1 ECTS Study Project of the Module Parallel Computing (BV440004) SS21'

CHAIR OF COMPUTATIONAL MODELING AND SIMULATION
DEPARTMENT OF CIVIL, GEO AND ENVIRONMENTAL ENGINEERING
TECHNICAL UNIVERSITY OF MUNICH
Arcisstraße 21
D-80333 München

Parallelization of a Sudoku Solver

- Final Report -

Hua-Ming Huang
huaming.huang.tw@gmail.com

Supervisor: Christoph Ertl

Contents

1 Introduction

1.1 What is Sudoku?

A Sudoku puzzle is a n-by-n grid that contains numbers from 1 to n, with box size $\sqrt{n} \times \sqrt{n}$. A standard Sudoku contains 81 cells, in a 9×9 grid, and has 9 boxes (3×3 grid), as shown in Figure 1.

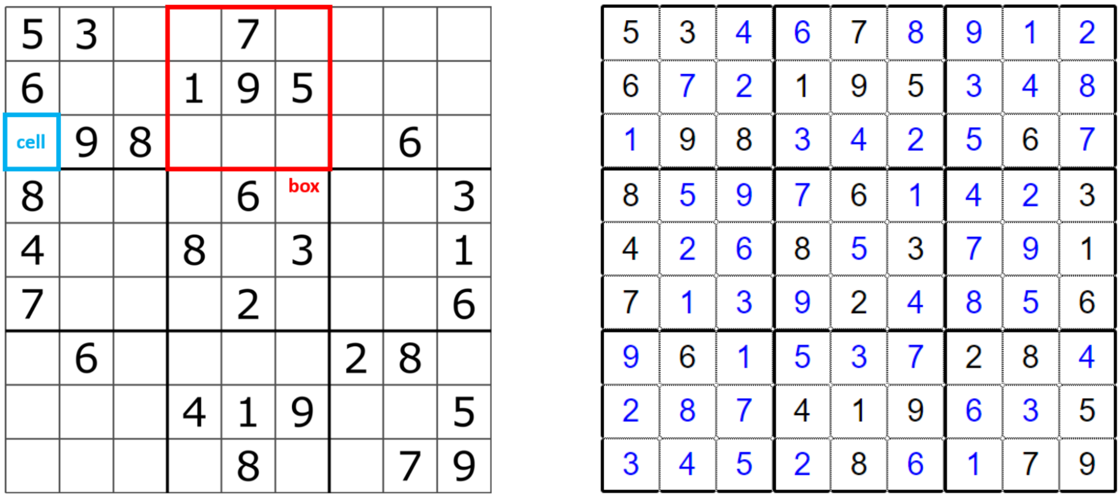


Figure 1: A typical Sudoku puzzle (*left*) and its solution (*right*)

The goal of the puzzle game is to fill in the empty cells on the board such that each column, row, and box (or called "subgrid", "region", "block") contains every number in the set $\{1, ..., 9\}$ exactly once. Sudoku puzzles usually come with a partially filled-in board (*clues*). The difficulty of the puzzle varies depending on how many numbers are given, as well as the location of the given numbers. There are many strategies to solve Sudoku puzzles (See Section Solving Algorithms). Solutions might not be unique, or might not even exist. However, a properly formulated Sudoku puzzle should have an unique solution that can be reached logically.

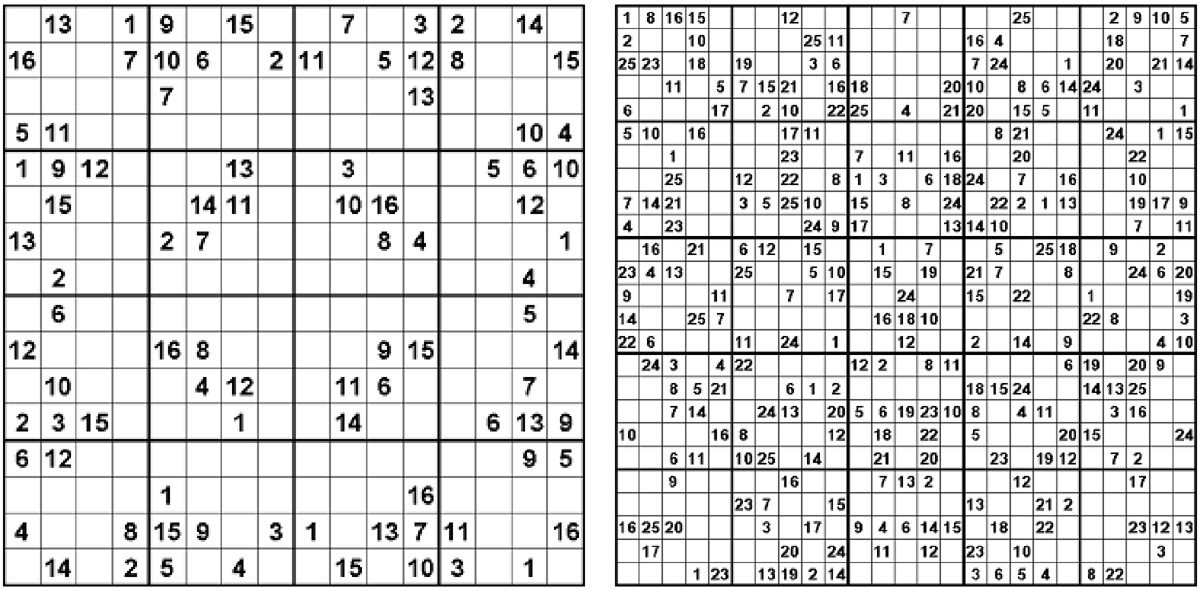
1.2 Rules of Sudoku

- Each number must appear exactly once in each row.
- Each number must appear exactly once in each column.
- Each number must appear exactly once in each box.

The above rules imply no *duplicate* numbers in any row, column and box.

2 Motivation

Thanks to the computational power provided by modern computers, there are already several algorithms (See Section Solving Algorithms) that will solve 9×9 Sudoku puzzles in fractions of a second. However, as the size of Sudoku puzzle n gets larger, e.g., 16×16, 25×25 (Figure 2), the *combinatorial explosion* occurs and thus leads to *exponential* growth of overall solving time. Combinatorial explosion creates limits to the properties of Sudokus that can be constructed, analyzed, and solved [1]. This also directly eliminates the possibility of solving Sudoku in a reasonable amount of time.



(a) A 16×16 Sudoku puzzle

(b) A 25×25 Sudoku puzzle

Figure 2: Large sudoku puzzles

Therefore, this project aims to implement a Sudoku solver that could solve **large** Sudoku puzzles as efficiently as possible by means of various parallelization techniques and algorithms.

3 Solving Algorithms

[3]

3.1 Naive brute-force algorithm

A brute-force algorithm would enumerate *all* possible combinations of numbers for each empty cell. This algorithm has an incredibly large search space to wade through. For instance, a blank n-by-n grid has a total of $n^{n \times n}$ different possible combinations of solutions. Solving Sudoku puzzles in this way is extremely slow and computationally intensive, but guarantees to find a solution eventually.

3.2 Backtracking algorithm

A common algorithm to solve Sudoku boards is called *backtracking*, which is a type of brute-force search. This algorithm is essentially a *depth-first search (DFS)* by completely exploring one branch to a possible solution before moving to another branch.

Given a partially filled-in (incomplete) board, the following illustrates the backtracking algorithm step by step: [2]

1. Finds the first empty cell on the given board.
2. Attempts to place the smallest possible number (namely 1) in that cell.
3. Checks if the inserted number is valid.
    - If the number is valid (i.e., does not violate the rules stated in Section Rules of Sudoku), proceed to the next empty cell and **recursively** repeat steps 1-3.
    - If the number is not valid, increment its number by 1 and repeat step 3. If a cell is discovered where none of the possible numbers is allowed, then reset the cell you just filled to zero/blank and backtrack to the most recently filled cell. The value in that cell is then incremented by 1 and repeat step 3. If the number cannot be incremented, then we backtrack again.
4. Repeats the procedure until either there are no more empty cells on the board, which means we have found the solution. Or we backtracked to the first unfilled cell. In this case, no solution exists for the given board.

Rather than trying to continue a solution that can never possibly work which we do with naive brute-force algorithm, we *only* continue solutions that currently work. If they don't work, we backtrack to the previous step and try other numbers again. This is going to be a lot faster than trying every single possible combination of solutions as naive brute-force algorithm did.

3.3 DLX algorithm

4 Implementation Details & Performance Results

4.1 Parallelization Challenges

5 Conclusion & Future Work

References

[1] *Combinatorial explosion of Sudoku puzzle — Wikipedia, The Free Encyclopedia*. URL: https://en.wikipedia.org/wiki/Combinatorial_explosion#Sudoku.

[2] *Python Sudoku Solver Backtracking — tech with tim*. URL: https://www.techwithtim.net/tutorials/python-programming/sudoku-solver-backtracking/.

[3] *Sudoku solving algorithms — Wikipedia, The Free Encyclopedia*. URL: https://en.wikipedia.org/wiki/Sudoku_solving_algorithms.