Model Equations for
"Future Climate Change Pathways" Matlab App*

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This document contains the carbon and climate equations underlying the Matlab app developed for Econ 150C2 at the University of Arizona.

*I appreciate assistance from Ernesto Rivera Mora in coding the app.

The user inputs emissions E_t at several future years, which are linearly interpolated to give emissions e_t at a 5-year timestep. The carbon cycle follows Joos et al. (2013, Table 5), as recommended and compiled by Dietz et al. (2021). That carbon cycle has

$$\boldsymbol{M}_{t+1} = \boldsymbol{\Lambda}^{\Delta} M_t + \boldsymbol{b} E_t \tag{Carbon reservoirs}$$

where $\boldsymbol{M}$ is a 4×1 vector of atmospheric carbon reservoirs. The coefficient matrices are:

$$\boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.9975 & 0 & 0 \\ 0 & 0 & 0.9730 & 0 \\ 0 & 0 & 0 & 0.7927 \end{bmatrix} \tag{Carbon transfer}$$

and

$$\boldsymbol{b} = \begin{bmatrix} 0.2173 \\ 0.2240 \\ 0.2824 \\ 0.2763 \end{bmatrix}. \tag{Emissions' fate}$$

The year 2015 values (in Gt C) are

$$\boldsymbol{M}_0 = \begin{bmatrix} 588 + 139.1 \\ 90.2 \\ 29.2 \\ 4.2 \end{bmatrix}, \tag{Carbon starting value}$$

where 588 Gt C is the stock of preindustrial carbon.

The parameters of the climate model come from Geoffroy et al. (2013), as compiled by Dietz et al. (2021). Additional atmospheric carbon dioxide (CO_2) increases radiative forcing $F_t(\boldsymbol{M}_t)$, which measures additional energy at the earth's surface due to CO_2 in the atmosphere. Forcing is

$$F_t(\boldsymbol{M}_t) = f_{2x} \frac{\ln\left(\sum_{i=1}^{4} M_t^i / 588\right)}{\ln(2)} + EF_t, \tag{Forcing}$$

where M_t^i indicates element i of $\boldsymbol{M}_t$, EF_t is exogenous forcing from non-CO_2 greenhouse gases (defined below), and f_{2x} is forcing induced by doubling CO_2. Surface temperature evolves as

$$T_{t+1}^s = T_t^s + \frac{\Delta}{5}\phi_1 \left[F_{t+1}(\boldsymbol{M}_{t+1}) - \lambda T_t^s - \phi_3 \left(T_t^s - T_t^o\right)\right]. \qquad \text{(Surface temperature)}$$

Ocean temperature evolves as

$$T_{t+1}^o = T_t^o + \frac{\Delta}{5}\phi_4 \left[T_t^s - T_t^o\right]. \qquad \text{(Ocean temperature)}$$

Steady-state warming from doubled carbon dioxide ("climate sensitivity") is $f_{2x}/\lambda = 3.1°\text{C}$. Exogenous forcing is

$$EF_{t+1} = EF_0 + (EF_{100} - EF_0)\min\{\Delta\, t/(5 * 17), 1\}. \qquad \text{(Non-CO}_2\text{ forcing)}$$

When the uncertainty switch is turned on, temperature is not deterministic. "Climate sensitivity" ($S = f_{2x}/\lambda$) is the equilibrium warming that results from doubling the atmospheric concentration of carbon dioxide. Gillingham et al. (2018) consider the merits of various distributions for climate sensitivity and ultimately choose to fit a lognormal distribution to results from Olson et al. (2012). They report a location parameter of 1.10704 and a scale parameter of 0.264. These parameters were the ones used in Lemoine (2021). The intervals depicted on the plot derive from the λ implied by a Monte Carlo sample of 10,000 draws for S.

Table 1: Parameters

Parameter	Value	Description
Δ	5	Timestep (years)
EF_0	0.5	Year 2015 non-CO_2 forcing (W/m^2)
EF_{100}	1	Year 2100 non-CO_2 forcing (W/m^2)
ϕ_1	0.386	Warming delay parameter
ϕ_3	0.73	Parameter governing transfer of heat from ocean to surface
ϕ_4	0.034	Parameter governing transfer of heat from surface to ocean
f_{2x}	3.503	Forcing from doubling CO_2 (W/m^2)
λ	1.13	Forcing per degree warming ($[W/m^2]/°C$)
$\boldsymbol{M}_0$	see text	Year 2015 carbon reservoirs (Gt C)
T_0^s	0.85	Year 2015 surface temperature (°C, wrt 1900)
T_0^o	0.0068	Year 2015 lower ocean temperature (°C, wrt 1900)

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