**NYU Computer Science Bridge to Tandon Course** Winter 2021

Homework 1

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# Question 1

A1) $10011011_2$

$= 1 + 1 \cdot 2^1 + 1 \cdot 2^3 + 1 \cdot 2^4 + 1 \cdot 2^7$
$= 1 + 2 + 8 + 16 + 128$
$= (155)_{10}$

A2) $456_7$

$= 6 \cdot 7^0 + 5 \cdot 7^1 + 4 \cdot 7^2$
$= 6 + 35 + 196$
$= (237)_{10}$

A3) $38A_{16}$

$= 10 \cdot 16_0 + 8 \cdot 16^1 + 3 \cdot 16^2$
$= 10 + 128 + 768$
$= (906)_{10}$

A4) $2214_5$

$= 4 \cdot 5_0 + 1 \cdot 5^1 + 2 \cdot 5^2 + 2 \cdot 5^3$
$= (309)_{10}$

B1) $69_{10}$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\underline{0}$ | $\underline{1}$ | $\underline{0}$ | $\underline{0}$ | $\underline{0}$ | $\underline{1}$ | $\underline{0}$ | $\underline{1}$ |
| $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |

$= 64 + 4 + 1$
$= 69$
$= 1000101_2$

B2) $485_{10}$

| 456 | 229 | 101 | 37 | 5 | 1 |
|---|---|---|---|---|---|
| -256 | -128 | -64 | -32 | -4 | -1 |
| 229 | 101 | 37 | 5 | 1 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| $2^8$ | $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
| 256 | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |

$= 111100101_2$

B3) $6D1A_{16}$

$= 10 \cdot 16^0 + 1 \cdot 16^1 + 13 \cdot 16^2 + 6 \cdot 6^3$
$= 10 + 16 + 3328 + 24576$
$= 27930_{10}$

| 27930 | 11546 | 3354 | 1306 | 282 | 26 | 10 | 2 |
|---|---|---|---|---|---|---|---|
| -16384 | -8192 | -2048 | -1024 | -256 | 16 | 8 | 2 |
| 11546 | 3354 | 1306 | 282 | 26 | 10 | 2 | 0 |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| $2^{15}$ | $2^{14}$ | $2^{13}$ | $2^{12}$ | $2^{11}$ | $2^{10}$ | $2^9$ | $2^8$ | $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
| 32768 | 16384 | 8192 | 4096 | 2048 | 1024 | 512 | 256 | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |

$= 0110110100011010_2$

C1) $1101011_2$

| | |
|---|---|
| 0110 | 1011 |
| $= 2^2 + 2^1$ | $= 2^3 + 2^1 + 2^0$ |
| $= 4 + 2$ | $= 8 + 2 + 1$ |
| $= 6$ | $= 11 \Longrightarrow B$ |

$= 6B_{16}$

C2) $895_10$

| | |
|---|---|
| $895 \div 16$ | $= 55$ R15 $\Longrightarrow F$ |
| $55 \div 16$ | $= 3$ R7 |
| $3 \div 16$ | $= 0$ R3 |

$= 37F_{16}$

## Question2

1)$7266_8 + 4515_8$

$$\begin{array}{l} \ \ 7566_8 \\ +4515_8 \\ \hline =14303_8 \end{array}$$

2) $10110011_2 + 1101_2$

$$\begin{array}{l} \ \ \ \ \ \ ^{111111} \\ \ \ 10110011_2 \\ +00001101_2 \\ \hline =11000002_2 \end{array}$$

3) $7\mathrm{A}66_16 + 45C5_16$

$$\begin{array}{l} \ \ 7\ 10\ 06\ 06_{16} \\ +4\ 05\ 12\ 05_{16} \\ \hline =12\ 0\ 02\ 11_{16} \end{array}$$

$\Longrightarrow C02B_{16}$

4) $3022_5 + 2433_5$

$$\begin{array}{l} \ \ 3\ 0\ 2\ 2_5 \\ \text{-}2\ 4\ 3\ 3_5 \\ \hline =0\ 0\ 3\ 4_5 \end{array}$$

$\Longrightarrow 34_5$

## Question 3

$A1) 124_{10}$

| | | | | |
|---|---|---|---|---|
| 124 | 60 | 28 | 12 | 4 |
| -64 | -32 | -16 | -8 | -4 |
| =60 | =28 | =12 | =4 | =0 |
| $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ |

$= 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0_{8-bit\ 2`s\ comp}$

A2) $-124_{10}$

$$\begin{array}{r} {\scriptstyle 11111\phantom{000}} \\ 01101100 \\ +1000\ 0100 \\ \hline =1000\ 0000_{8-bit\ 2`s\ comp} \end{array}$$

A3) $109_{10}$

| | | | | |
|---|---|---|---|---|
| 109 | 45 | 18 | 5 | 1 |
| -64 | -32 | -8 | -4 | -1 |
| =45 | =13 | =5 | =1 | =0 |
| $2^6$ | $2^5$ | $2^3$ | $2^2$ | $2^0$ |

$= 0\ 1\ 1\ 0\ 1\ 1\ 0\ 1_{8-bit\ 2`s\ comp}$

A4) $-79_{10}$

$\implies convert\ to\ 79_{10}$

| | | | | |
|---|---|---|---|---|
| 799 | 15 | 7 | 3 | 1 |
| -64 | -8 | -4 | -2 | -1 |
| =15 | =7 | =3 | =1 | =0 |
| $2^6$ | $2^3$ | $2^1$ | $2^1$ | $2^0$ |

$= 0\ 1\ 0\ 0\ 1\ 1\ 1\ 1$

$$\begin{array}{r} 0\ 1\ 0\ 0\ 1\ 1\ 1\ 1 \\ +1\ 0\ 1\ 1\ 0\ 0\ 0\ 1 \\ \hline 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \end{array}$$

$= 1\ 0\ 1\ 1\ 0\ 0\ 0\ 1_{8-bit\ 2`s\ comp}$

B1) $00011110_{8-bit\ 2`s\ comp}$

$= 1{\cdot}2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4$
$= 2 + 4 = 8 + 16$
$= 30_{10}$

B2) $11100110_{8-bit\ 2`s\ comp}$

$$\begin{array}{r} 1\ 1\ 1\ 0\ 0\ 1\ 1\ 0 \\ +0\ 0\ 0\ 1\ 1\ 0\ 1\ 0 \\ \hline 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \end{array}$$

$0\ 0\ 0\ 1\ 1\ 0\ 1\ 0_{8-bit\ 2`s\ comp=}$

$= 1{\cdot}2^1 + 1 \cdot 2^3 + 1 \cdot 2^4$
$= 2 + 8 + 16$
$= 26_{10}$

$1\ 1\ 1\ 0\ 0\ 1\ 1\ 0_{8-bit\ 2`s\ comp=}$

$=$ -$26_{10}$

B3) $00101101_{8-bit\ 2`s\ comp}$

$= 1{\cdot}2^0 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^5$
$= 1 + 4 + 8 + 32$
$= 45_{10}$

B4)$10011110_{8-bit\ 2`s\ comp}$

$$\begin{array}{r} 1\ 0\ 0\ 1\ 1\ 1\ 1\ 0 \\ +0\ 1\ 1\ 0\ 0\ 1\ 0 \\ \hline 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \end{array}$$

$0\ 1\ 1\ 0\ 0\ 0\ 1\ 0_{8-bit\ 2`s\ comp=}$

$= 1{\cdot}2^1 + 1 \cdot 2^5 + 1 \cdot 2^6$
$= 2 + 32 + 64$
$= 98_{10}$

$1\ 0\ 0\ 1\ 1\ 1\ 1\ 0_{8-bit\ 2`s\ comp=}$

$=$ -$98_{10}$

## Question 4

1b) $\neg(p \vee q)$

| p | q | $\neg(p \vee q)$ |
|---|---|---|
| T | T | $\neg T \vee T \implies F$ |
| T | F | $\neg T \vee F \implies F$ |
| F | T | $\neg F \vee T \implies F$ |
| F | F | $\neg F \vee F \implies T$ |

1c) r $\vee(p \wedge \neg q)$

| p | q | r | (p $\wedge \neg q$) | r$\vee(p \wedge \neg q)$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | T | F | F | F |
| T | F | T | T | T |
| T | F | F | T | T |
| F | T | T | F | T |
| F | T | F | F | F |
| F | F | T | F | T |
| F | F | F | F | F |

2b) (p $\rightarrow q) \rightarrow (q \rightarrow p)$

| p | q | (p$\rightarrow q$) | (q $\rightarrow p$) | (p$\rightarrow q) \rightarrow (q \rightarrow p)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | F |
| F | F | T | T | T |

2d) (p $\leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

| p | q | (p$\leftrightarrow q$) | (p $\leftrightarrow \neg q$) | (p$\leftrightarrow q) \oplus (p \leftrightarrow \neg q)$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | T | F | T |

## Queston 5

1b) The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

$= (\boldsymbol{B} \wedge \boldsymbol{D}) \vee (\boldsymbol{B} \wedge \boldsymbol{M}) \vee (\boldsymbol{D} \wedge \boldsymbol{M})$

1c) Applicant must present either a birth certificate or both a driver's license and a marriage license.

$= \boldsymbol{B} \vee (\boldsymbol{D} \wedge \boldsymbol{M})$

2b) A person can park in the school parking lot if they are a senior or at least seventeen years of age.

$= \mathrm{p} \rightarrow (s \vee y)$

2c) Being 17 years of age is a necessary condition for being able to park in the school parking lot.

$= \mathrm{p} \rightarrow y$

2d) A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

$= \mathrm{p} \leftrightarrow (s \wedge y)$

2e) Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

$= \mathrm{p} \rightarrow (s \vee y)$

3c) The applicant can enroll in the course only if the applicant has parental permission.

$= \mathrm{c} \rightarrow p$

3d) Having parental permission is a necessary condition for enrolling in the course.

$= \mathrm{c} \rightarrow p$

## Question 6

1b) Maintaining a B average is necessary for Joe to be eligible for the honors program.

If Joe wants to be eligible for the program, then maintaining a B average is necessary.

1c) Rajiv can go on the roller coaster only if he is at least four feet tall.

= If Rajiv goes on the roller coaster, then he is at least four feet tall.

1d) Rajiv can go on the roller coaster if he is at-least four feet tall.

= If Rajiv is at-least four feet tall, then he can go on the roller coaster.

2c) (p $\vee r) \leftrightarrow (q \wedge r)$
$p : T$
$q : F$
$r : Unknown$

| p | q | r | (p $\vee r$) | (q $\wedge r$) | (p $\vee r) \leftrightarrow (q \wedge r)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | T | F | T | F | F |
| T | F | T | T | F | F |
| T | F | F | T | F | F |
| F | T | T | T | T | T |
| F | T | F | T | F | F |
| F | F | T | T | F | F |
| F | F | F | T | F | F |

False.

2d) (p $\wedge r) \leftrightarrow (q \wedge r)$

| p | q | r | (p $\wedge r$) | (q $\wedge r$) | (p $\wedge r) \leftrightarrow (q \wedge r)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | T | F | F | F | F |
| T | F | T | T | F | F |
| T | F | F | F | F | F |
| F | T | T | F | T | T |
| F | T | F | F | F | F |
| F | F | T | F | F | F |
| F | F | F | F | F | F |

Unknown. When "r" is T, then conclusion is F; When "r" is F, then the conclusion is T.

2e) p $\rightarrow (r \vee q)$

| p | q | r | (r $\wedge q$) | p $\rightarrow (r \vee q)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | F | F |
| T | F | T | F | F |
| T | F | F | F | F |
| F | T | T | T | T |
| F | T | F | F | T |
| F | F | T | F | T |
| F | F | F | F | T |

Unknown. When "r" is T, then conclusion is T; When "r" is F, then the conclusion is T.

2f) (p $\wedge q) \rightarrow r$

| p | q | r | (p $\wedge q$) | (p $\wedge q) \rightarrow r$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | T | F |
| T | F | T | F | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | T | F | F | T |
| F | F | T | F | T |
| F | F | F | F | T |

True. The hypothesis is true and the conclusion is True.

## Question 7

- j: Sally got the job.
- l: Sally was late for her interview
- r: Sally updated her resume.

1b) If Sally did not get the job, then she was late for interview or did not update her resume.
$\neg j \rightarrow (l \vee \neg r)$

If Sally updated her resume and was not late for her interview, then she got the job.
(r $\wedge \neg l) \rightarrow j$

| j | l | r | $\neg j \rightarrow (l \vee \neg r)$ | (r $\wedge \neg l) \rightarrow j$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | T | T |
| T | F | T | T | T |
| T | F | F | T | T |
| F | T | T | T | T |
| F | T | F | T | T |
| F | F | T | F | F |
| F | F | F | T | T |

$\neg j \rightarrow (l \vee \neg r) \equiv (r \wedge \neg l) \rightarrow j$

1c) If Sally got the job then she was not late for her interview.
(j $\rightarrow \neg l)$

If Sally did not get the job, then she was late for her interview.
$(\neg j \rightarrow l)$

| j | l | (j $\rightarrow \neg l)$ | $(\neg j \rightarrow l)$ |
|---|---|---|---|
| T | T | F | T |
| T | F | T | T |
| F | T | T | T |
| F | F | T | F |

(j $\rightarrow \neg l) \not\equiv (\neg j \rightarrow l)$

1c) If Sally updated her resume or she was not late for her interview, then she got the job.
(r $\vee \neg l) \rightarrow j$

If Sally got the job, then she updated her resume and was not late for her interview.
j $\rightarrow (r \wedge \neg l)$

| j | l | r | (r $\vee \neg l) \to j$ | j $\to (r \wedge \neg l)$ |
|---|---|---|---|---|
| T | T | T | T | F |
| T | T | F | F | T |
| T | F | T | T | T |
| T | F | F | F | T |
| F | T | T | T | F |
| F | T | F | F | T |
| F | F | T | T | F |
| F | F | F | T | T |

(r $\vee \neg l) \not\equiv$ j $\to (r \wedge \neg l)$

## Question 8

1c) (p $\rightarrow q) \land (p \rightarrow r) \equiv p \rightarrow (q \land r)$

| | |
|---|---|
| $(\neg p \lor q) \land (\neg p \lor r)$ | Conditional Identity |
| $(\neg p \lor (q \land r))$ | Distributive Law |
| p $\rightarrow (q \land r)$ | Conditional Identity |

1f) $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg r$

| | |
|---|---|
| $\neg p \land \neg(\neg p \land q)$ | De Morgan's Law |
| $\neg p \land (\neg\neg p \lor \neg q)$ | De Morgan's Law |
| $\neg p \land (p \lor \neg q)$ | Double Negation |
| $(\neg p \land p) \lor (\neg p \land \neg q)$ | Distributive Law |
| False $\lor(\neg p \land \neg q)$ | Complement Law |
| $(\neg p \land \neg q) \lor F$ | Commutative Law |
| $(\neg p \land \neg q)$ | Identity Law |

1i) (p $\land q) \rightarrow r \equiv (p \land \neg r) \rightarrow \neg q$

| | |
|---|---|
| $\neg(p \land q) \lor r$ | Conditional Identity |
| $(\neg p \land \neg q) \lor r$ | De Morgan's Law |
| r $\lor(\neg p \lor \neg q)$ | Commutative Law |
| (r $\lor\neg p) \lor \neg q$ | Associative Law |
| $\neg\neg(r \lor \neg p) \lor \neg q$ | Double Negation Law |
| $\neg(r \lor \neg p) \rightarrow \neg q$ | Conditional Identity |
| $\neg(\neg p \lor r) \rightarrow \neg q$ | Commutative Law |
| $(\neg\neg p \land \neg r) \rightarrow \neg q$ | De Morgan's Law |
| (p $\land\neg r) \rightarrow \neg q$ | Double Negation Law |

2c) $\neg r \lor (\neg r \rightarrow p)$

| | |
|---|---|
| $\neg r \lor (\neg\neg r \rightarrow p)$ | Conditional Identity |
| $\neg r \lor (r \rightarrow p)$ | Double Negation Law |
| $((\neg r \lor r) \lor p)$ | Associative Law |
| ((r $\lor\neg r) \lor p)$ | Commutative Law |
| T $\lor p$ | Complement Law |
| p $\lor T$ | Commutative Law |
| T | Domination Law |

2d) $\neg(p \rightarrow q) \rightarrow \neg q$

| | |
|---|---|
| $\neg(\neg p \vee q) \to \neg q$ | Conditional Identity |
| $\neg\neg(\neg p \vee q) \vee \neg q$ | Conditional Identity |
| $(\neg p \vee q) \vee \neg q$ | Double Negation Law |
| $\neg p \vee (q \vee \neg q)$ | Associative Law |
| $\neg p \vee T$ | Complement Law |
| T | Domination Law |

## Question 9

1c) There is a number that is equal to its square.

$\exists x(x = x^2)$

1d) Every number is less than or equal to its square.

$\forall x(x \leq x^2)$

- S(x): x was sick yesterday
- W(x): x went to work yesterday
- V(x): x was on vacation yesterday

2b) Everyone was well and went to work yesterday.

$\forall x(\neg S(x) \wedge W(x))$

2c) Everyone who was sick yesterday did not go to work.

$\forall x(S(x) \rightarrow \neg W(x))$

2d) Yesterday someone was sick and went to work.

$\exists x(S(x) \wedge W(x))$

# 1 Question 10

1c) $\exists x((x = c) \to P(x))$

False.

1d) $\exists x(Q(x) \wedge R(x))$

TRUE.

1e) Q(a) $\wedge P(d)$

TRUE.

1f) $\forall x((x \neq b) \to Q(x))$

TRUE.

1g) $\forall x(P(x) \vee R(x))$

False. Counter-example: c.

1h) $\forall x(R(x) \to P(x))$

TRUE.

1i) $\exists x(Q(x) \vee R(x))$

TRUE.

2b) $\exists x \forall y Q(x, y)$

TRUE.

2c) $\exists x \forall y P(y, x)$

TRUE.

2d) $\exists x \exists y S(x, y)$

False.

2e) $\forall x \exists y Q(x, y)$

False.

2f) $\forall x \exists y P(x, y)$

TRUE.

2g) $\forall x \forall y P(x, y)$

False.

2h) $\exists x \exists y Q(x, y)$

TRUE.

2i) $\forall x \forall y \neg S(x, y)$

TRUE.

## Question 11

1c) There are two numbers whose sum is equal to their product.

$\exists x \exists y(x + y = xy)$

1d) The ratio of every two positive numbers is also positive.

$\forall x \forall y(((x > 0) \wedge (y > 0)) \to (x/y > 0)$

1e) The reciprocal of every positive number less than one is greater than one.

$\forall x(((x > 0) \wedge (x < 1)) \to (1/x > 1))$

1f) There is no smallest number.

$\neg \exists x \forall y(x < y)$

1g) Every number besides 0 has a multiplicative inverse.

$\forall x \exists y((x \neq 0) \to (xy = 1))$

- P(x, y): x knows y's phone number. (A person may or may not know their own phone number.)
- D(x): x missed the deadline.
- N(x): x is a new employee.

2c) There is at least one new employee who missed the deadline.

$\exists x(N(x) \wedge D(x))$

2d) Sam knows the phone number of everyone who missed the deadline.

$\forall y(D(y) \to P(Sam, y))$

2e) There is a new employee who knows everyone's phone number.

$\exists x \forall y(N(x) \wedge P(x, y))$

2f) Exactly one new employee missed the deadline.

$\exists x \forall y((N(x) \wedge D(x)) \wedge (((x \neq y \wedge N(y)) \to \neg D(y)))$

3c) Every student has taken at least one class besides Math 101.

$\forall x \exists y (T(x, y) \land (y \neq Math101))$

3d) There is a student who has taken every math class besides Math 101.

$\exists x \forall y ((y \neq Math101) \rightarrow T(x, y)$

3e) Everyone besides Sam has taken at least two different math classes.

$\forall x \exists y \exists z (((x \neq Sam)) \rightarrow ((y \neq z) \land T(x, y) \land T(x, z)))$

3f) Sam has taken exactly two math classes.

$\exists y \exists z \forall w (((y \neq z) \land T(Sam, y) \land T(Sam, z)) \land (((w \neq y) \land (w \neq z)) \rightarrow \neg T(Sam, w)$

## Question 12

- P(x): x was given the placebo
- D(x): x was given the medication
- M(x): x had migraines

1b) Every patient was given the medication or the placebo or both.

$$\begin{aligned} &\forall x(D(x) \vee P(x)) \\ \text{Negation:}\quad & \neg\forall x(D(x) \vee P(x)) \\ \text{Applying De Morgan's Law:}\quad & \exists x(\neg D(x) \wedge P(x)) \\ \text{English:}\quad & \text{Some patient was not given medication and not given placebo} \end{aligned}$$

1c) There is a patient who took the medication and had migraines.

$$\begin{aligned} &\exists x(D(x) \wedge M(x)) \\ \text{Negation:}\quad & \neg\exists x(D(x) \wedge M(x)) \\ \text{Applying De Morgan's Law:}\quad & \forall x(\neg D(x) \vee \neg M(x)) \\ \text{English:}\quad & \text{Every patient was either not given medication or not had migraines(or both).} \end{aligned}$$

1d) Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, p $\rightarrow q \equiv \neg p \vee q$.)

$$\begin{aligned} &\forall x(P(x) \rightarrow M(x)) \\ \text{Negation:}\quad & \neg\forall x(P(x) \rightarrow M(x)) \\ \text{Applying De Morgan's Law:}\quad & \exists x(\neg P(x) \wedge \neg M(x)) \\ \text{English:}\quad & \text{Some patient who took the placebo and did not have migraines.} \end{aligned}$$

1e) There is a patient who had migraines and was given the placebo.

$$\begin{aligned} &\exists x(M(x) \wedge P(x)) \\ \text{Negation:}\quad & \neg\exists x(M(x) \wedge P(x)) \\ \text{Applying De Morgan's Law:}\quad & \forall x(\neg M(x) \vee \neg P(x)) \\ \text{English:}\quad & \text{Every patient either not had migraines or was not given the placebo (or both).} \end{aligned}$$

2c) $\exists x \forall y(P(x,y) \rightarrow Q(x,y))$

$\forall x \exists y(P(x,y) \wedge \neg Q(x,y))$

2d) $\exists x \forall y(P(x,y) \leftrightarrow P(y,x))$

$\forall x \exists y((P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y)))$

2e) $\exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y)$

$\forall x \forall y \neg P(x,y) \vee \exists x \exists y \neg Q(x,y)$