2.3 Derivation

DE fluid Energy conservation

$$\dot{\rho} + 3H(\rho + P) = 0 \qquad P = \omega\rho$$

$$\frac{d\rho}{dt} = -3\frac{da}{dt}\frac{1}{a}(1+\omega)\rho$$

$$\int_{\rho_0}^{\rho}\frac{d\rho'}{\rho'} = -3\int_{a_0}^{a}(1+\omega)\frac{da'}{a'}$$

Substitution $\quad (1+z) = \frac{a_0}{a} \Rightarrow d\ln(1+z) = -\frac{da}{a}$

$$\Omega = \rho/\rho_c$$

$$\Rightarrow \int_{\Omega_0}^{\Omega}\frac{d\Omega'}{\Omega'} = -3\int_{\ln(1+z)}^{0}(1+\omega)\,d\ln(1+z)$$

$$\Rightarrow \ln\left(\frac{\Omega}{\Omega_0}\right) = -3\int_{\ln(1+z)}^{0}(1+\omega)\,d\ln(1+z)$$

$$\Rightarrow \Omega = \Omega_0 \exp\left[-3\int(1+\omega)\,d\ln(1+z)\right]$$

2.4 soln

$$\omega(z) = \omega_0 + \omega_a\frac{z}{1+z} \qquad \frac{d\ln(1+z)}{dz}dz = \frac{1}{1+z}dz$$

$$\Rightarrow \text{Integral}: \int_{\ln(1+z)}^{0}\left\{1+\omega_0+\omega_a\frac{z'}{1+z'}\right\}\frac{1}{1+z'}dz'$$

$$= -(1+\omega_0)\ln(1+z) + \omega_a\int_{z}^{0}\frac{z'}{(1+z')^2}dz'$$

$$\frac{1}{1+z} - \frac{1}{(1+z)^2}$$

$$\Rightarrow \ln(1+z)^{-(1+\omega_0)} + \omega_a \left[\ln(1+z') + \frac{1}{1+z'} \right]_z^0$$

$$= \ln(1+z)^{-(1+\omega_0)} + \omega_a \left[1 - \ln(1+z) - \frac{1}{1+z} \right]$$

$$= \ln(1+z)^{-(1+\omega_0+\omega_a)} + \frac{\omega_a z}{1+z}$$

subbing into 2.3

$$\Omega(z) = \Omega_0 \exp\left[-3 \left\{ \ln(1+z)^{-(1+\omega_0+\omega_a)} + \frac{\omega_a z}{1+z} \right\} \right]$$

$$= \Omega_0 (1+z)^{3(1+\omega_0+\omega_a)} \exp\left[-\frac{3\omega_a z}{1+z} \right]$$

* different from text