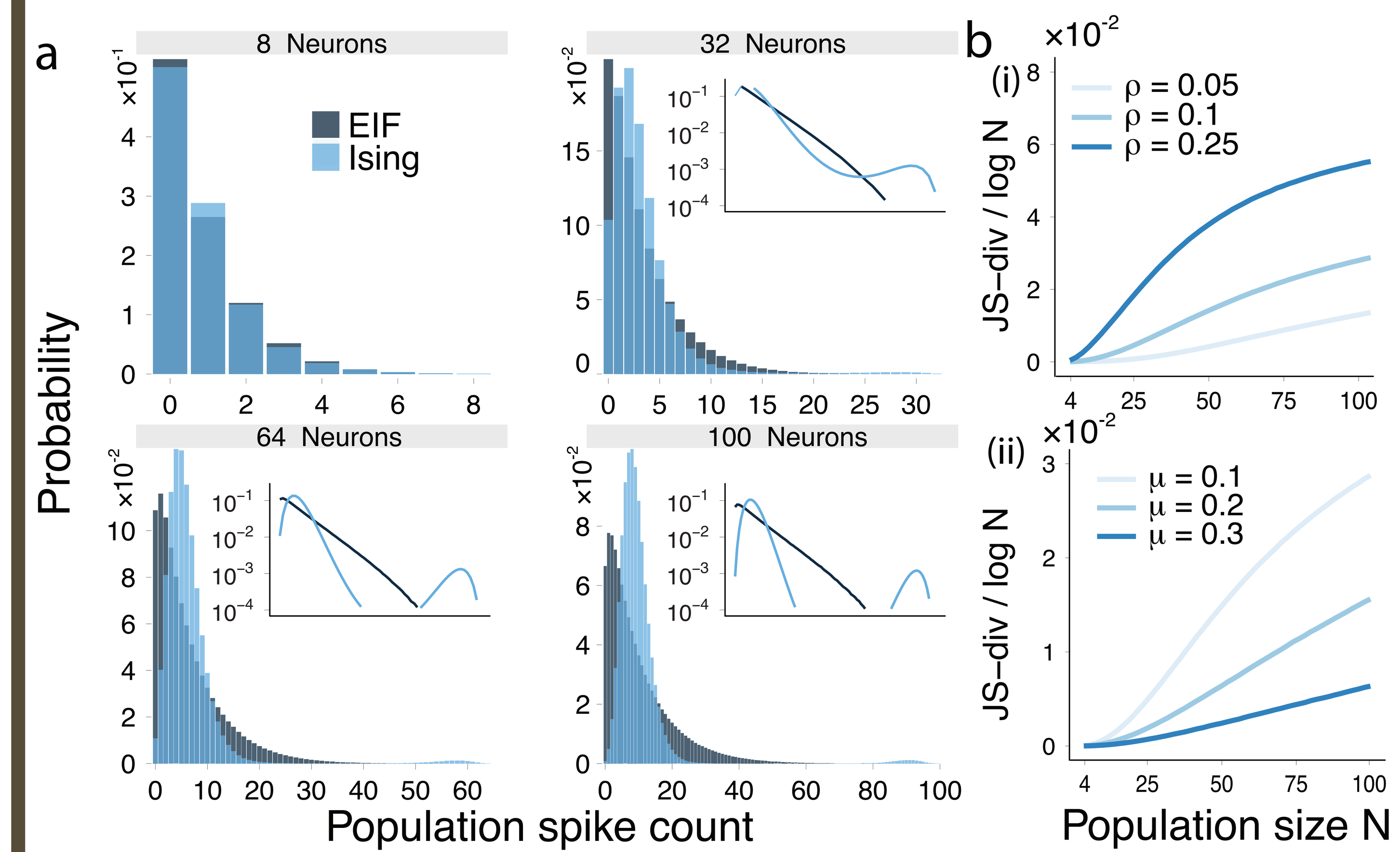


- The voltage of the EIF neurons evolves according to the eqn: $\tau_m V_i' = -V_i + \psi(V_i) + I(t)$

where $\psi(V_i) = \Delta_T \exp\left((V_i - V_S)/\Delta_T\right)$ defines the EIF neuron and the input current is:

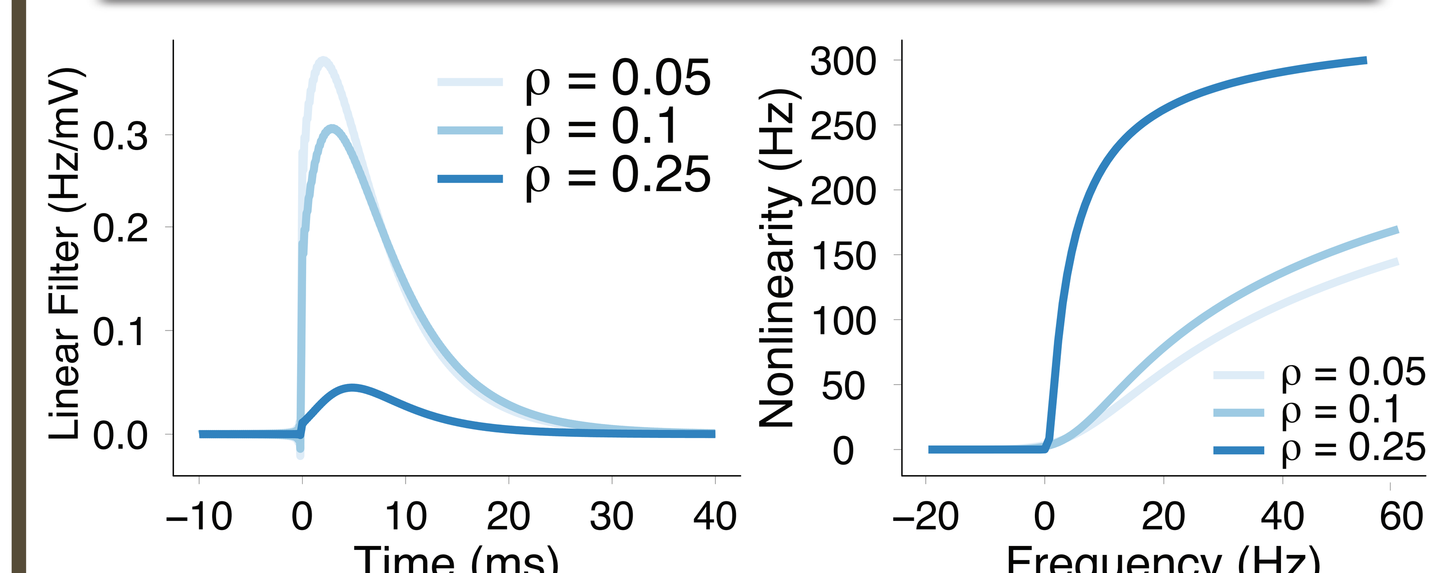
$$I(t) = \gamma + \sqrt{\sigma^2\tau}\left[\sqrt{1-\lambda}\,\xi_i(t) + \sqrt{\lambda}\,\xi_c(t)\right]$$

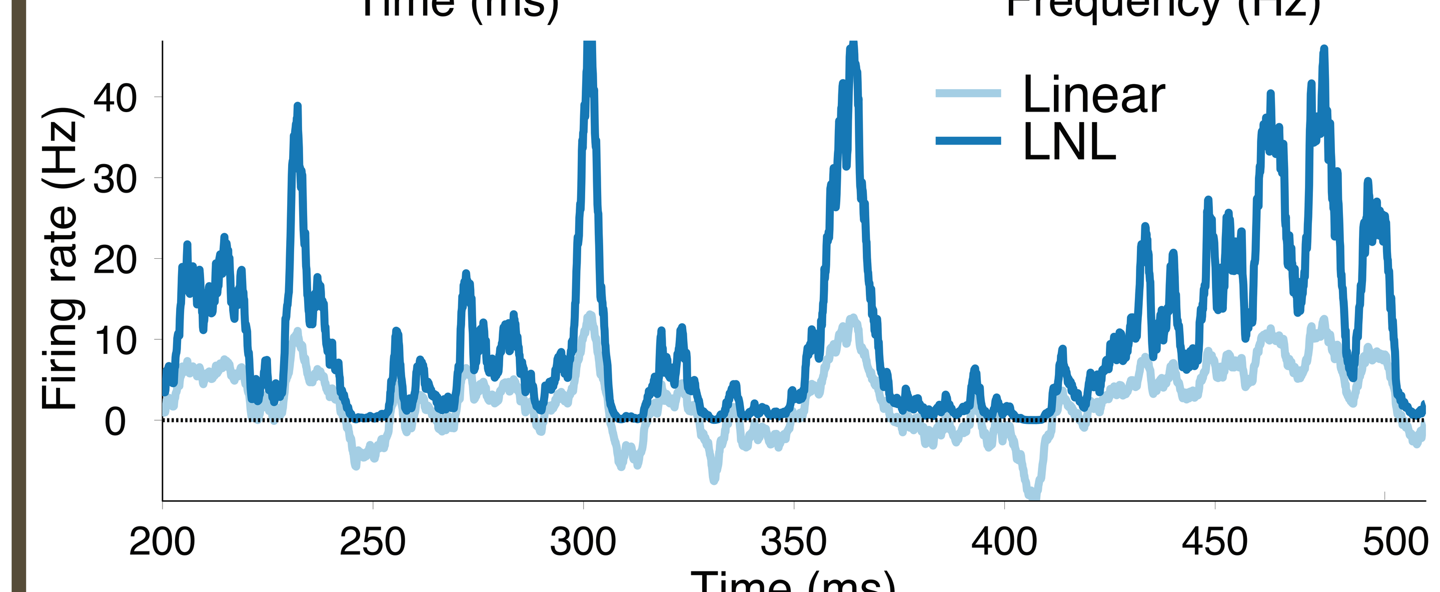
- Time is divided into bins of 10ms. A spike occurs when the voltage reaches a threshold of 20mV. The voltage is reset to the rest potential of -60mV and the neuron remains silent for the refractory period of 3ms.
- Mean firing rate is measured as the mean number of spikes per bin: $\mu = 0.1$
Correlation coefficient is: $\rho = 0.1$



- **Exponential integrate-and-fire neurons receiving common input give rise to higher-order correlations**
i.e. not well described by pairwise maximum entropy model.

Linear-nonlinear (LNL) cascade





- The firing rate is estimated as:

$$r(t) = F\left(r_0 + A * c(t)\right)$$

where $A(t)$ is the linear filter and F is the static non-linearity and c(t) is the common input.
- For an inhomogeneous poisson process with rate r(t) and common input c(t) the probability of a spike occuring in the interval $[t, t+\Delta t]$:

$$P(\text{spike} \in \Delta t|c) = 1 - \exp\left(-\int_0^{\Delta t} r(s+t)\mathrm{d}s\right)$$

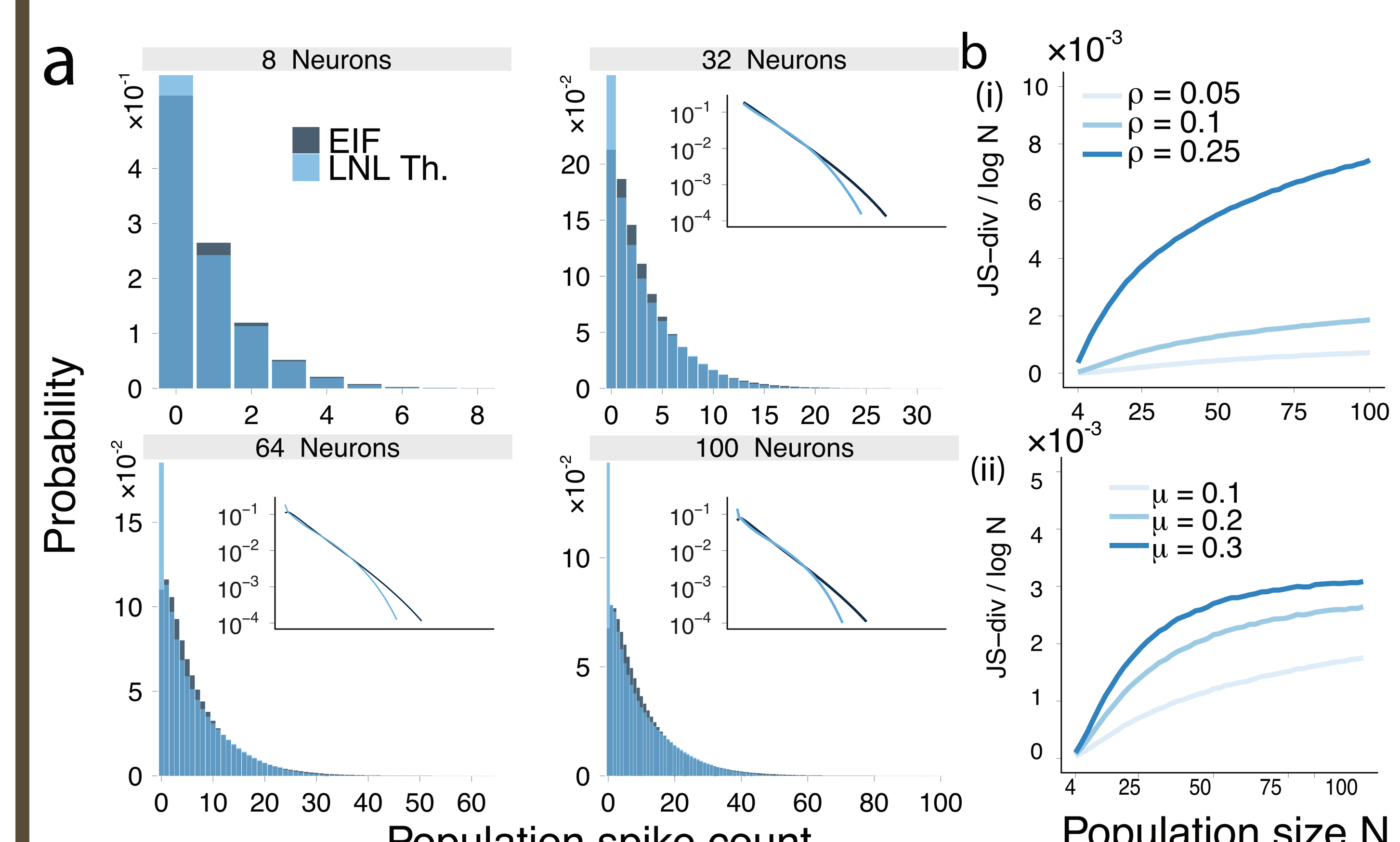
- By defining a new variable: $\mathcal{S} = \int_0^{\Delta t} r(s+t)\mathrm{d}s$

and the function: $\tilde{L}(\mathcal{S}) = 1 - \exp(-\mathcal{S})$

we can write the probability of observing k spikes occuring out of a homogeneous population of n neurons as:

$$P_{\mathrm{LF}}(k) = \binom{n}{k}\int_{-\infty}^{\infty} \phi_{LF}(\mathcal{S})\left(1-\tilde{L}(\mathcal{S})\right)^{n-k}\tilde{L}(\mathcal{S})^k \mathrm{d}\mathcal{S}$$

where $\phi_{LF}(\mathcal{S})$ is the pdf of the new variable.



- **LNL-cascade does an order of magnitude better job than the pairwise maximum entropy model.**

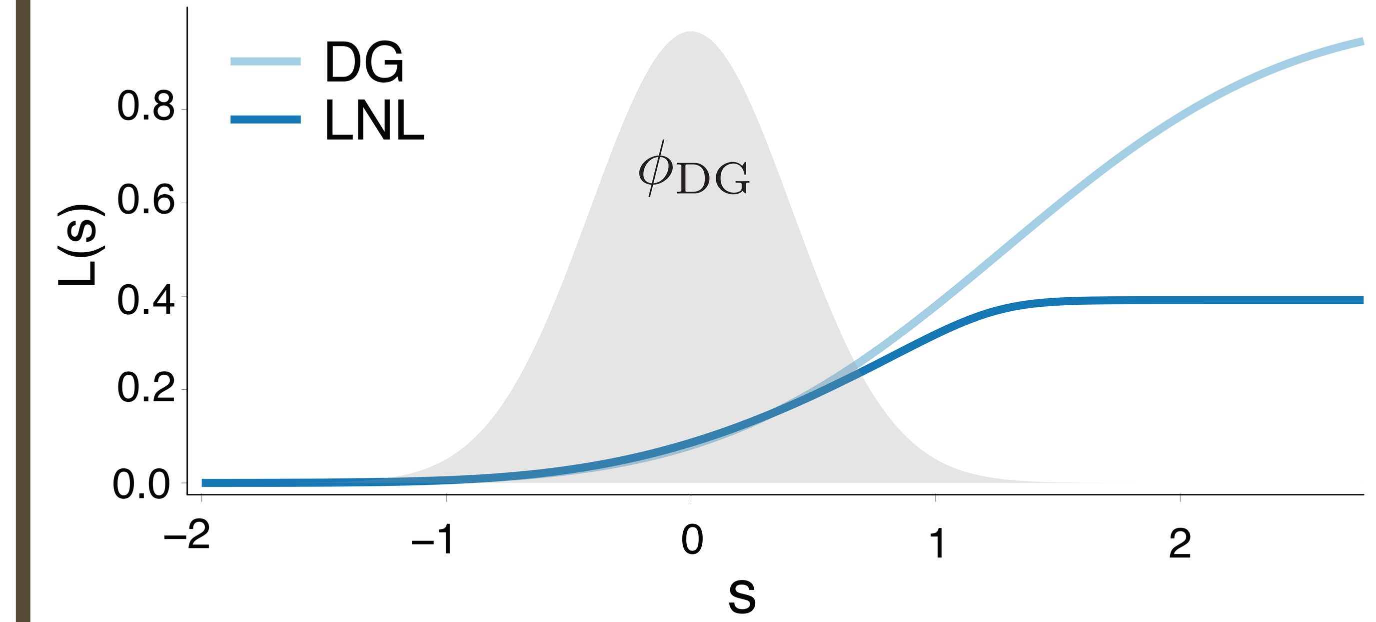
Comparison of LNL and DG

- DG is a model of a binary neuron receiving a correlated Gaussian input. The neuron spikes if the input is positive and is silent otherwise.
- We can write the input as:

$$Z_i = \gamma + \sqrt{1-\lambda}\,T_i + \sqrt{\lambda}\,s$$

- And so we can write the L(s) function like the previous section, using the CDF:

$$L(s) = \Phi\left(\frac{s+\gamma}{\sqrt{1-\lambda}}\right)$$



- By a change of variables, writing the LNL picture in terms of the DG, we can compare the L(s) functions over the support of the DG: ϕ_{DG}

Conclusions

- Exponential-IF neurons receiving common input give rise to higher-order correlations.
- The simpler, linear-nonlinear cascade model gives an order of magnitude better description of the population spike count distribution than the pairwise maximum entropy model.
- The linear-nonlinear cascade gives some insight as to why the Dichotomized Gaussian model has been so successful in the literature.

[1] J. Macke, et. al. Common Input Explains Higher-Order Correlations and Entropy in a Simple Model of Neural Population Activity. **PRL 106 May 2011**
[2] S. Yu, et. al. Higher-Order Interactions Characterized in Cortical Activity **J. Neuroscience 31(48): 17514 Nov 2011**
[3] S. Ostojic, N. Brunel. From Spiking Neuron Models to Linear-Nonlinear Models **PLoS Comput Biol 7(1) 2011**
[4] M. Richardson. Firing-rate response of linear and nonlinear integrate-and-fire neurons... **PRE 76 (2007)**

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