Homework 1

For the first four homework assignments, I will also be giving 1 bonus point if you turn in a printed pdf of your LaTex edited homework solution. I will provide the source code for each homework to give you an idea of how LaTex works. On Canvas, there is a file ''LaTex-Getting started" to help you get it all initiated (thanks, Bennet Gloeckner for this file!).

1 Problem

Read page 22 and page 23 'Writing Guidelines' in Sundstrom's text about how to write a proof. Whenever you write a proof I want you to follow this scheme. Points from this come indirectly, as there will be 2 points given for organization and neatness.

2 Problem (20 points)

Prove each of the following. Write the proof in table form, just as in Example 1.2 of Taggart's and Conroy's text. Every change must be justified by the EPI's (cite which one you are using each time)

(a) *If a, b are integers with $a + b = a$, then $b = 0$.*

(b) *If a, b, c, d are integers, it holds that $(a + b) + (c + d) = (a + c) + (b + d)$.*

(c) *If a, b, c are integers with $ac = bc$ and $c \neq 0$, then $a = b$.*

(d) *If a, b are integers, it holds that $(a + b)^2 = a^2 + 2ab + b^2$. (Here, for any integer x, we introduce the notation $x^2 = x \cdot x$ and $x + x = 2x$.)*

3 Problem (10 points)

I mentioned that our EPI's are not redundant. I will have you prove two of them with the help of other axioms.

(a) *Using only part I of the EPI's prove that for any integer a it is true that $a \cdot 0 = 0$. You can use table format or written text.*

(b) *Using* **only** *Part I of the EPI's and (a) prove that for an integer a it holds that $-a = (-1) \cdot a$.* **to be clear: you cannot use $-1 \cdot a = -a$ in any form. $-a$ is the symbol for the additive inverse of a, so you can only use the property**.

4 Problem (9 points)

Mark if a statement is true or false. If it is false, find a counterexample. Don't prove if it true.

(a) *If a is an integer and $a < -5$, then $|a| > 5$.*

(b) *Suppose a, b are integers. If $b > 0$, then $|a - b| < |a|$.*

(c) *Let a, b be integers. If $a \mid b$ and $a \mid c$, then $a \mid (b + c)$.*

(d) *Let a, b be integers. If $a \mid b$, then $a \leq b$.*

(e) *If a is an integer, then $2a$ and $3a$ have opposite parity.*

(f) *If a is an odd integer, then $2a$ and $3a$ have opposite parity.*

5 Problem (3 points)

*Two prime numbers p and q are known as twin primes if $q = p + 2$. For example, $p = 29$ and $q = 31$ are twin primes; so are $p = 101$ and $q = 103$. It is conjectured that there an in
finitely many twin primes.*
We'll call three prime numbers p, q, and r triplet primes if $q = p + 2$ and $r = q + 2$. For example, $p = 3, q = 5$, and $p = 7$ are triplet primes. Are there any other triplet primes? Either provide an example, or explain (no formal proof, but get to the base of it) why no other triplet primes exist.

6 Problem (3+3 points)

A **conjecture** *is a statement that we believe is true (maybe because of many experiments we did). Let $n \in \mathbb{N}$. (In other words, n is a natural number, which is a positive integer.)*

(a) *Make some conjectures about the values of n for which $3^n - 1$ is prime.*

(b) *Make some conjectures about the values of n for which $3^n - 2^n$ is prime. For this, try a bunch of n's out and see if you can find a pattern. You do not need to prove your conjectures.*