A CLAS Proposal for PAC44

Transition Form Factors of the η' and ϕ Mesons with CLAS12

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Abstract

Dalitz decays are radiative decays in which the photon is virtual and subsequently produces an electron positron pair, $P \to l^+l^-X$. It is an important tool used to reveal the internal structure of hadrons and the interaction mechanisms between photons and hadrons. Furthermore, assuming point-like particles, the electromagnetic interaction is calculable within QED by the Kroll-Wada formula. Transition form factors are deviations from the QED decay rate. They characterize modifications of the point-like photon-meson vertex due to the structure of the meson. For the η' meson this deviation represents the internal structure of the meson, while for the ϕ meson the deviation represents the transition from $\phi \to \eta$. The transition form factor can be characterized as $\left|F(q^2)\right|$, where q^2 is the square of the invariant mass of the lepton pair, and can be determined by comparing QED to what is measured experimentally.

Measurements with the highest scientific impact on the determination of the transition form factor have been performed in the space-like region ($q^2 < 0$) in collider experiments. However, due to experimental limitations (e.g. $\pi^\pm$ contamination in lepton sample, low branching fractions, external conversion contamination), transition form factors in the time-like region ($q^2 > 0$) have not yet been precisely determined. Recent measurements of the time-like transition form factor for $\eta' \to e^+e^-\gamma$ have been performed by the BESIII collaboration with insufficient statistical precision, therefore the proper theoretical description cannot be determined.

From previous CLAS analyses using the g12 data set, it was preliminarily shown that measurements of the time-like transition form factor were achievable, but without the statistical precision needed to be completive. Therefore, we propose to use CLAS12 to focus on the dilepton decay channels from the reactions $ep \to e'p\eta'$ and

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$ep \to e'p\phi$, where $\eta' \to e^+e^-\gamma$ and $\phi \to \eta e^+e^-$. The CLAS12 detector will be used to identify and measure the e^+e^- decay products by means of the High Threshold Cherenkov Counter (HTCC), Pre-Calorimeter (PCAL) and Electromagnetic Calorimeter (EC). The combination of HTCC+PCAL+EC can provide a rejection factor for single $e^\pm/\pi^\pm$ of up to 10^6 for momenta less than 4.9 GeV/c with $\approx$ 100% efficiency. For dileptons (e^+e^- pairs), this rejection factor will be $\approx 10^{12}$, which enables dilepton studies for branching ratios $\approx 10^{-9}$. Precise determination of momenta and angles of the e^+e^- decay products are the key features available to CLAS12. Preliminary studies using the CLAS12 simulation suite have shown that a beam time of 100 days, at full luminosity, will accumulate a data sample at least one order of magnitude larger in statistics, than the most current $\eta' \to e^+e^-\gamma$ and $\phi \to \eta e^+e^-$ measurement.

1 Appendix

1.1 η' Decays

The main decays studied for this analysis are when a pseudoscalar meson, $P_p(\pi^0,\ \eta,\ \eta')$, decays via 2 photons $\gamma\gamma$ or a photon γ and a dilepton (l^+l^-) pair, which are the two most prevalent decays of η' as shown in

1.2 Dalitz Decay

When a pseudoscalar meson decays via a photon γ and a dilepton (l^+l^-) pair, it is known as a Dalitz decay or a so-called single off-shell decay. The Dalitz decay is related to the two photon decay. However, in the Dalitz decay, one of the photons is off-shell (γ^*) and decays into a dilepton pair. Since the Dalitz decay is related to the two photon decay, the form factor of the Dalitz decay, for P$(\pi^0,\ \eta,\ \eta')$, will be similar to the form factor of the two photon decay of P$(\pi^0,\ \eta,\ \eta')$, except there will be an effective mass dependence for the Dalitz decay. Figure **??** depicts the Feymann diagram of the Dalitz decay.

The amplitude for the decay $P_P \to \gamma^\star(p)\gamma(k) \to l^+(p_+)l^-(p_-)\gamma(k)$ is given by the following expression:

$$\mathcal{M}(P \to l^+(p_+,s_+)l^-(p_-,s_-)\gamma) = M_P(p^2,k^2=0)\varepsilon_{\mu\nu\rho\sigma}\frac{1}{q^2}e\bar{u}(p_-,s_-)\gamma^\mu v(p_+,s_-)q^\nu\epsilon^\rho k^\sigma. \tag{1}$$

Comparing the amplitudes of Eq. 1 and Eq. **??** it is seen that the polarization of the off-shell photon turned into the current $e\bar{u}(p_-,s_-)\gamma^\mu v(p_+,s_-)$ of the lepton pair. The parameters $s_\pm$ are the spin helicities of the outgoing leptons $l^\pm$ and as in Eq. **??**, ϵ is the polarization of the outgoing photon.

1.2.1 *Squared Matrix Element*

$|\mathcal{M}(P \to l^{+}(p_{+}, s_{+})l^{-}(p_{-}, s_{-})\gamma)|^2 =$
$\frac{e^2}{q^4} |M|^2 \varepsilon_{\mu\nu\rho\sigma}\varepsilon_{\mu'\nu'\rho'\sigma'}\bar{u}(p_{-}, s_{-})\gamma^{\mu}v(p_{+}, s_{+})\bar{v}(p_{+}, s_{+})\gamma^{\mu'}u(p_{-}, s_{-})q^{\nu}\epsilon^{\rho}k^{\sigma}q^{\nu'}\epsilon^{\rho'}k^{\sigma'}$.
using an equation found between equation 5.3 and 5.4 found in [?] $\sum_{s_{-},s_{+}} \bar{u}(p_{-}, s_{-})\gamma^{\mu}\nu(p_{+}, s_{+})\bar{\nu}(p_{+}, s_{+})$

$Tr\left[(p_{-} + m)\gamma^{\mu}(p_{+} - m)\gamma^{\mu'}\right]$
$= 2q^2\left[-(g_{\mu\mu'} - \frac{p_{\mu}p_{\mu'}}{q^2}) - \frac{(p_{+}-p_{-})_{\mu}(p_{+}-p_{-})_{\mu'}}{q^2}\right]$ where the identity $q = p_{+} + p_{-}$
was used. Substituting Eq. 1.2.1 into Eq. 1.2.1 $|\mathcal{M}|^2 = \frac{2e^2|M_P|^2}{q^2}\varepsilon_{\mu\nu\rho\sigma}\varepsilon_{\mu'\nu'\rho'\sigma'}\left[-g^{\mu\mu'} - \frac{(p_{+}-p_{-})^{\mu}(p_{+}-p_{-})}{q^2}\right.$
Substituting $k = P - q$ and $p_{-} = q - p_{+}$ into Eq. 1.2.1 $|\mathcal{M}|^2 = \frac{2e^2|M_P|^2}{q^2}\varepsilon_{\mu\nu\rho\sigma}\varepsilon_{\mu'\nu'\rho'\sigma'}\left[-g^{\mu\mu'} - \frac{(2p_{+}-q)^{\mu}(2}{q^2}\right.$
$\times (-g^{\nu\nu'})(q^{\rho}P^{\sigma} - q^{\rho}q^{\sigma})(q^{\rho}P^{\sigma'} - q^{\rho'}q^{\sigma'})$ Applying properties of $-g^{\mu\mu'}$ and $-g^{\nu\nu'}$ onto Eq. 1.2.1 $|\mathcal{M}|^2 = \frac{2e^2|M_P|^2}{q^2}\left[\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{\mu\nu}{}_{\rho'\sigma'}q^{\rho}P^{\sigma}q^{\rho'}P^{\sigma'} + \frac{4}{q^2}\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{\mu}{}_{\nu'\rho'\sigma'}p_{+}^{\nu}p_{+}^{\nu'}q^{\rho}q^{\rho'}P^{\sigma}P^{\sigma'}\right]$
Switching to the rest frame of the pseudoscalar meson, P_p, the 4-momenta is transformed to $P^{\sigma} = m_p\delta^{\sigma 0}$. The squared amplitude of Eq. 1.2.1 reads; $|\mathcal{M}|^2 = \frac{2e^2|M_P|^2}{q^2}m_p^2\left[\varepsilon_{\mu\nu\rho}\varepsilon^{\mu\nu}{}_{\rho'}q^{\rho}q^{\rho'} - \frac{4}{q^2}\varepsilon_{\mu\nu\rho}\varepsilon^{\mu}{}_{\nu'\rho'}p_{+}^{\nu}p_{+}^{\nu'}q^{\rho}q^{\rho'}\right]$ The sign change is due to $g^{\sigma\sigma'} = -\delta^{\sigma\sigma'}$. Using the antisymmetric tensor properties $\varepsilon_{\mu\nu\rho}\varepsilon^{\mu\nu}{}_{\rho'} = 2\delta_{\rho\rho'}$ and $\varepsilon_{\mu\nu\rho}\varepsilon^{\mu}{}_{\nu'\rho'} = \delta_{\nu\nu'}\delta_{\rho\rho'} - \delta_{\nu\rho'}\delta_{\rho\nu'} = (\hat{e}_{\nu} \times \hat{e}_{\rho}) \cdot (\hat{e}_{\nu'} \times \hat{e}_{\rho'})$, Eq. 1.2.1 is reduced to $|\mathcal{M}|^2 = \frac{2e^2|M_P|^2}{q^2}m_p^2\left[2|\mathbf{q}|^2 - \frac{4}{q^2}|\mathbf{q}|^2|\mathbf{p}_{+}|^2\sin^2(\theta_{p_{+}q})\right]$

1.2.2 *Decay rate*

The decay rate of a three-body decay is given in Equation 46.19 of [?] as $\mathrm{d}\Gamma = \frac{1}{(2\pi)^5}\frac{1}{16m_p^2}|\mathcal{M}|^2|\mathbf{p_1^*}||\mathbf{p_3}|d\Omega_1^* d\Omega_3 dm_{12}$, where $(|\mathbf{p_1^*}|, \Omega_1^*)$ is the momentum of particle 1 in the rest frame of 1 and 2, and Ω_3 is the angle of particle 3 in the rest frame of the decaying particle m_p [?]. Relating Eq. 1.2.2 to the variables in Eq. 1.2.1, where $(|\mathbf{p_1^*}|, \Omega_1^*) = (|\mathbf{p}_{+}|, \Omega_{p_{+}q})$, $m_{12} = q$ and $(|\mathbf{p_3}|, \Omega_3) = (|\mathbf{p_k}|, \Omega_k)$, reads; $\mathrm{d}\Gamma = \frac{1}{(2\pi)^5}\frac{1}{16m_p^2}|\mathcal{M}|^2|\mathbf{p}_{+}||\mathbf{p_k}|d\Omega_{+}d\Omega_k dq$, In the rest from of the decaying particle m_p, the 3-momenta $|\mathbf{p_k}| = |\mathbf{q}|$ and the solid angle $\Omega_k = \Omega_q$. Substituting the square matrix element from Eq. 1.2.1 into Eq. 1.2.2 yields; $\mathrm{d}\Gamma = \frac{1}{(2\pi)^5}\frac{1}{16m_p^2}\frac{2e^2|M_P|^2}{q^2}m_p^2\left[2|\mathbf{q}|^2 - \frac{4}{q^2}|\mathbf{q}|^2|\mathbf{p}_{+}|^2\sin^2(\theta_{p_{+}q})\right]|\mathbf{p}_{+}||\mathbf{q}|d\Omega_{p_{+}q}d\Omega_q dq$. The variables $|\mathbf{q}|$ and $|\mathbf{p}_{+}|$ can be redefined, by means of Eq. 46.20b and Eq. 46.20a of [?], as $|\mathbf{q}| = \frac{m_p^2 - q^2}{2m_p}$

$|\mathbf{p}_{+}| = \frac{\sqrt{q^2 - 4m_l^2}}{2} = \frac{q\sqrt{1 - \frac{4m_l^2}{q^2}}}{2} = \frac{q\mathcal{K}}{2}$, where $\mathcal{K} = \sqrt{1 - \frac{4m_l^2}{q^2}}$. Replacing the variables calculated in Eq. 1.2.2 and Eq. 1.2.2 into Eq. 1.2.2 and collecting terms yields; $\mathrm{d}\Gamma = \frac{1}{(2\pi)^5}\frac{1}{16m_p^2}|M_P|^2\left[\frac{2e^2m_p^2}{8}\left(\frac{m_p^2 - q^2}{2m_p}\right)^3\right]\left(2 - \mathcal{K}^2\sin^2(\theta_{p_{+}q})\right)\frac{\mathcal{K}}{4q^2}dq^2 d\Omega_{p_{+}q}d\Omega_q$, where the identity $qdq = \frac{dq^2}{2}$. Performing the integration of $\Omega_{p_{+}q}d\Omega_q$ and replacing $e^2 = 4\pi\alpha$ transforms Eq. 1.2.2 into; $d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32}\frac{4\pi\alpha}{3}\left|M_P\right|^2\left[\frac{m_p^6\left(1-\frac{q^2}{m_p^2}\right)^3}{m_p^3}\right](3-\mathcal{K}^2)\,\frac{\mathcal{K}}{q^2}dq^2$,

which can be simplified further to; $d\Gamma = \left(\frac{1}{64\pi}\left|M_P\right|^2 m_P^3\right)\frac{2\alpha}{3\pi}\frac{1}{q^2}\left(1-\frac{q^2}{m_p^2}\right)^3\left(1+\frac{2m_l^2}{q^2}\right)\left(1-\frac{4m_l^2}{q^2}\right)^{\frac{1}{2}}dq^2$. The form factor $M_P(p^2,k^2=0)$ can be written as follows: $\mathrm{M}_P \to M_P \times \left|F(q^2)\right|$, where M_p is the decay constant of two photons mentioned in Sec. **??** and $\left|F(q^2)\right|$ is called the transition form factor, which defines the electromagnetic space structure of the meson.

It can be seen that the first set of variables in parenthesis in Eq. 1.2.2 is Eq. **??**, therefore; $\frac{d\Gamma}{\Gamma_{\gamma\gamma}dq^2 = \frac{2\alpha}{3\pi}\frac{1}{q^2}\left(1-\frac{q^2}{m_p^2}\right)^3\left(1+\frac{2m_l^2}{q^2}\right)\left(1-\frac{4m_l^2}{q^2}\right)^{\frac{1}{2}}\left|F(q^2)\right|^2}$, which is the Kroll-Wada equation founded in [**?**].

The value of $\left|F(q^2)\right|$ can be directly measured by comparison of the differential cross section with that of Q.E.D. pointlike differential cross section i.e. $\frac{d\sigma}{dq^2 = \left[\frac{d\sigma}{dq^2}\right]_{pointlike}\left|F(q^2)\right|^2}$, or by performing a line shape analysis on the l^+l^- invariant system using assumptions on the structure of $\left|F(q^2)\right|$. One such assumption for $\left|F(q^2)\right|$ is the dipole approximation in which $\mathrm{F(q^2)} = [1-\frac{q^2}{\Lambda^2}]^{-1}$

1.3 Summary

The two photon decay and the Dalitz decay have different branching ratios. This difference is attributed to the factor of α along with a q^2 dependence calculated in the Dalitz decay. However, due to the probability of a photon converting into an electron-positron pair in ℓH_2, the total amount of pairs produced via photon conversion is about the same as via Dalitz decay.