Dynamic Substructuring using Virtual Point Transformation

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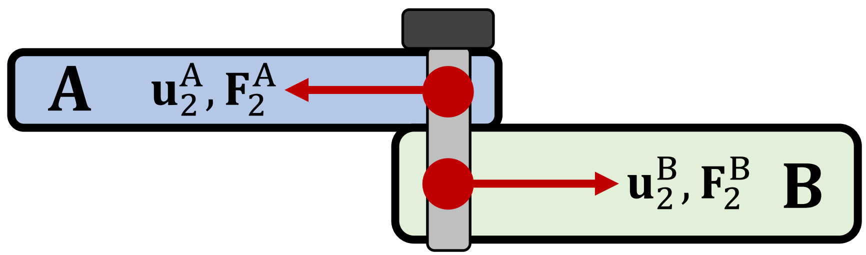
Purpose

- My future research interests = ‘Robotics’.
- My current research interest fields have expanded to "Vibration".
- My purpose of internship
  1. Enhance ability of Modeling dynamic mechanical system.
  2. Experience both Experiment & Theory.
  3. Study further about theory of Mechanical Vibration.

FBS with Rigid body assumption

▶ FBS(Frequency Based Substructuring) : A frequency based method that combines FRF(Frequency Response Functions) of substructures and use it to form a FRF of a large dynamic system response.

▶ Combine FRFs using the Compatibility and Force Equilibrium expressions at the joining point. Assuming the joint is rigid

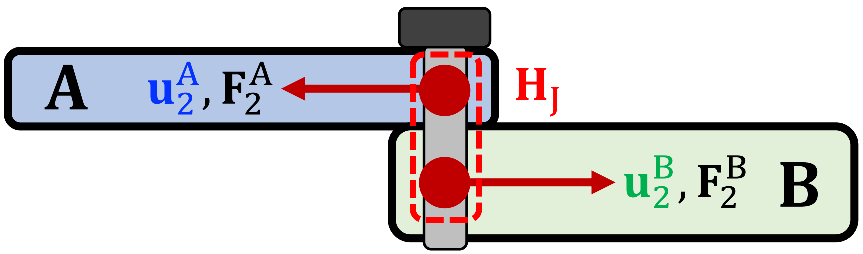


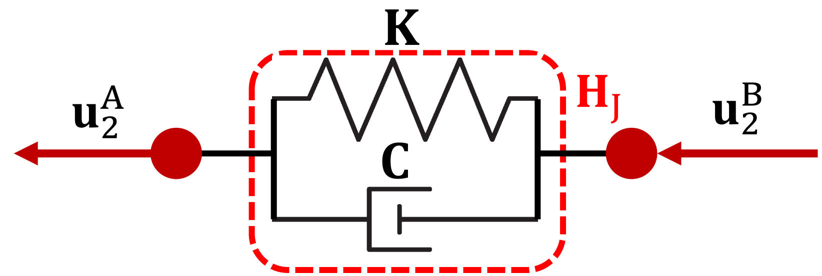
Equilibrium : $\mathbf{F}_2^A + \mathbf{F}_2^B = 0$ **Compatibility** : $\mathbf{u}_2^A = \mathbf{u}_2^B$

$$\mathbf{Y}^{AB} = \begin{pmatrix} \mathbf{Y}_{11}^A - \mathbf{Y}_{12}^A \mathbf{Y}_2^{-1} \mathbf{Y}_{21}^A & \mathbf{Y}_{12}^A \mathbf{Y}_2^{-1} \mathbf{Y}_{23}^B \\ \mathbf{Y}_{32}^A \mathbf{Y}_2^{-1} \mathbf{Y}_{21}^B & \mathbf{Y}_{33}^B - \mathbf{Y}_{32}^B \mathbf{Y}_2^{-1} \mathbf{Y}_{23}^B \end{pmatrix} \qquad \mathbf{Y}_2 = \mathbf{Y}_{22}^A + \mathbf{Y}_{22}^B$$

FBS with Joint Property (1)

▶ When a rigid assumption of a single joint is failed (i.e., shows dynamic behavior), using assumed Damping ($\mathbf{C}$), Stiffness ($\mathbf{K}$) matrices, we can model Joint Property $\mathrm{H_J}$.





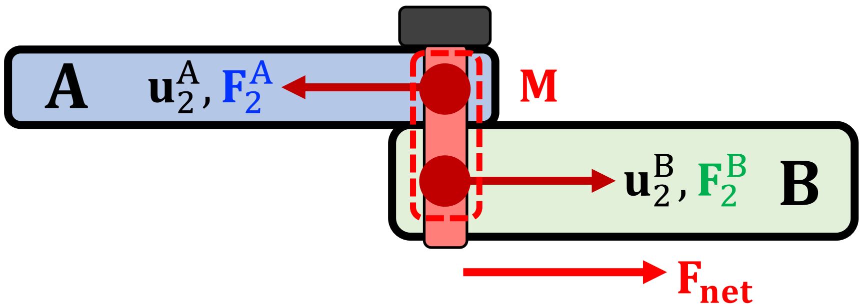
Equilibrium : $\mathbf{F}_2^A + \mathbf{F}_2^B = \mathbf{0}$ **Compatibility :** $\mathbf{u}_2^A - \mathbf{u}_2^B = \mathbf{H_J}\mathbf{F}_2^A$ [1]

$$\mathbf{Y}^{AB} = \begin{pmatrix} \mathbf{Y}_{11}^A - \mathbf{Y}_{12}^A \mathbf{Y}_2^{-1} \mathbf{Y}_{21}^A & \mathbf{Y}_{12}^A \mathbf{Y}_2^{-1} \mathbf{Y}_{23}^B \\ \mathbf{Y}_{32}^A \mathbf{Y}_2^{-1} \mathbf{Y}_{21}^B & \mathbf{Y}_{33}^B - \mathbf{Y}_{32}^B \mathbf{Y}_2^{-1} \mathbf{Y}_{23}^B \end{pmatrix} \qquad \mathbf{Y}_2 = \mathbf{Y}_{22}^A + \mathbf{Y}_{22}^B + \mathbf{H_J}$$

[1] Tsai, J.-S., and Y.-F. Chou. "The Identification of Dynamic Characteristics of a Single Bolt Joint." *Journal of Sound and Vibration*, vol. 125, no. 3, 1988, pp. 487-502.

FBS with Joint Property (2)

▶When a joint is assumed to be rigid, using assumed Mass ($\mathbf{M}$) matrix, we can model the Joint Property.



$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & J_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & J_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & J_{zz} \end{bmatrix}$$

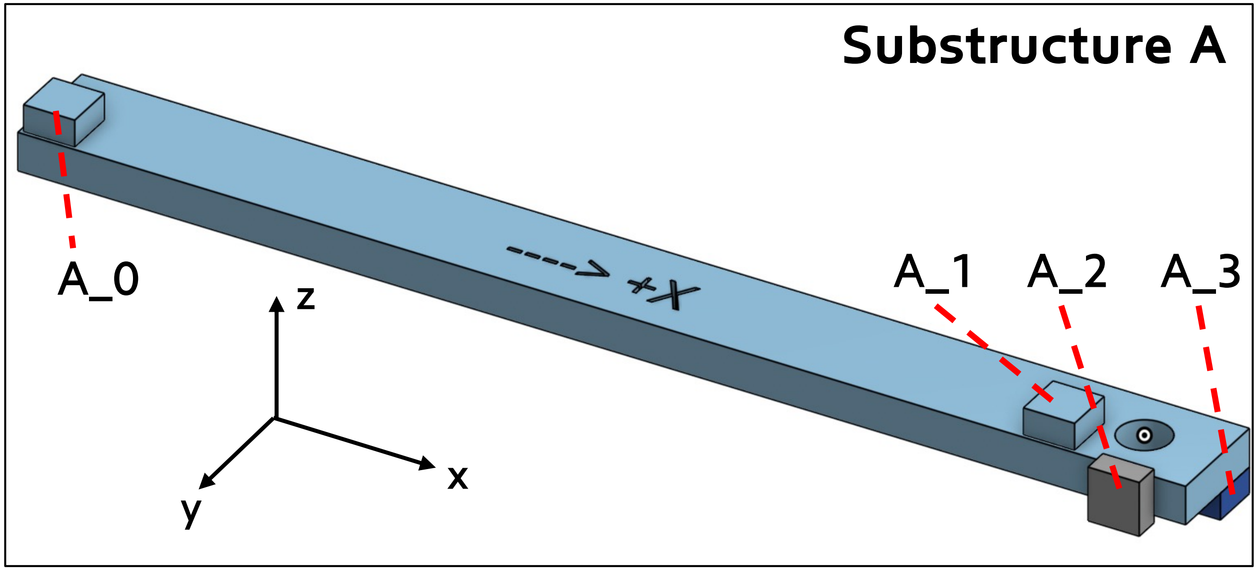
Equilibrium : $\mathbf{F}_2^{A} + \mathbf{F}_2^{B} = -\mathbf{M}\omega^2\mathbf{u}_2^{A}$ **Compatibility** : $\mathbf{u}_2^{A} = \mathbf{u}_2^{B}$

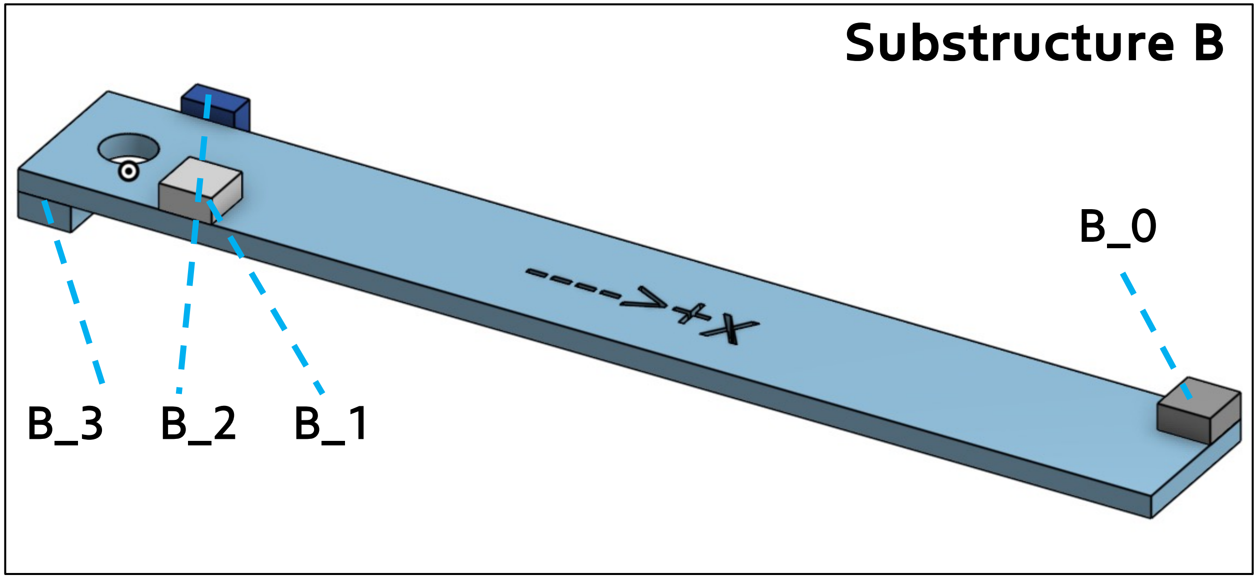
$$\mathbf{Y}^{\mathrm{AB}} = \begin{pmatrix} \mathbf{Y}_{11}^{\mathrm{A}} - \mathbf{Y}_{12}^{\mathrm{A}}\mathbf{Y}_2^{-1}\mathbf{Y}_{21}^{\mathrm{A}} & \mathbf{Y}_{12}^{\mathrm{A}}\mathbf{Y}_2^{-1}\mathbf{Y}_{23}^{\mathrm{B}} \\ \mathbf{Y}_{32}^{\mathrm{A}}\mathbf{Y}_2^{-1}\mathbf{Y}_{21}^{\mathrm{B}} & \mathbf{Y}_{33}^{\mathrm{B}} - \mathbf{Y}_{32}^{\mathrm{B}}\mathbf{Y}_2^{-1}\mathbf{Y}_{23}^{\mathrm{B}} \end{pmatrix}$$

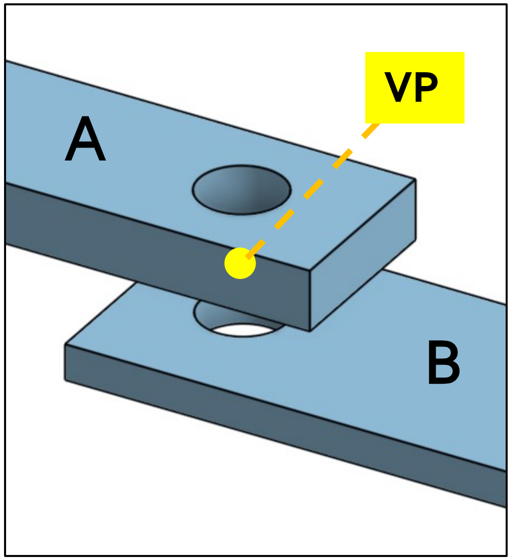
$$\mathbf{Y}_2 = \mathbf{Y}_{22}^{\mathrm{A}} + \mathbf{Y}_{22}^{\mathrm{A}} + \mathbf{Y}_{22}^{\mathrm{A}}\mathbf{M}\omega^2\mathbf{Y}_{22}^{\mathrm{B}} \quad \text{(Receptance)}$$

$$\mathbf{Y}_2 = \mathbf{Y}_{22}^{\mathrm{A}} + \mathbf{Y}_{22}^{\mathrm{A}} - \mathbf{Y}_{22}^{\mathrm{A}}\mathbf{M}\mathbf{Y}_{22}^{\mathrm{B}} \quad \text{(Inertance)}$$

Experimental Setup







VP: The midpoint of Interface between A & B.

Result (1) : Obtaining Joint Property

▶**Result 1.** Compute the Joint Property with the Least Square Method

$$\mathbf{Y}^{\mathrm{AB}} = \begin{pmatrix} \mathbf{Y}_{11}^{\mathrm{A}} - \mathbf{Y}_{12}^{\mathrm{A}}\mathbf{Y}_{2}^{-1}\mathbf{Y}_{21}^{\mathrm{A}} & \mathbf{Y}_{12}^{\mathrm{A}}\mathbf{Y}_{2}^{-1}\mathbf{Y}_{23}^{\mathrm{B}} \\ \mathbf{Y}_{32}^{\mathrm{A}}\mathbf{Y}_{2}^{-1}\mathbf{Y}_{21}^{\mathrm{B}} & \mathbf{Y}_{33}^{\mathrm{B}} - \mathbf{Y}_{32}^{\mathrm{B}}\mathbf{Y}_{2}^{-1}\mathbf{Y}_{23}^{\mathrm{B}} \end{pmatrix}$$

$$\mathbf{Y}_2 = \mathbf{Y}_{22}^{\mathrm{A}} + \mathbf{Y}_{22}^{\mathrm{B}} + (\mathbf{K} + j\omega\mathbf{C})^{-1}$$

→ Joint Property $(\mathbf{C}, \mathbf{K})$

▶ $(\mathbf{K} + j\omega\mathbf{C})(\mathbf{u}_2^{A} - \mathbf{u}_2^{B}) = \mathbf{f}_2^{A}$

● $\mathbf{C}$

254	1588	-3065	44274	-18905	-84831
2482	79627	8861	-527528	-1244658	-3998438
623	-114256	-98192	1541422	-2242944	7308024
-69354	-122181	1491262	-17610556	46150094	-10741408
-203021	-5018155	1837132	-4393057	48084870	262990060
-45661	1562817	2097951	-52415639	18707176	-93354612

Negative values exist in the term.
Improved modeling is needed.

● $\mathbf{K}$

1953	-45686	-33107	910326	-415095	2602017
-7760	806335	436128	731581	46034418	-62203814
22132	-927111	-733067	9160861	-45801352	68330767
-213027	5064646	6686704	-262229057	-136294039	-219391688
-496268	11013527	20998719	-969794490	-1.186E+09	-108866815
318949	2146482	-8031996	79249722	-247728924	-24527886

Observed that
beam has a large stiffness along a short length
i.e., (y, z) direction.

Obtained Joint Properties can be used to improve the accuracy of the Virtual Point Transformation by this joint.

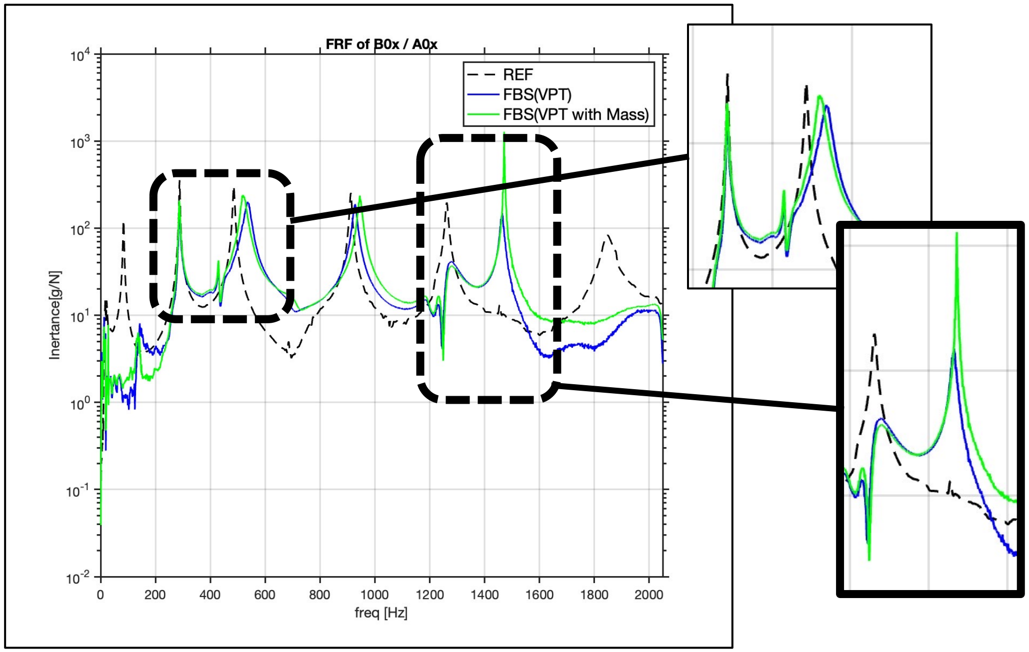
Result (2) : Modeling using Mass matrix

▶**Result 2.** Use Mass matrix to improve the accuracy of the FRF.

$$\mathbf{Y}^{AB} = \begin{pmatrix} \mathbf{Y}_{11}^{A} - \mathbf{Y}_{12}^{A}\mathbf{Y}_{2}^{-1}\mathbf{Y}_{21}^{A} & \mathbf{Y}_{12}^{A}\mathbf{Y}_{2}^{-1}\mathbf{Y}_{23}^{B} \\ \mathbf{Y}_{32}^{A}\mathbf{Y}_{2}^{-1}\mathbf{Y}_{21}^{B} & \mathbf{Y}_{33}^{B} - \mathbf{Y}_{32}^{B}\mathbf{Y}_{2}^{-1}\mathbf{Y}_{23}^{B} \end{pmatrix}$$

$$\mathbf{Y}_2 = \mathbf{Y}_{22}^{A} + \mathbf{Y}_{22}^{A} - \mathbf{Y}_{22}^{A}\mathbf{M}\mathbf{Y}_{22}^{B} \qquad \text{(Inertance)}$$

→ Improved $\mathbf{Y}^{AB}$



- At Low Frequency:

Dynamic property of system heavily influenced by **damping** and **stiffness**.

- At High Frequency:

Influenced by **mass**.

At high frequency range, modeling of a joint mass improve the accuracy.

Result (3) : Using Weight Matrix

▶**Result 3.** Use weight matrix to improve the results of VPT

<u>Example</u>

$$u = Rq + \mu, \qquad u = \begin{bmatrix} 1 \\ 2 \\ 10 \end{bmatrix}, R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, q = ?$$

Using Left-inverse of R,

$$q = \begin{bmatrix} 10/3 \\ 13/3 \end{bmatrix}$$

Now, let weight matrix as

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mathbf{0} \end{bmatrix}$$

Then,

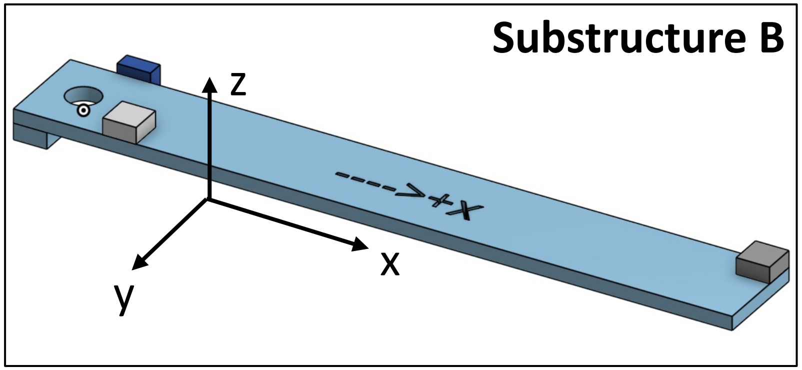
$$q = \begin{bmatrix} \mathbf{1} \\ \mathbf{2} \end{bmatrix}$$

From this example, weight matrix can be used to

unwanted signals from inaccuate sensor.

Result (3) : Using Weight Matrix

- In this experiment, the coherence of the FRFs is in the order of $\mathrm{z} < \mathrm{x} < \mathrm{y}$ (see figure below).
  - Use the weight matrix to lessen the signals from z-direction.
  - Expectation : improved VPT results particularly when the driven force is along y axis.



- Use Weight Matrix only for FBS for displacement, not for FBS for force.

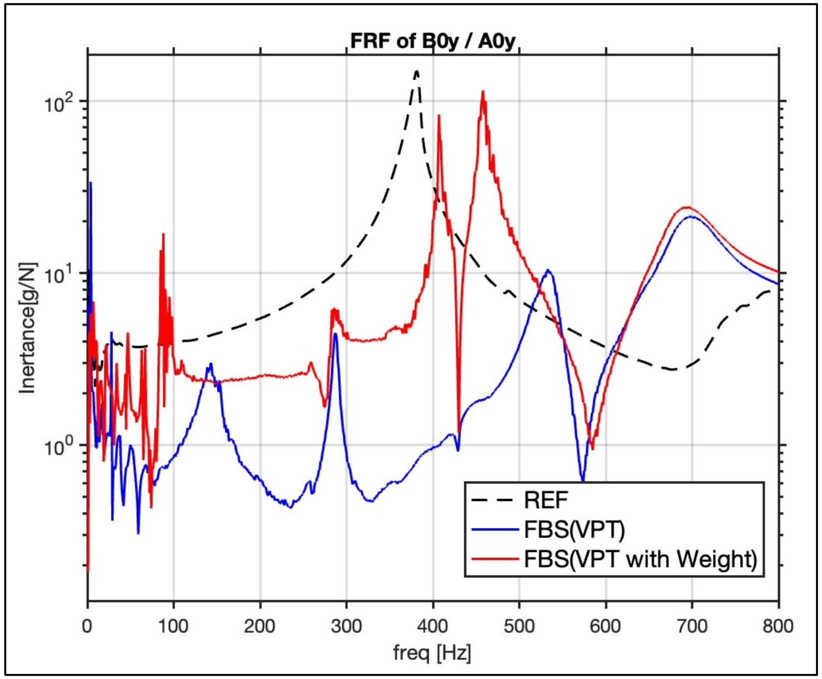
$$\underset{(3n\times 1)}{\mathbf{u}} = \underset{(3n\times 6)}{\mathbf{R}_{\mathrm{u}}}\underset{(6\times 1)}{\mathbf{q}} + \underset{(3n\times 1)}{\boldsymbol{\mu}}$$

$$\mathbf{q} = \left(\mathbf{R}_{\mathrm{u}}^{\mathrm{T}}\mathbf{R}_{\mathrm{u}}\right)^{-1}\mathbf{R}_{\mathrm{u}}^{\mathrm{T}}\mathbf{u} = \mathbf{T}_{\mathrm{u}}\mathbf{u} \qquad \mathbf{W} = \mathbf{I}.$$

$$\mathbf{q} = \left(\mathbf{R}_{\mathrm{u}}^{\mathrm{T}}\mathbf{W}\mathbf{R}_{\mathrm{u}}\right)^{-1}\mathbf{R}_{\mathrm{u}}^{\mathrm{T}}\mathbf{W}\mathbf{u} = \mathbf{T}_{\mathrm{u}}\mathbf{u} \qquad \mathbf{W} \neq \mathbf{I}.$$

Result (3) : Using Weight Matrix

▶ Set the weight of noisy signal channel (z direction) to be less than 1.



$W_{(3,3)} = 0.5$

$W_{(6,6)} = 0.4$

$\boxed{W_{(9,9)}} = 0.3$

Otherwise, 1

This corresponds to the z direction.

Weight matrix successfully worked and
Accuracy improved comparing with the conventional VPT.
More improvements when the driven force is along the y-axis.

Performance Indicator (FRAC)

▶ Calculate the Frequency Response Assurance Criterion (FRAC) to measure the correlation between VPT results and reference results.

- Validation

[2]

$$▶ \quad FRAC_{pq} = \frac{|\sum_{\omega=\omega_1}^{\omega_2} H_{pq}(\omega)\widehat{H}_{pq}^{*}(\omega)|^2}{\sum_{\omega=\omega_1}^{\omega_2} H_{pq}(\omega)H_{pq}^{*}(\omega)\sum_{\omega=\omega_1}^{\omega_2} \widehat{H}_{pq}(\omega)\widehat{H}_{pq}^{*}(\omega)|^2}$$

$$\mathbf{FRAC}_{\mathbf{VPT,REF}} = \begin{bmatrix} \boxed{\mathbf{0.60}} & 0.04 & 0.16 & 0.34 & 0.02 & 0.21 \\ 0.01 & 0.14 & 0.18 & 0.04 & 0.06 & 0.05 \\ 0.11 & 0.06 & 0.14 & 0.20 & 0.04 & 0.17 \\ \boxed{\mathbf{0.68}} & 0.02 & 0.17 & 0.23 & 0.08 & 0.18 \\ 0.01 & 0.02 & 0.08 & 0.01 & 0.18 & 0.17 \\ 0.10 & 0.02 & 0.11 & 0.24 & 0.02 & 0.23 \end{bmatrix}$$

Correlation between VPT and Reference.

Calculated FRAC is very small except two driven/measured points (Uncorrelated)

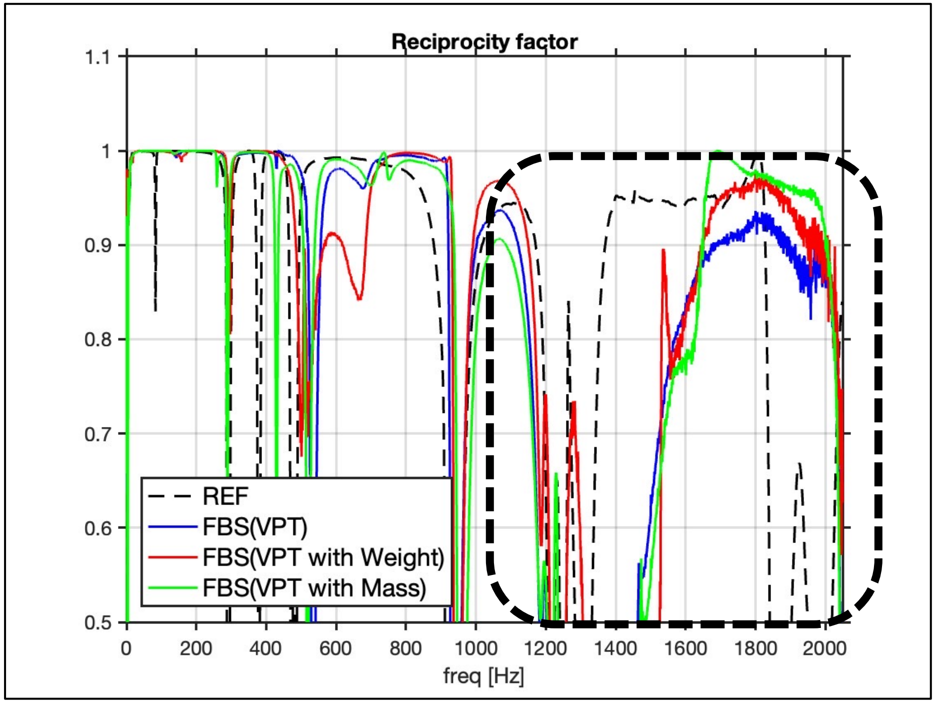
[2] Allemang, Randall J. "The Modal Assurance Criterion - Twenty Years of Use and Abuse." Sound and Vibration, vol. 37, no. 8, 2003, p. 14.

Performance Indicator (Reciprocity)

▶ Calculate the Reciprocity factor by assuming the Joint Property and Weight Matrix

● Validation of modeling of mass is physically justified.

▶ $$x_{ij} = \frac{(H_{ij} + H_{ji})(H_{ij}^* + H_{ji}^*)}{2(H_{ij}^* H_{ij} + H_{ji}^* H_{ji})}$$: function of frequency.



When considering Mass in the joint property, the reciprocity of the VPT results can be improved in high frequency range than the traditional VPT results. Close to the value of 1 (*i.e,* physically correct)

Future Work

Additional Experiments

Justify measured Damping and Stiffness Matrix $(\mathbf{C}, \mathbf{K})$ can improve VPT results.

Weight matrix used in Force FBS.

Weight Matrix is a vague concept that is determined through the experiment.

I don't understand why previous research use weight matrix for Force FBS.

Further research on the weight matrix is needed

Modeling

More precise modeling than other three modeling options outlined today.

I failed to complete linear modeling that satisfies the Equilibrium and Compatibility conditions.

Further research on the modeling of system is needed