Homework 4

Context

This assignment reinforces ideas in Module 4: Constrained Optimization. We focus specifically on implementing quantile regression and LASSO.

Due date and submission

Please submit (via Canvas) a PDF containing a link to the web address of the GitHub repo containing your work for this assignment; git commits after the due date will cause the assignment to be considered late. Due date is Wednesday, 4/2 at 10:00AM.

Points

Problem	Points
Problem 0	20
Problem 1	20
Problem 2	30
Problem 3	30

Dataset

The dataset for this homework assignment is in the file `cannabis.rds`. It comes from a study conducted by researchers at the University of Colorado who are working to develop roadside tests for detecting driving impairment due to cannabis use. In this study, researchers measured levels of THC—the main psychoactive ingredient in cannabis—in participants' blood and then collected other biomarkers and had them complete a series of neurocognitive tests. The goal of the study is to understand the relationship between performance on these neurocognitive tests and the concentration of THC metabolites in the blood.

The dataset contains the following variables:

- `id`: subject id
- `t_mmr1`: Metabolite molar ratio—a measure of THC metabolites in the blood. This is the outcome variable.
- `p_*`: variables with the `p_` prefix contain measurements related to pupil response to light.
- `i_*`: variables with the `i_` prefix were collected using an iPad and are derived from neurocognitive tests assessing reaction time, judgment, and short-term memory.
- `h_*`: Variables related to heart rate and blood pressure.

Problem 0

This "problem" focuses on structure of your submission, especially the use git and GitHub for reproducibility, R Projects to organize your work, R Markdown to write reproducible reports, relative paths to load data from local files, and reasonable naming structures for your files.

To that end:

- Create a public GitHub repo + local R Project; I suggest naming this repo / directory bios731_hw4_YourLastName (e.g. bios731_hw4_wrobel for Julia)
- Submit your whole project folder to GitHub
- Submit a PDF knitted from Rmd to Canvas. Your solutions to the problems here should be implemented in your .Rmd file, and your git commit history should reflect the process you used to solve these Problems.

Problem 1: Exploratory data analysis

Perform some EDA for this data. Your EDA should explore the following questions:

- What are n and p for this data?
- What is the distribution of the outcome?
- How correlated are variables in the dataset?

Summarize key findings from your EDA in one paragraph and 2-3 figures or tables.

```
##          id t_mmr1       p_fpc1      p_fpc2      p_fpc3      p_fpc4     p_fpc5
## 1   003-062 1.1060   -1.8211162  31.2313128 -10.6693836   2.7442271  1.0314427
## 2   003-063 1.1173   40.4466647 -12.9756688  15.4684626  -3.5677034  6.0107910
## 3   003-065 1.0931   78.3098485 -38.9890359  21.2331346  -3.8911474  5.3591818
## 4   003-066 0.6205   79.5907424  19.8607925  11.5744150  -0.9544689  2.1109962
## 5   003-067 0.8493  -44.6759997 -21.1724208  15.8689102  -8.7858449  4.4601135
## 6   003-071 0.3176   -0.1558157 -31.4700300  -9.1539762   6.3502493  8.4365323
## 7   003-074 0.9883   15.8511286  -5.9559856 -10.8295617  -3.2763318  0.1895255
## 8   003-075 1.1716   67.4402429   0.3364498  -4.9373142  -4.3321599  2.4385379
## 9   003-076 0.9957  -83.5271958 -24.7436593   6.8695266 -14.4042731  3.8951045
## 10  003-078 1.3592  102.6609230  25.1630892  -1.0119763   1.1154723  4.6851621
## 11  003-079 1.0669   90.2481532  49.4662711   2.0620516  -2.5269178 -5.8198896
## 12  003-080 0.6023    2.7591547  11.7747759  18.2287624  -4.5345087  9.4415042
## 13  003-081 2.0981   36.4596719  33.2582148  -2.8147333  -6.8358503 -8.1238832
## 14  003-082 0.8347  124.2106807  37.7027830  25.2144975   0.5467373  5.8555231
## 15  003-083 0.8510   72.5120789 -16.5667572  -1.7221799  -2.7710587  2.9185083
## 16  003-084 0.9529  -88.4254444  13.0818508   4.4952973  10.3090863  7.4592976
## 17  003-086 1.0327  -27.8297246   0.6702393 -10.6483122 -10.3174353 -2.6068585
## 18  003-087 0.9773   90.8934455  28.2673602  11.1911768  -1.0514149  0.1519068
## 19  003-088 0.5377   54.8651720 -33.0802385  -5.3055680  -7.8329838  3.1983064
## 20  003-089 1.9131  -45.9421618  21.1199157   9.0920367  -6.0950206 -0.2395883
## 21  003-090 2.0686  -14.4890312 -10.4762127  17.7377922   0.7532385 -3.8403306
## 22  003-091 0.0000 -120.1704693  -9.6266439  -2.9547274   2.6918050 -1.1367078
## 23  003-093 0.0000  -10.2568702  -2.4468166  -6.6049314  -5.7241131  3.8162358
## 24  003-094 0.0000  -55.8651823 -21.3462666  11.8291957  13.1202978  2.3042356
## 25  003-095 0.0000   66.8419657  -9.1934024   6.3239597  -5.0000866  1.8553546
## 26  003-096 0.0000  -63.5741502 -26.3608689 -15.5144571  -3.7097125 -1.5559544
## 27  003-097 0.0000    6.3240461  -3.5433982   3.0645768  -3.0521202  3.5556406
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## 28  003-098 0.0000 -106.0439487 -18.8115462  13.7568515  -5.0414568  -6.2597750
## 29  003-099 0.0000  -61.3685049 -43.3926756   1.0295897 -12.5623126 -20.7383420
## 30  003-100 0.0000   -5.2449184   2.8451780   0.2099581  -8.6600784   2.4666823
## 31  003-101 0.0000   45.6094783 -19.4380707  -4.9543956  10.5696351  10.6803246
## 32  003-104 0.0000  -16.7733012  12.7941537  11.4151756  -3.9895015   3.6103907
## 33  003-105 0.0000   31.2154782  -4.0672232   0.7535465   2.7175299  -0.5777874
## 34  003-106 0.0000    3.6344888  19.0419365 -19.9889397  10.5128865   6.0895879
## 35 003-1061 2.2999   30.5415960  -9.8740425   0.6179542  -8.2293352  -2.1746067
## 36 003-1069 1.5965  -97.7004164   8.6241176 -14.0160854   2.9905001   1.9848708
## 37  003-107 0.0000   87.2828606  18.0534305 -19.5708654  -2.0752695  -6.9676230
## 38 003-1070 0.5728   91.7063983   3.6541484  17.7954743   5.8788939  -0.7203922
## 39 003-1073 0.4061   18.4489781 -20.0993177 -10.9505365  -4.5806685  -1.0467096
## 40 003-1077 1.7797    9.4020812 -22.4824730  13.0027178  -0.4527400   2.0375407
## 41  003-108 0.0000   38.9033136 -11.4297094  -6.7131628  -5.0890189  -0.3113591
## 42 003-1085 1.4711  100.8316966 -75.8686730  -0.3711939  12.8778687 -12.2618331
## 43  003-109 0.0000  -24.1798227   8.7593336   7.7833051  -8.8344531   4.9553768
## 44 003-1092 0.0000  -99.4148386  23.0131322  -7.1238140   1.9295373  -8.9496181
## 45 003-1096 0.0000  -45.9683695   6.3036625 -12.2290849  -5.1556930   7.5425747
## 46  003-110 0.0000  -97.4913861  -3.8240677  26.2737239  -2.4758000  13.6188625
## 47 003-1102 0.0000   -0.7010158  -7.3380488 -14.4864206  10.8401500 -15.8422074
## 48 003-1103 0.0000    8.3719518 -20.2580802  13.7272166  14.8134555  -8.4909828
## 49 003-1119 0.0000  -53.7397730 -11.2154809 -13.6738384  -0.3379389 -12.8044544
## 50  003-113 0.0000  -66.1110578  38.5162689  -0.6374550 -13.5756101  -6.8576238
## 51  003-114 0.0000  -50.8249427  54.4731753  -2.1353033  -5.3610386  -2.7418629
## 52  003-115 0.0000  -17.7438764 -15.2577297   9.9550615  -5.8943409   2.8596029
## 53  003-116 0.0000  -39.9590114 -22.7392346   5.8054859  -0.7243319   1.3755688
## 54  003-117 0.0000   -9.1910297  -6.2814593   0.5617281  -8.5255966   4.2950371
## 55  003-118 0.0000  -78.1069330  14.0892442  12.8330819  -6.9333106  -0.9755250
## 56  003-120 0.0000  -64.4687349 -10.4213505 -14.5309482   1.0833440  -0.5904779
## 57 003-2064 2.8165    6.0948362  -5.0752611  -8.1118517  -7.6486121  -3.7540167
##         p_fpc6 p_change    p_auc i_prop_false_timeout i_prop_failed1
## 1  -2.78397805 -39.6005 4.612579          0.024877996     0.07730656
## 2  -2.51031257 -37.5525 3.159882          0.055119931     0.07730656
## 3  -0.50444747 -38.8625 2.984965          0.024877996     0.18639747
## 4  -0.17804853 -32.5775 2.940459          0.055119931     0.16821566
## 5  -8.87420918 -44.0495 4.450467          0.024877996     0.16821566
## 6  -0.11313557 -44.3605 4.237557          0.055119931     0.16821566
## 7   4.19307070 -41.3860 3.992279          0.055119931    -0.01360253
## 8  -6.18474617 -38.0450 3.355522          0.024877996     0.08639747
## 9   2.34178660 -51.5320 5.668523          0.055119931    -0.01360253
## 10 -1.10638044 -29.3685 2.797095          0.055119931     0.40306414
## 11  0.23161869 -28.9560 2.631331          0.083529022     0.07730656
## 12  0.74761198 -41.4605 4.701674          0.024877996    -0.01360253
## 13  4.06953359 -35.4640 3.499287          0.055119931     0.16821566
## 14  0.43853704 -26.7435 2.463968          0.083529022     0.16821566
## 15 -2.33542303 -36.9315 2.488439          0.083529022     0.23639747
## 16 -8.19302206 -45.1745 5.496087          0.083529022     0.25912475
## 17 -3.50765204 -44.8005 4.751305          0.024877996     0.16821566
## 18 -2.60679335 -30.8755 2.968198          0.055119931     0.16821566
## 19  2.44855108 -40.5720 3.537008          0.024877996     0.07730656
## 20 -0.56793335 -43.4240 5.304655          0.110266990     0.25912475
## 21  2.92275098 -39.9535 3.374157          0.135477074     0.15306414
## 22 -4.15669753 -51.2970 5.973513          0.083529022     0.16821566
## 23 -5.27115099 -44.2615 5.011917          0.024877996     0.07730656
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## 24  4.01567831 -47.5540  4.863039          0.083529022     0.25912475
## 25 -2.75838931 -37.1385  3.628735          0.024877996    -0.01360253
## 26 -3.29493586 -51.0835  5.769163          0.024877996     0.07730656
## 27 -1.64622729 -42.9655  3.389351         -0.007380069    -0.01360253
## 28 -0.39595926 -50.8760  6.048001          0.055119931     0.07730656
## 29 -3.47400334 -49.4390  3.951179         -0.007380069    -0.01360253
## 30  0.80304642 -53.8840 11.572640         -0.007380069     0.28639747
## 31  3.70372683 -40.9500  3.945099         -0.007380069     0.08639747
## 32 -1.89699705 -42.3185  4.364456          0.024877996     0.07730656
## 33 -4.06736558 -38.3940  3.149497          0.055119931    -0.01360253
## 34 -8.58670475 -39.1430  3.007768          0.055119931     0.07730656
## 35  0.18543582 -39.9725  4.974833          0.055119931     0.15306414
## 36 -9.71028190 -47.2700  5.350628          0.055119931     0.07730656
## 37  1.92263112 -32.9855  2.935934          0.055119931     0.07730656
## 38  2.19478114 -35.5625  3.195690          0.260912614     0.41496890
## 39 -2.75285802 -42.9500  3.909394          0.024877996     0.08639747
## 40 -0.46710250 -42.0975  3.334271          0.024877996     0.16821566
## 41  1.20227002 -41.4800  3.396440          0.024877996     0.07730656
## 42  5.37007398 -58.1730  8.746051          0.024877996     0.07730656
## 43  8.27189119 -43.1490  5.045051          0.024877996     0.07730656
## 44 -8.59432359 -44.5045  5.380730          0.024877996    -0.01360253
## 45 -0.01436351 -46.1245  3.673991         -0.007380069     0.08639747
## 46  7.90232633 -49.9570  6.505343          0.083529022     0.21716670
## 47 -0.04484710 -40.5260  4.761744          0.055119931     0.15306414
## 48  6.19813031 -40.3540  4.630392          0.055119931     0.18639747
## 49  4.65333816 -47.6750  4.803467          0.110266990     0.08639747
## 50 -2.05168811 -45.0695  5.529803          0.055119931     0.07730656
## 51  0.38867314 -40.5285  5.447642          0.055119931     0.23639747
## 52  4.52701784 -45.2595  3.799721          0.083529022    -0.01360253
## 53 -2.08194589 -46.2810  3.610533          0.055119931     0.25912475
## 54 11.75506795 -45.5440  4.575363         -0.007380069    -0.01360253
## 55 -0.52988521 -44.8495  6.083528          0.083529022     0.15306414
## 56  2.20332470 -48.7380  5.253996          0.083529022     0.18639747
## 57  2.83765873 -43.1235  3.683714         -0.007380069     0.08639747
##    i_prop_failed2 i_judgement_time1 i_judgement_time2 i_time_outside_reticle
## 1     0.104123028         2.0425384          2.181320                86.3471
## 2     0.242218266         1.4679495          1.713505                67.4071
## 3     0.289837313         1.4292301          1.992735                69.7671
## 4     0.242218266         1.5510811          1.927601                90.5071
## 5     0.054123028         2.3168185          2.200560                85.6871
## 6     0.242218266         1.4201257          1.764344                72.9271
## 7     0.185941209         2.7054249          4.705862                91.6471
## 8     0.194599218         1.7290852          2.453758                74.6871
## 9     0.276850300         1.7833510          1.855359                55.1671
## 10    0.104123028         1.8732941          2.433432                67.7271
## 11    0.231395755         1.9123911          2.005059                62.3271
## 12    0.146980170         1.7589637          1.899595                89.3871
## 13    0.099361123         1.7228055          2.253858                45.8671
## 14    0.322304846         1.5668490          1.980444                57.2271
## 15    0.385075408         1.4976394          1.560194                76.8671
## 16    0.185941209         1.5391295          1.874605                59.1271
## 17    0.054123028         2.1427780          2.552795               112.3881
## 18    0.051742075         2.4879429          3.091939                69.2071
## 19    0.054123028         1.4991420          1.625021                64.5871
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## 20   0.351949114       1.7807143        2.132290              97.0271
## 21   0.221514332       1.8701211        2.213456              51.9871
## 22   0.367759391       1.2379519        1.359632              92.0271
## 23   0.154123028       1.8409841        2.294033              75.2671
## 24   0.276850300       1.3937740        1.869285              92.0871
## 25   0.194599218       1.5253943        2.053864              90.6871
## 26   0.054123028       1.6261459        2.009238              58.8271
## 27   0.104123028       2.1597202        2.012721              79.3871
## 28   0.099361123       2.0163360        2.030557              72.6471
## 29   0.204123028       1.4210044        1.643242             100.2681
## 30   0.254123028       2.0789429        2.083909              68.8471
## 31   0.054123028       2.0744397        2.231037              82.6071
## 32   0.104123028       1.7005046        2.147479              61.0471
## 33   0.185941209       3.2215917        3.115062              82.6271
## 34   0.194599218       1.9307331        2.196651              83.0871
## 35   0.054123028       1.5151706        1.899893              48.1871
## 36   0.099361123       1.5923364        1.882359              50.8071
## 37   0.146980170       1.7703297        1.808255              85.9471
## 38   0.670789694       1.3027024        2.449825              59.4071
## 39   0.194599218       1.6512536        1.948031              81.5071
## 40   0.004123028       1.3807838        1.683807              67.0671
## 41   0.154123028       1.9903299        2.363125              82.2671
## 42   0.104123028       1.9968060        1.858063              81.4071
## 43   0.254123028       0.8939648        1.180202              53.5071
## 44   0.146980170       1.4849496        1.711772              69.8871
## 45   0.104123028       1.9549828        3.150737              99.6271
## 46   0.104123028       1.4619597        2.003340              63.8271
## 47   0.104123028       1.3877304        1.667908              99.4471
## 48   0.413213937       1.5846403        1.637861              72.4471
## 49   0.379123028       1.7463111        1.818980              87.3271
## 50   0.242218266       1.9934294        2.268524              84.2271
## 51   0.104123028       1.8187444        2.129077              81.2271
## 52   0.438905636       1.4902916        1.641259              89.8271
## 53   0.194599218       1.5956151        1.749682              51.2471
## 54   0.254123028       1.6275586        1.801289              82.2871
## 55   0.099361123       1.5663961        1.841905              87.1271
## 56   0.264992593       1.3394260        1.735525              81.4871
## 57   0.104123028       2.1134706        1.925280              58.4071
##    i_time_on_edge i_prop_hit i_correct_reaction2 i_reaction_time_max2
## 1        2.369032 0.03792502           0.9970116            0.7067039
## 2        1.849032 0.06495205           0.9970116            0.4872479
## 3        1.629032 0.10549259           0.9061025            0.6378119
## 4        4.149032 0.10549259           0.9493925            0.5705579
## 5        2.289032 0.09197908           0.9970116            0.5647169
## 6        2.009032 0.03792502           0.9970116            0.5867559
## 7        3.129032 0.02441151           0.9970116            0.9398119
## 8        3.329032 0.11900610           0.9061025            0.6682069
## 9        5.109032 0.07846556           0.9017735            0.6687899
## 10       1.629032 0.06495205           0.9970116            0.6057609
## 11       1.629032 0.18657367           0.9970116            0.5009199
## 12       1.629032 0.06495205           0.9970116            0.6685699
## 13       2.429032 0.10549259           0.9970116            0.7555149
## 14       1.749032 0.07846556           0.9470116            0.5706909
## 15       2.709032 0.15954664           0.9470116            0.5856979
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## 16       3.169032 0.11900610          0.9970116           0.9218169
## 17      11.369032 0.06495205          0.9470116           0.6199409
## 18       1.769032 0.05143854          0.9970116           0.8709439
## 19       2.809032 0.06495205          0.9970116           0.5700729
## 20       5.589032 0.03792502          0.8151934           0.7081819
## 21       2.869032 0.24062772          0.8970116           0.5479619
## 22       2.709032 0.02441151          0.9017735           0.6509809
## 23       3.229032 0.05143854          0.9470116           0.7525539
## 24       2.889032 0.10549259          0.9970116           0.5756819
## 25       4.509032 0.06495205          0.9970116           0.7837699
## 26       2.109032 0.02441151          0.9970116           0.8048449
## 27       1.869032 0.06495205          0.9493925           0.6695209
## 28       1.629032 0.07846556          0.9470116           0.6209449
## 29      11.669032 0.03792502          0.9970116           0.8568719
## 30       1.669032 0.06495205          0.8541544           0.5897089
## 31       1.969032 0.02441151          0.9970116           0.6060809
## 32       3.229032 0.02441151          0.9493925           0.9059479
## 33       2.189032 0.06495205          0.9970116           0.6058339
## 34       2.449032 0.05143854          0.9970116           0.7519159
## 35       3.749032 0.03792502          0.9970116           0.7088219
## 36       4.529032 0.06495205          0.9493925           0.8567169
## 37       2.269032 0.10549259          0.9970116           0.6014869
## 38       2.249032 0.10549259          0.8665768           0.7860879
## 39       1.629032 0.05143854          0.9493925           0.5875789
## 40       3.069032 0.01089799          0.9970116           0.4857909
## 41       2.569032 0.05143854          0.9970116           0.6318069
## 42       3.129032 0.01089799          0.9970116           0.9061889
## 43       1.909032 0.09197908          0.9970116           0.5521699
## 44       3.649032 0.02441151          0.9970116           0.9025249
## 45       2.329032 0.06495205          0.8606480           0.7064369
## 46       3.709032 0.05143854          0.9493925           0.5291649
## 47       6.569032 0.02441151          0.9493925           0.5733419
## 48       6.969032 0.03792502          0.9493925           0.5092119
## 49       4.109032 0.11900610          0.9061025           0.5602229
## 50       1.629032 0.02441151          0.9970116           0.8374609
## 51       2.289032 0.07846556          0.9470116           0.6378839
## 52       7.809032 0.02441151          0.9970116           0.4893999
## 53       1.729032 0.10549259          0.9493925           0.8700909
## 54       1.729032 0.07846556          0.9970116           0.5536949
## 55       1.829032 0.02441151          0.9470116           0.9033769
## 56       4.409032 0.05143854          0.8970116           0.7197779
## 57       2.669032 0.07846556          0.9017735           0.8709499
##    i_reaction_time2 i_rep_shapes12 i_rep_shapes34 i_memory_time12
## 1         0.4757025       5.774194      8.7741935        3.429678
## 2         0.4078134       5.774194      5.7741935        3.122255
## 3         0.4850229       5.774194      4.7741935        4.497671
## 4         0.4658847       6.774194     10.7741935        4.048908
## 5         0.4538427       7.774194      6.7741935        3.266354
## 6         0.4008222       7.774194     13.7741935        3.328021
## 7         0.5789992       5.774194      2.7741935        3.136153
## 8         0.5309624       5.774194      0.7741935        5.283331
## 9         0.4967265       6.774194      5.7741935        3.907191
## 10        0.5471573       7.774194      5.7741935        3.638236
## 11        0.4341575       5.774194      5.7741935        2.841710
```
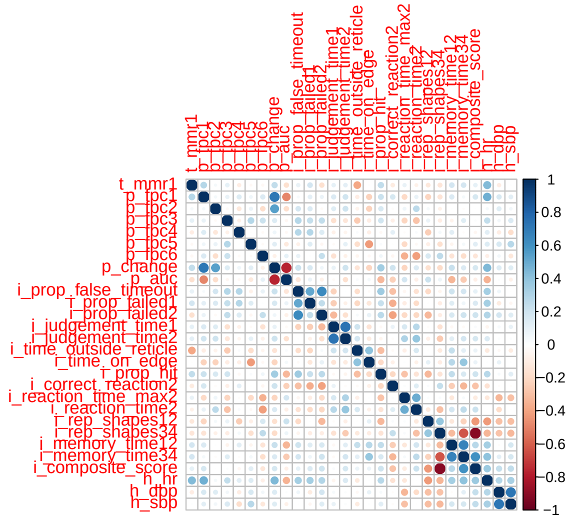

```
## 12        0.5263437       6.774194      7.7741935        2.714405
## 13        0.5174524       7.774194     14.7741935        3.184903
## 14        0.5045017       7.774194      2.7741935        3.259269
## 15        0.4730478       3.774194      0.7741935        4.390233
## 16        0.5421488       5.774194      6.7741935        4.110404
## 17        0.4637707       7.774194      5.7741935        4.267852
## 18        0.5160792       7.774194      6.7741935        4.724653
## 19        0.4583461       7.774194      2.7741935        3.173006
## 20        0.4417032       7.774194      4.7741935        4.845613
## 21        0.4087126       7.774194      7.7741935        4.690278
## 22        0.5214911       7.774194      7.7741935        3.733862
## 23        0.4527890       7.774194     10.7741935        2.787701
## 24        0.4423840       6.774194      9.7741935        3.910439
## 25        0.5571245       7.774194     12.7741935        3.535803
## 26        0.4676254       7.774194     12.7741935        2.761508
## 27        0.5365965       7.774194     11.7741935        3.259900
## 28        0.5065523       7.774194      5.7741935        3.471808
## 29        0.4877143       7.774194      7.7741935        3.518020
## 30        0.4661365       7.774194      4.7741935        2.289443
## 31        0.4647178       7.774194      8.7741935        2.960975
## 32        0.5047758       6.774194      4.7741935        2.576139
## 33        0.4573178       6.774194     10.7741935        3.610685
## 34        0.5909765       5.774194      3.7741935        4.246280
## 35        0.4632924       7.774194      7.7741935        3.637354
## 36        0.5233035       6.774194     10.7741935        3.358070
## 37        0.5187446       7.774194     14.7741935        4.093477
## 38        0.4551417       6.774194     10.7741935        3.842554
## 39        0.4701870       5.774194     14.7741935        3.195443
## 40        0.4090367       7.774194     14.7741935        3.244414
## 41        0.5462173       7.774194      8.7741935        4.246932
## 42        0.4923538       6.774194     11.7741935        3.371349
## 43        0.4243286       7.774194     13.7741935        3.969437
## 44        0.5085567       7.774194      9.7741935        3.250444
## 45        0.5075751       7.774194      6.7741935        4.943933
## 46        0.4076595       7.774194     12.7741935        3.059644
## 47        0.4867397       7.774194      7.7741935        5.077424
## 48        0.4211613       6.774194      9.7741935        3.191049
## 49        0.4557111       5.774194     11.7741935        3.021014
## 50        0.4907262       7.774194     12.7741935        3.293118
## 51        0.5053581       7.774194     10.7741935        3.269649
## 52        0.4506646       7.774194      9.7741935        4.446578
## 53        0.4455662       7.774194     14.7741935        2.851596
## 54        0.4075373       7.774194     12.7741935        2.607169
## 55        0.4838153       7.774194      8.7741935        2.267751
## 56        0.4307729       6.774194      6.7741935        3.419646
## 57        0.4452946       7.774194     12.7741935        3.162909
##    i_memory_time34 i_composite_score h_hr h_dbp h_sbp
## 1         3.085852        0.34723899  102    86   129
## 2         3.277464        0.40265008  100    92   137
## 3         5.769673        0.43372608  130    66   126
## 4         5.298267        0.31376304  125    88   123
## 5         3.757668        0.37807672   80    70   130
## 6         3.357675        0.14627954   82    76   122
## 7         4.906614        0.64464797   91    82   119
```

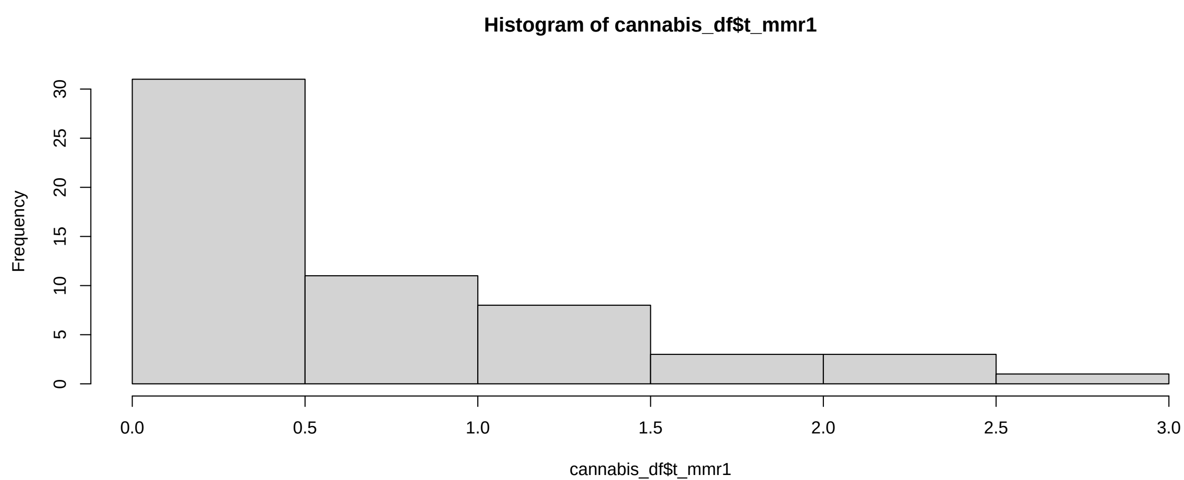
```
## 8          6.777051         0.72245379  86    85   126
## 9          6.813843         0.59973256  98    93   140
## 10         4.383347         0.47496454  94    69   114
## 11         3.358378         0.47895922 101    94   129
## 12         4.180772         0.27789492  81    91   123
## 13         2.904177         0.07646019  85    77   127
## 14         5.625092         0.62315491  97    74   120
## 15         5.703390         0.78596592 121    97   150
## 16         5.704878         0.41008723 100    74   126
## 17         5.928912         0.38811002 101    82   112
## 18         4.945369         0.22561039 132    88   143
## 19         4.356081         0.68131115  87    93   130
## 20         6.543360         0.39853018  74    83   122
## 21         4.980374         0.22656969  93    77   120
## 22         3.674429         0.45303732  77    64   115
## 23         3.130487         0.24586741  67   102   127
## 24         3.908670         0.32575643  64    71   110
## 25         3.011242         0.16469832  67    60    99
## 26         4.118355         0.13785219  53    80   116
## 27         4.194431         0.13592609  64    85   130
## 28         4.797056         0.45127056  60    66   113
## 29         5.601424         0.35509707  58    62   109
## 30         2.466948         0.48117700  71    97   143
## 31         3.781201         0.25471375  77    74    99
## 32         3.930807         0.31850087  63    97   135
## 33         3.533379         0.20108454  51    80   128
## 34         5.197804         0.38354358  55    72   118
## 35         4.577384         0.41287263  82    75   116
## 36         3.736490         0.18217328  92    67   116
## 37         3.879693         0.20048013  55    72   114
## 38         5.018183         0.30739040 142    78   104
## 39         3.137835         0.08628805  96    86   143
## 40         3.336901         0.07111082 100    77   109
## 41         4.279615         0.21470266  86    78   124
## 42         3.251638         0.16376385  80    66   104
## 43         3.668781         0.14725507  52    78   118
## 44         4.888513         0.18685552  74    83   113
## 45         5.908538         0.50895861  85    65   118
## 46         3.866787         0.11790361  51    81   122
## 47         4.273228         0.34123766  69    83   111
## 48         5.185436         0.40299334  94    88   121
## 49         3.484645         0.22943829  71    80   107
## 50         2.996573         0.12140902  55    72    93
## 51         3.295798         0.19904690  65    76   127
## 52         4.293125         0.16637723  86   105   173
## 53         2.481974         0.09807329  51    61   111
## 54         2.822329         0.14585502  62    72   101
## 55         3.468070         0.28107428  60    62   107
## 56         3.892531         0.49138104  76    81   109
## 57         3.288303         0.14502419  64    54    96
```

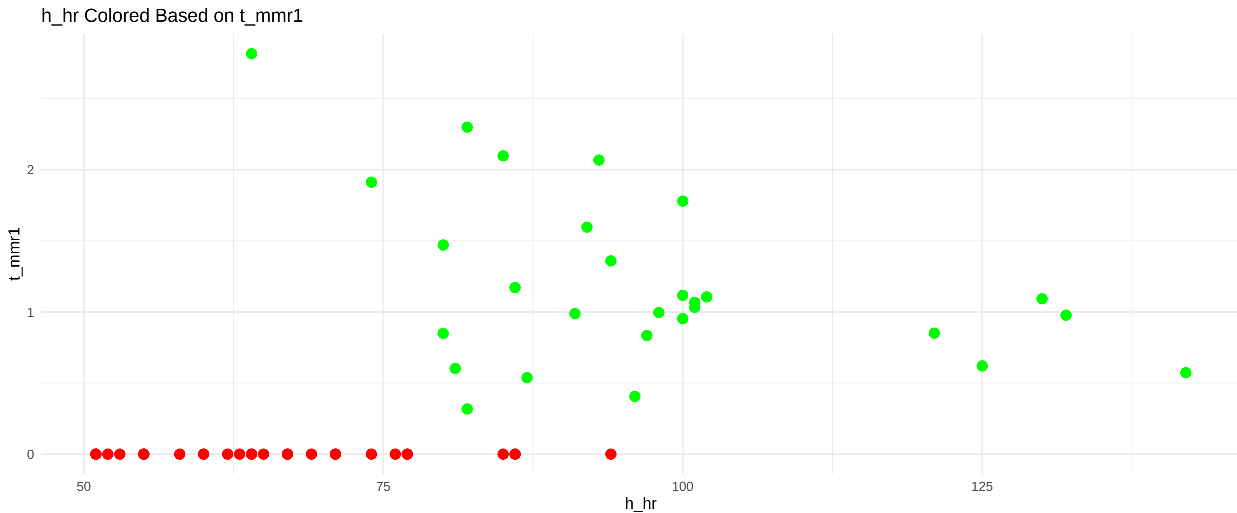


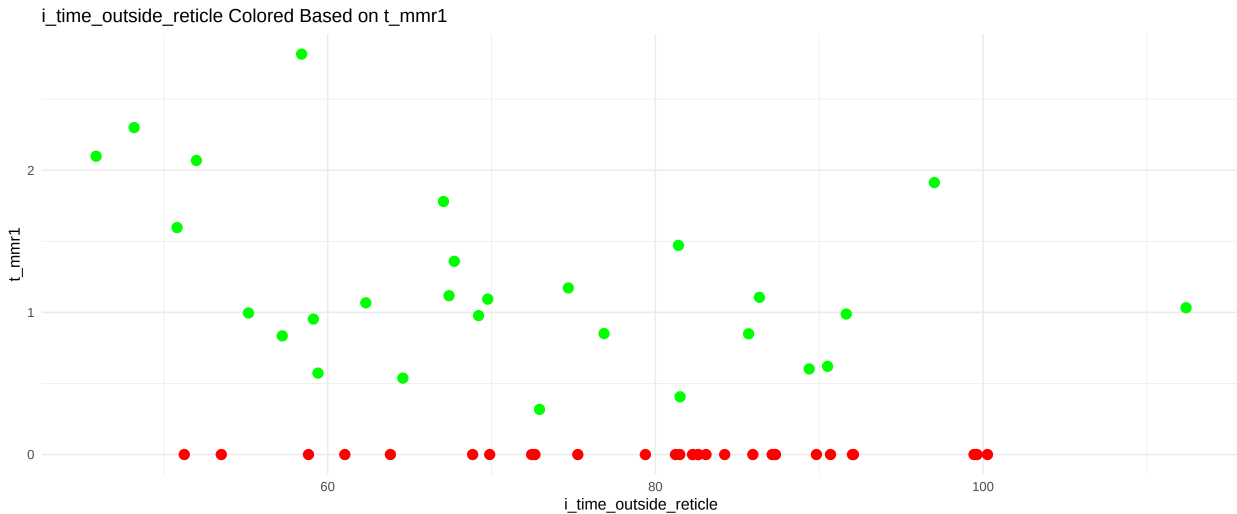
var1	var2	correlation
p__fpc1	p__change	0.7166244
p__fpc2	p__change	0.5491598
p__change	p__auc	-0.7610100
i__prop__false__timeout	i__prop__failed1	0.5120145
i__prop__false__timeout	i__prop__failed2	0.6399012
i__judgement__time1	i__judgement__time2	0.7398758
i__rep__shapes34	i__memory__time34	-0.6207037
i__memory__time12	i__memory__time34	0.6629413
i__rep__shapes34	i__composite__score	-0.9010134
i__memory__time34	i__composite__score	0.6029671
h__dbp	h__sbp	0.7215615

```
## [1] 57
```

```
## [1] 27
```







n is 57 for the 57 individuals in the dataset p is 27 for the number of predictors (columns in the dataset)

The distribution of the outcome is highly right skewed - looking at the histogram, it looks like the majority of values are clustered by a metabolite molar ratio of THC of 0.

Problem 2: Quantile regression

Use linear programming to estimate the coefficients for a quantile regression. You need to write a function named `my_rq`, which takes a response vector y, a covariate matrix X and quantile τ , and returns the estimated coefficients. Existing linear programming functions can be used directly to solve the LP problem (for example, `simplex` function in the `boot` package, or `lp` function in the `lpSolve` package).

- Use your function to model `t_mmr1` from the cannabis data using `p_change` (percent change in pupil diameter in response to light), `h_hr` (heart rate), and `i_composite_score` (a composite score of the ipad variables) as variables.
- Compare your results with though estimated using the `rq` function in R at quantiles $\tau \in \{0.25, 0.5, 0.75\}$.
- Compare with mean obtain using linear regression
- Summarize findings

When explaining your results, be sure to explain what LP method you used for estimating quantile regression.

My Implementation

```
## # A tibble: 3 x 5
##     tau intercept      x2      x3     x4
##   <dbl>     <dbl>   <dbl>   <dbl>  <dbl>
## 1  0.25    -0.150 0.0114  0.00818  0.384
## 2  0.5     -0.283 0.0122  0.0136   0.198
## 3  0.75    -1.11  0.00418 0.0273  -0.581
```

Using rq() in R

```
## # A tibble: 3 x 5
##     tau `(Intercept)` p_change    h_hr i_composite_score
##   <dbl>         <dbl>    <dbl>   <dbl>             <dbl>
## 1  0.25        -0.150  0.0114  0.00818             0.384
## 2  0.5         -0.283  0.0122  0.0136              0.198
## 3  0.75        -1.11   0.00418 0.0273             -0.581
```

OLS

```
##                      Estimate  Std. Error    t value    Pr(>|t|)
## (Intercept)       -0.17879290 0.927695788 -0.1927279 0.847908663
## p_change           0.00766679 0.016245872  0.4719223 0.638919448
## h_hr               0.01435096 0.004922448  2.9154103 0.005195576
## i_composite_score -0.23822747 0.556914927 -0.4277628 0.670556997
```

```
## # A tibble: 1 x 4
##   `(Intercept)` p_change   h_hr i_composite_score
##           <dbl>    <dbl>  <dbl>             <dbl>
## 1        -0.179  0.00767 0.0144            -0.238
```

Problem 3: Implementation of LASSO

As illustrated in class, a LASSO problem can be rewritten as a quadratic programming problem.

1. Many widely used QP solvers require that the matrix in the quadratic function for the second order term to be positive definite (such as `solve.QP` in the `quadprog` package). Rewrite the quadratic programming problem for LASSO in matrix form and show that the matrix is not positive definite, thus QP solvers like `solve.QP` cannot be used.

From slide 27 of the lecture slides, we are trying to max:

$$\max_{\beta_j^+,\beta_j^-} -\sum_i \left(y_i - \sum_j \beta_j^+ x_{ij} + \sum_j \beta_j^- x_{ij}\right)^2 \text{ s.t.} \sum_j \left(\beta_j^+ + \beta_j^-\right) \leq \lambda, \beta_j^+, \beta_j^- \geq 0$$

If we factor the x_{ij} , we get

$$-\sum_i \left(y_i - \left[x_{ij}(\sum_j (\beta_j^+ + \beta_j^-)\right]\right)^2$$

Putting it into matrix form and redistributing, we get:

$$(Y - \left[X(\beta_j^+ - \beta_j^-)\right])^T(Y - \left[X(\beta_j^+ - \beta_j^-)\right])$$

$$(Y - \left[X\beta_j^+ - X\beta_j^-\right])^T(Y - \left[X\beta_j^+ - X\beta_j^-\right])$$

Defining new matrices:

$$X' = \begin{bmatrix} X \\ -X \end{bmatrix}$$

$$\beta' = \begin{bmatrix} \beta_j^+ \\ \beta_j^- \end{bmatrix}$$

we can rewrite as:

$$(Y - X'\beta')^T(Y - X'\beta')$$

Expanding, we can see that $\beta'^T X'^T X'\beta'$ is the second order term. Substituting back in our matrix definition of X', we see that:

$$\begin{aligned}(Y - X'\beta')^T(Y - X'\beta') &= Y^TY - 2Y^TX'\beta' + \beta'^TX'^TX'\beta' \\ &= Y^TY - 2Y^TX'\beta' + \beta'^T \begin{bmatrix} X^TX & -X^TX \\ -X^TX & X^TX \end{bmatrix} \beta'\end{aligned}$$

We can see that the determinant of this matrix is going to be 0 - by definition, the determinant of a positive definite matrix is not 0, hence, this matrix is not positive definite.

2. The `LowRankQP` function in the `LowRankQP` package can handle the non positive definite situation. Use the matrix format you derived above and `LowRankQP` to write your own function `my_lasso()` to estimate the coefficients for a LASSO problem. Your function needs to take three parameters: Y (response), X (predictor), and *lambda* (tuning parameter), and return the estimated coefficients.

- Use your function to model `log(t_mmr1)` from the cannabis data using all other variables as potential covariates in the model
- Compare your results with those estimated using the `cv.glmnet` function in R from the `glmnet` package
- Summarize findings

The results will not be exactly the same because the estimation procedures are different, but trends (which variables are selected) should be similar.